Target Tracking in Glint Noise Environment Using Nonlinear Non-Gaussian Kalman Filter

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Abstract— **The problem of nonlinear non-Gaussian target tracking with glint measurement noise is addressed in this work. The heavy-tailed glint noise distribution is modeled by mixture of two Gaussians. A new nonlinear Gaussian mixture Kalman filter (NL-GMKF), is applied to this problem. The tracking performance of the NL-GMKF is evaluated and compared to the particle filter (PF) and the extended Kalman filter (EKF) via simulations. It is shown that the NL-GMKF outperforms both the PF and the EKF.**

I. Motivation

The problem of target tracking has been intensively investigated [1]-[2] for application in military surveillance systems, sonar and air-traffic control systems. Most of the tracking algorithms use dynamic state-space (DSS) modeling approach [3]. The unobserved system state in the DSS model, which usually characterizes the time-varying target dynamics, can be effectively estimated according to the Bayesian approach via posterior probability density function (PDF) [4]. The Kalman filter (KF) is widely used in tracking problems. It optimally estimates in the minimum-mean-squared error (MMSE) sense, the target dynamics from noisy measurements in linear, Gaussian systems [1]-[2]. However, the Gaussianity and linearity assumptions are restrictive and extension of the KF to non-Gaussian nonlinear models has been intensively studied [1]-[2], [5] and [6].

Practical radar tracking systems are rarely Gaussian due to many factors. One of them is changes in the aspect toward the radar, which can cause irregular electromagnetic wave reflection, resulting in significant variations of radar reflections [7]. This phenomenon gives rise to outliers in angle tracking, and it is referred to as target glint. The concept of angular glint was initially proposed in [8], and was explained as the tilt of wave-front normal, resulting from the distortion of target echo signal phase front. It was found that glint has a long-tailed PDF [7].

The statistical characteristics of the glint noise and its mathematical models have been studied in [7], [9] and [10]. The glint has been modeled by a Student's t distribution in [10], based on theoretical studies. A mixture model of two zero-mean Gaussians for glint noise has been proposed in [7], based on the statistical analysis. This model consists of one Gaussian with high probability and small variance and another with small probability of occurrence and very high variance. In [9], the glint noise was alternatively modeled by mixture of zero-mean, small-variance Gaussian with heavy-tailed Laplacian. The last two models are commonly used for glint noise modeling [5].

Tracking precision of the KF can be greatly degraded due to glint noise [11] with non-Gaussian PDF, tracking complex and large targets at short ranges. Many researches addressed the problem of filtering in non-Gaussian models, however only few techniques have been found to be effective. One of them is Masreliez's algorithm [11]. This algorithm is based on the score function and was used in [12] in the target tracking problem with glint noise. The main disadvantage of this approach is that it involves a complicated score function calculation.

Recently, two new filtering approaches were proposed. In 90's, a new class of filtering methods was proposed based on the sequential Monte Carlo (MC) approach for nonlinear non-Gaussian problems, as an alternative to linearized Kalman-type filters (see for example [4]). In these techniques the filtering is performed recursively generating MC samples of the state variables. The most popular realization of the MC approach is the particle filters (PF), which approximate the posterior distribution by a set of random samples with associated weights, rather than using an analytic model [4]. In [13], the PF was applied to the problem of tracking in glint noise environment.

In [14], the idea of interacting multiple model (IMM) was utilized in the problem of target tracking with glint noise and the IMM algorithm was implemented with two extended Kalman filters (EKF). The IMM approach with EKF in the tracking problem with glint provides poor performance due to the linearization, involved in the EKF. The IMM approach was further studied in many later works [5] and was extended to the problem of maneuvering target tracking in the presence of glint [12], where Masreliez's filter was used instead of EKF. The IMM approach with Masreliez's filter involves complicated score function calculation and therefore has limited practical implementation.

Recently, the Gaussian mixture Kalman filter (GMKF) was proposed in [15]. This algorithm was shown to be optimal under the minimum-mean-square error (MMSE) criterion for linear non-Gaussian problems. In this DSS model, the PDFs of the system initial state, system noise, and the posterior state are assumed to be described by the Gaussian mixture model (GMM).

In the present work, the GMKF, presented in [15] was generalized to nonlinear non-Gaussian problems and was applied to the problem of radar target tracking with glint noise, modeled by heavy-tailed angle measurement noise. Using the property that any PDF can be approximated by a mixture of finite number of Gaussians [2], the distributions of the system initial state, measurement noise and the posterior state, were modeled by GMMs. In this work, the glint noise was modeled by mixture of two zero-mean Gaussians. The properties of the GMM are used in the proposed algorithm, thus, the expectation-maximization (EM) algorithm was used for GMM order reduction. The proposed algorithm does not involve any complicated calculations. This algorithm does not involve linearization and therefore is expected to outperform any EKF-based algorithms. The tracking performance of the proposed NL-GMKF is compared to the PF and to the EKF via simulations.

The paper is organized as follows. The problem of target tracking with glint noise is stated in Section II. The NL-GMKF, used in this work is described in Section III. Tracking performances of the proposed algorithm are presented in Section IV. Our conclusions are summarized in Section V.

II. Problem formulation

Consider a state sequence $\{\mathbf{s}[n],\ n = 0,1,2,\ldots\}$ and observations $\{\mathbf{x}[n],\ n = 0,1,2,\ldots\}$ whose time evolution and observation equations are described by the following nonlinear non-Gaussian DSS model

$$\begin{aligned} \mathbf{s}[n] &= \mathbf{a}(\mathbf{s}[n-1], \mathbf{u}[n]) \ , \quad (1) \\ \mathbf{x}[n] &= \mathbf{h}(\mathbf{s}[n], \mathbf{w}[n]) \ , \quad (2) \end{aligned}$$

where the nonlinear transition function, $\mathbf{a}(\cdot,\cdot)$, and the observation function, $\mathbf{h}(\cdot,\cdot)$, are known. The initial state $\mathbf{s}[-1]$, the zero-mean measurement noise $\mathbf{w}[n]$ and the zero-mean driving noise $\mathbf{u}[n]$ are independent with the following distributions

$$\begin{aligned} \mathbf{s}[-1] &\sim GMM(\alpha_{\mathbf{s}l}[-1], \boldsymbol{\mu}_{\mathbf{s}l}[-1], \boldsymbol{\Gamma}_{\mathbf{s}l}[-1]; l = 1,\ldots,L) \ , \\ \mathbf{w}[n] &\sim GMM(\alpha_{\mathbf{w}k}[n], \boldsymbol{\mu}_{\mathbf{w}k}[n], \boldsymbol{\Gamma}_{\mathbf{w}k}[n]; k = 1,\ldots,K) \ , \\ \mathbf{u}[n] &\sim \mathcal{N}(\boldsymbol{\mu}_{\mathbf{u}}[n], \boldsymbol{\Gamma}_{\mathbf{u}}[n]) \ , \end{aligned}$$

where $GMM(\alpha_m, \boldsymbol{\mu}_m, \boldsymbol{\Gamma}_m, m = 1,\ldots,M)$ denotes an M-order Gaussian mixture distribution with weights, $\{\alpha_m\}_{m=1}^M$, mean vectors, $\{\boldsymbol{\mu}_m\}_{m=1}^M$, and covariance matrices, $\{\boldsymbol{\Gamma}_m\}_{m=1}^M$. Distribution of the measurement noise $\mathbf{w}[n]$ is $f_{\mathbf{w}}(\mathbf{w})$. The driving noise and the measurement noise are temporally independent, i.e. $\mathbf{u}[n]$ and $\mathbf{u}[n']$, and $\mathbf{w}[n]$ and $\mathbf{w}[n']$ are mutually independent for any time instances $n = 0,1,2,\ldots;\ n' = 0,1,2,\ldots;\ n \neq n'$. Addressing the problem of non-maneuvering target tracking, the Gaussian distribution was assumed for the driving noise.

The PDF of a GMM-distributed random vector $\mathbf{y} \sim GMM(\alpha_{\mathbf{y}m}, \boldsymbol{\mu}_{\mathbf{y}m}, \boldsymbol{\Gamma}_{\mathbf{y}m}; m = 1,\ldots,M)$ is given by

$$f_{\mathbf{y}}(\mathbf{y}) = \sum_{m=1}^{M} \alpha_{\mathbf{y}m} \Phi(\mathbf{y}; \boldsymbol{\theta}_m) \ , \quad (3)$$

where $\Phi(\mathbf{y}; \boldsymbol{\theta}_m)$ is a complex Gaussian PDF and $\boldsymbol{\theta}_m$ contains the mean vector, $\boldsymbol{\mu}_{\mathbf{y}m}$ and the covariance matrix, $\boldsymbol{\Gamma}_{\mathbf{y}m}$.

The non-maneuvering target tracking problem with glint noise can be modeled by nearly-constant target motion model [3] where the transition function in (1) is

$$\begin{aligned} \mathbf{a}(\mathbf{s}[n-1], \mathbf{u}[n]) &= \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{s}[n-1] \\ &+ \begin{bmatrix} \frac{T^2}{2} & 0 \\ T & 0 \\ 0 & \frac{T^2}{2} \\ 0 & T \end{bmatrix} \mathbf{u}[n] \ , \quad (4) \end{aligned}$$

in which T is the sampling interval and $\{\mathbf{s}[n],\ n = 0,1,2,\ldots\}$ is a state sequence which consists of two-dimensional target position $[r_x[n]\ \ r_y[n]]^T$ and velocity $[\dot{r}_x[n]\ \dot{r}_y[n]]^T$: $\mathbf{s}[n] = [r_x[n]\ \dot{r}_x[n]\ r_y[n]\ \dot{r}_y[n]\ ]^T$.

We assume that the radar is placed at the origin $[0\ 0]^T$ and therefore, the radar measurements: range $r[n] = [r_x[n]\ r_y[n]]^T$ and bearing $\beta[n]$ of the target are described by the measurement function

$$\begin{aligned} \mathbf{x}[n] &= h(\mathbf{s}[n], \mathbf{w}[n]) = \begin{bmatrix} r[n] \\ \beta[n] \end{bmatrix} + \mathbf{w}[n] \\ &= \begin{bmatrix} (r_x^2[n] + r_y^2[n])^{\frac{1}{2}} \\ \arctan\left(\frac{r_y[n]}{r_x[n]}\right) \end{bmatrix} + \mathbf{w}[n] \ , \end{aligned}$$

where $\mathbf{w}[n]$ is the glint noise.

As mentioned in [7], the glint noise has a non-Gaussian distribution. A mixture approach is widely used in modeling the non-Gaussian glint noise. It was obtained that the glint is Gaussian-like around the mean (at origin) and has a non-Gaussian, long-tailed nature in the tail region. The data at the tail region represent outliers, caused by the glint spikes. The outliers with low occurrence probability have a significant influence on the conventional target tracking algorithms, such as the KF. In [9], it was proposed that the heavy-tailed behavior of the glint noise is the best modeled as the mixture of a zero-mean Gaussian noise with high occurrence probability and a Laplacian noise with low occurrence probability. In [7], [12] and [13], the glint noise was modeled as a mixture of two zero-mean Gaussians, where the outliers were represented by a zero-mean Gaussian with large (comparing to the thermal noise) covariance matrix:

$$f_{\mathbf{w}[n]}(\mathbf{w}) = \alpha_{\mathbf{w}} \Phi(\mathbf{w}, \boldsymbol{\theta}_{\mathbf{w}1}) + (1-\alpha_{\mathbf{w}}) \Phi(\mathbf{w}, \boldsymbol{\theta}_{\mathbf{w}2}) \ , \qquad n = 0,1,2,\ldots \ ,$$

where $\alpha_{\mathbf{w}}$ is the glint probability. In the proposed tracking algorithm, the non-Gaussian distributions are modeled by GMM. Therefore, GMM with two Gaussians was chosen to model the glint noise.

III. Nonlinear Gaussian mixture Kalman filter

Implementation of the NL-GMKF involves the following recursion.

1. Initialization:
Initialize the L-order GMM parameters of the state vector at time instance $n = -1$.

$$\begin{aligned}\alpha_{\mathbf{s}}[-1|-1, \eta_{\mathbf{s}l}[-1]] &= \alpha_{\mathbf{s}l}[-1] \, , \\ \boldsymbol{\mu}_{\mathbf{s}}[-1|-1, \eta_{\mathbf{s}l}[-1]] &= \boldsymbol{\mu}_{\mathbf{s}l}[-1] \, , \\ \boldsymbol{\Gamma}_{\mathbf{s}}[-1|-1, \eta_{\mathbf{s}l}[-1]] &= \boldsymbol{\Gamma}_{\mathbf{s}l}[-1] \, .\end{aligned}$$

Set $n = 0$.

2. Mixture parameters of nonlinear function:

- Generate an artificial data set $\mathcal{D}$ from the conditional distribution of $\begin{bmatrix} \mathbf{s}[n-1] \\ \mathbf{u}[n] \\ \mathbf{w}[n] \end{bmatrix}$, given $\mathcal{X}[n-1]$, according to the PDF of $\mathbf{s}[n-1]|\mathcal{X}[n-1]$ from the previous step and PDFs of $\mathbf{u}[n]$ and $\mathbf{w}[n]$.
- Apply the nonlinear function $\mathbf{G}(\cdot) = \begin{bmatrix} \mathbf{a}(\cdot,\cdot) \\ \mathbf{h}(\mathbf{a}(\cdot,\cdot),\cdot) \end{bmatrix}$ on $\mathcal{D}$ and obtain a new artificial data set $\mathcal{D}' = \mathbf{G}(\mathcal{D})$.
- Model the conditional distribution of $\begin{bmatrix} \mathbf{s}[n] \\ \hat{\mathbf{x}}[n] \end{bmatrix}$ given $\mathcal{X}[n-1]$ using the new artificial data $\mathcal{D}'$ by GMM of order M, obtained using a model order selection algorithm such as the minimum description length (MDL) [16]. The following parameters are obtained in this process

$$\begin{aligned}&\boldsymbol{\theta}_{\hat{\mathbf{x}}m}[n|n-1]=\{\boldsymbol{\mu}_{\hat{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]], \boldsymbol{\Gamma}_{\hat{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]]\} \, , \\ &\boldsymbol{\theta}_{\mathbf{s}m}[n|n-1]=\{\boldsymbol{\mu}_{\mathbf{s}}[n|n-1,\tilde{\eta}_m[n]], \boldsymbol{\Gamma}_{\mathbf{s}}[n|n-1,\tilde{\eta}_m[n]]\} \, , \\ &\alpha_{\hat{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]] \, , \boldsymbol{\Gamma}_{\mathbf{s}\hat{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]].\end{aligned}$$

3. Innovation:
The measurement prediction is calculated using these parameters as follows

$$\hat{\mathbf{x}}[n|n-1] = \sum_{m=1}^{M} \alpha_{\hat{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]] \boldsymbol{\mu}_{\hat{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]] \, ,$$

and the innovation is calculated according to

$$\tilde{\mathbf{x}}[n] = \mathbf{x}[n] - \hat{\mathbf{x}}[n|n-1] \, .$$

The parameters of the innovation PDF, modeled by GMM are:

$$\boldsymbol{\theta}_{\tilde{\mathbf{x}}m}[n]=\{\boldsymbol{\mu}_{\tilde{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]], \boldsymbol{\Gamma}_{\tilde{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]]\} \, .$$

Obtain these parameters as follows:

$$\begin{aligned}\alpha_{\tilde{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]] &= \alpha_{\hat{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]] \, , \\ \boldsymbol{\mu}_{\tilde{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]] &= \boldsymbol{\mu}_{\hat{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]] - \hat{\mathbf{x}}[n|n-1] \, , \\ \boldsymbol{\Gamma}_{\tilde{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]] &= \boldsymbol{\Gamma}_{\hat{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]] \\ \boldsymbol{\Gamma}_{\mathbf{s}\tilde{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]] &= \boldsymbol{\Gamma}_{\mathbf{s}\hat{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]] \, .\end{aligned}$$

The conditional distribution of $\begin{bmatrix} \mathbf{s}[n] \\ \tilde{\mathbf{x}}[n] \end{bmatrix}$ given $\mathcal{X}[n-1]$ is obtained in this process.

4. Kalman gain:

$$\mathbf{K}_m[n] \stackrel{\triangle}{=} \boldsymbol{\Gamma}_{\mathbf{s}\tilde{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]] \boldsymbol{\Gamma}_{\tilde{\mathbf{x}}}^{-1}[n|n-1,\tilde{\eta}_m[n]].$$

5a. Estimated state mixture parameters:

$$\begin{aligned}\alpha_{\mathbf{s}}[n|n,\tilde{\eta}_m[n]] &= \frac{\alpha_{\tilde{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]]\Phi(\tilde{\mathbf{x}}[n];\boldsymbol{\theta}_{\tilde{\mathbf{x}}m}[n])}{\sum_{m'=1}^{M} \alpha_{\tilde{\mathbf{x}}}[n|n-1,\tilde{\eta}_{m'}[n]]\Phi(\tilde{\mathbf{x}}[n];\boldsymbol{\theta}_{\tilde{\mathbf{x}}m'}[n])} \, , \\ \boldsymbol{\mu}_{\mathbf{s}}[n|n,\tilde{\eta}_m[n]] &= \boldsymbol{\mu}_{\mathbf{s}}[n|n-1,\tilde{\eta}_m[n]] \\ &\quad + \mathbf{K}_m[n] \left(\tilde{\mathbf{x}}[n] - \boldsymbol{\mu}_{\tilde{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]] \right) \, , \\ \boldsymbol{\Gamma}_{\mathbf{s}}[n|n,\tilde{\eta}_m[n]] &= \boldsymbol{\Gamma}_{\mathbf{s}}[n|n-1,\tilde{\eta}_m[n]] \\ &\quad - \mathbf{K}_m[n] \boldsymbol{\Gamma}_{\tilde{\mathbf{x}}\mathbf{s}}[n|n-1,\tilde{\eta}_m[n]] \, , \\ &\quad \forall \quad m = 1, \ldots, M \, .\end{aligned}$$

5b. Estimation:

$$\hat{\mathbf{s}}[n|n] = \sum_{m=1}^{M} \alpha_{\mathbf{s}}[n|n,\tilde{\eta}_m[n]] \boldsymbol{\mu}_{\mathbf{s}}[n|n,\tilde{\eta}_m[n]] \, .$$

6. Set $n \to n+1$, go to step 2.
This NL-GMKF is schematically presented in Fig. 1.

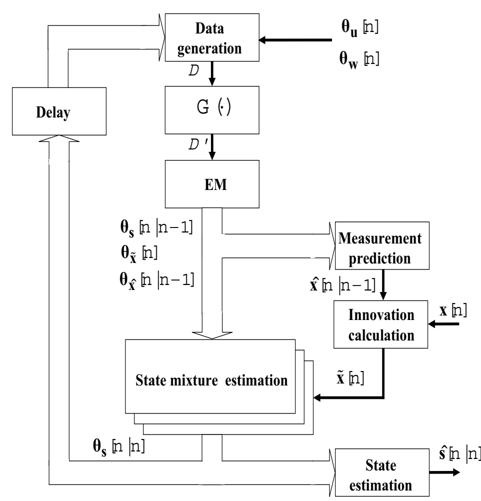

Fig. 1. The NL-GMKF schematic diagram.

IV. Simulation results

In this section, the performance of the NL-GMKF, applied to the problem of radar target tracking with glint noise, is evaluated and compared to the PF and the EKF. The PF was based on the standard sampling importance resampling (SIR) [4] with 10000 particles.

Samples of the non-maneuvering target with glint noise were simulated for $N = 100$ time instances with sampling interval $T = 1$ sec. For the glint noise PDF, the following mixture parameters were used:

$$\begin{aligned}\boldsymbol{\theta}_{\mathbf{w}1} &= \left\{ \boldsymbol{\mu}_{\mathbf{w}1} = \mathbf{0}, \boldsymbol{\Gamma}_{\mathbf{w}1} = diag\left(\left[0.25 \; km^2 \;\; 0.4 \; mrad^2 \right] \right) \right\} \, , \\ \boldsymbol{\theta}_{\mathbf{w}2} &= \left\{ \boldsymbol{\mu}_{\mathbf{w}2} = \mathbf{0}, \boldsymbol{\Gamma}_{\mathbf{w}2} = diag\left(\left[2.5 \; km^2 \;\; 400 \; mrad^2 \right] \right) \right\} \, .\end{aligned}$$

The covariance matrix of the zero-mean Gaussian driving noise was assumed to be

$$\mathbf{\Gamma_u}[n] = \quad \left([5 \times 10^{-4}\ km^2 \ \ 5\ m^2/sec^4]\right)\ . \tag{5}$$

The conditional distribution of the state vector $\mathbf{s}[n]$, given $\mathcal{X}[n]$ was assumed to be GMM of order $L = 20$. The NL-GMKF is initialized at time instance $n = -1$ for $m = 1, \ldots, M$ with

$$\begin{aligned} \alpha_{\mathbf{s}}[-1|-1, \eta_{\mathbf{s}m}[-1]] &= \frac{1}{M}\ , \\ \boldsymbol{\mu}_{\mathbf{s}}[-1|-1, \eta_{\mathbf{s}m}[-1]] &= \mathbf{0}\ , \\ \mathbf{\Gamma_s}[-1|-1, \eta_{\mathbf{s}m}[-1]] &= 10000\mathbf{I}\ , \\ \mathbf{s}[-1|-1] &= \mathbf{0}\ . \end{aligned} \tag{6}$$

The following initial target position and velocity in the two-dimensional space was chosen in simulations:

$$\mathbf{s}[-1|-1] \ = \ [10\ km\ 300\ m/sec\ 10\ km\ -100\ m/sec]^T\ .$$

Fig. 2 shows target trajectories in the two-dimensional space with glint noise. Target tracking scenarios with glint noise probability of $\alpha_{\mathbf{w}} = 0.15$, and $\alpha_{\mathbf{w}} = 0.25$ are shown in Figs. 2(a) and 2(b), respectively. Each of these tracks was randomly selected from 100 simulation trials for different glint noise probabilities. These figures show that the NL-GMKF tracks the target in the glint environment for any probability of the glint noise with small errors comparing to the PF and the EKF.

For performance evaluation the root-mean-squared error (RMSE) of the estimate of the two-dimentional position $[r_x[n]\ r_x[n]]$ and the two-dimentional velocity $[\dot{r}_x[n]\ \dot{r}_y[n]]$ were obtained in 100 independent trials. Tracking performances of the NL-GMKF, PF and EKF with glint probability of $\alpha_{\mathbf{w}} = 0.1$, are presented in Figs. 3 and 4. Fig. 3 shows the target tracking performance of the two dimentional position and Fig. 4 shows the target tracking performance of the two dimentional velocity. It can be observed that the PF outperforms the EKF, and the NL-GMKF outperforms both of them.

The tracking performance as a function of the glint probability at time instance $n = 50$ is presented in Figs. 5 and 6. Fig. 5 shows the target tracking performance of the two dimentional position and Fig. 5 shows the target tracking performance of the two dimentional velocity. It can be observed that the NL-GMKF outperforms both the PF and the EKF for different values of glint probability. These figures show that the estimation performance of the NL-GMKF remains almost constant as the probability of the glint noise increases. These figures also show that the proposed NL-GMKF ouperforms both the PF and the EKF for all glint noise probabilities.

V. Conclusion

A new NL-GMKF was applied to the problem of target tracking with non-Gaussian glint noise. The glint noise was modeled by mixture of two Gaussians. The outliers were represented by a large variance Gaussian. Tracking performance of the NL-GMKF for various glint noise probabilities were evaluated. It was shown that the NL-GMKF outperforms both PF and EKF.

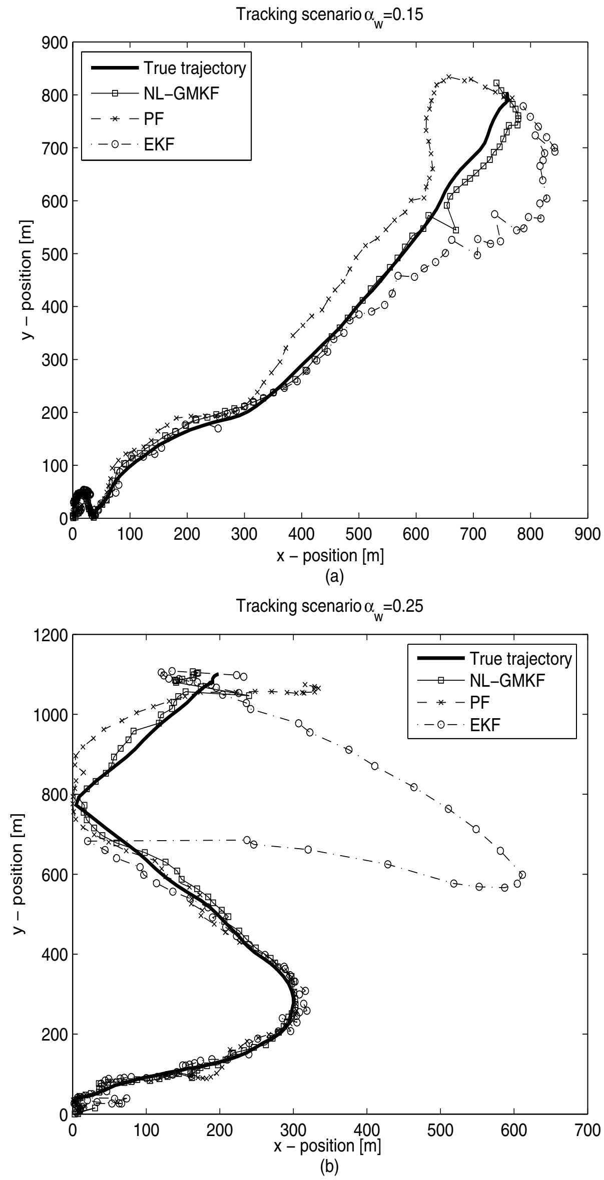

Fig. 2. Nonlinear non-Gaussian vs. PF and EKF tracking in the two-dimensional space with the glint noise probabilities of (a) $\alpha_{\mathbf{w}} = 0.15$ and (b) $\alpha_{\mathbf{w}} = 0.25$.

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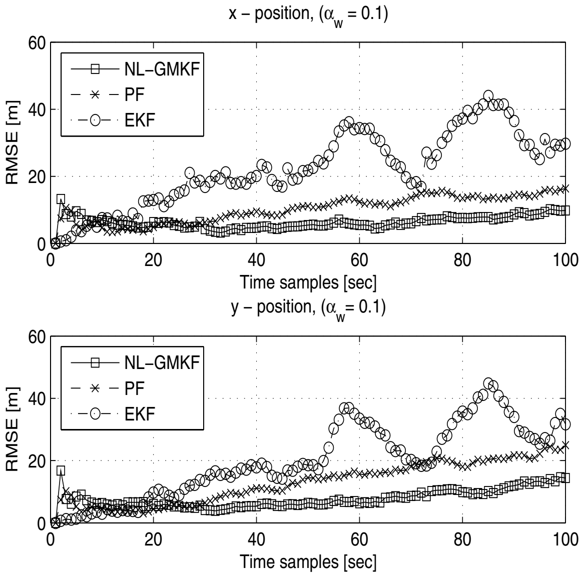

Fig. 3. Nonlinear non-Gaussian vs. PF and EKF two-dimentional position tracking performance with glint noise probability of $\alpha_{\mathbf{w}} = 0.25$.

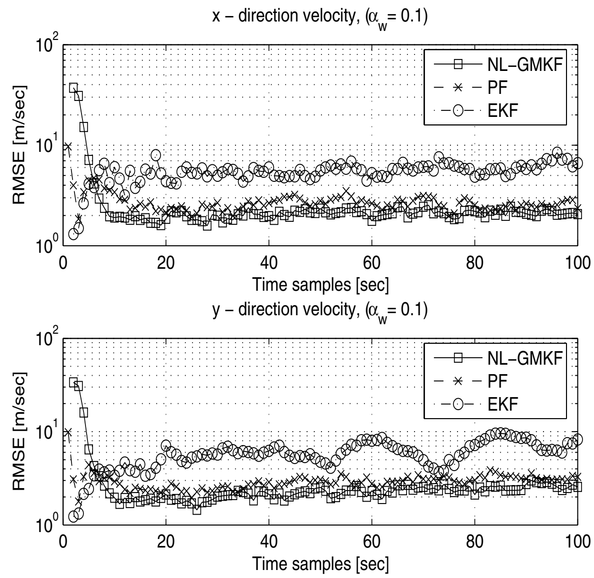

Fig. 4. Nonlinear non-Gaussian vs. PF and EKF two-dimentional velocity tracking performance with glint noise probability of $\alpha_{\mathbf{w}} = 0.25$.

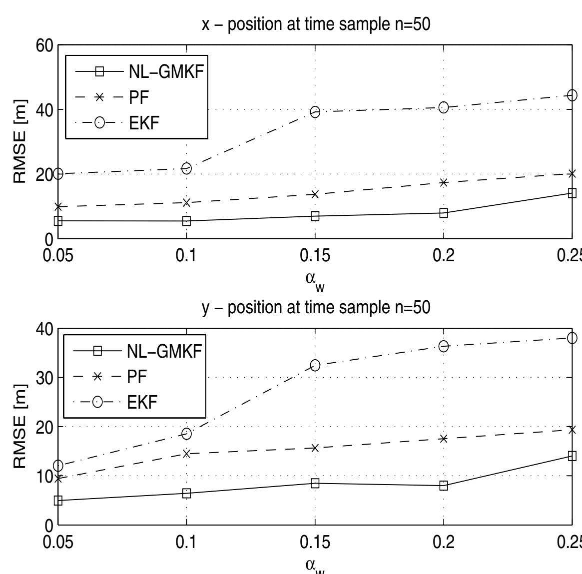

Fig. 5. Nonlinear non-Gaussian vs. PF and EKF two-dimentional position tracking performance at time instance $n = 50$, for variouse glint probabilities.

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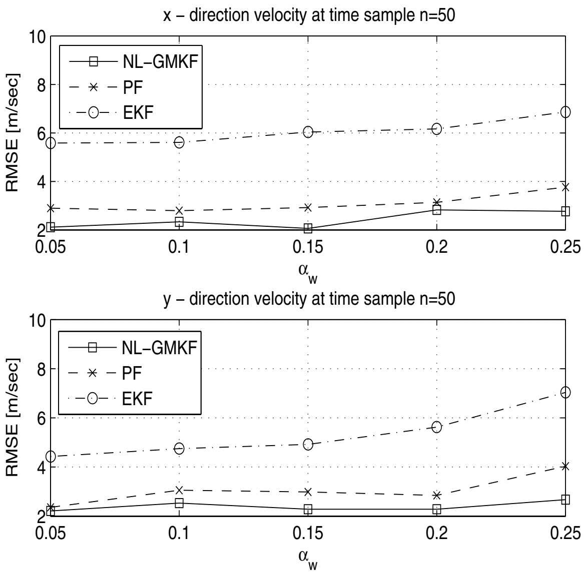

Fig. 6. Nonlinear non-Gaussian vs. PF and EKF two-dimentional velocity tracking performance at time instance $n = 50$, for variouse glint probabilities.