Microwave Resonators

1. Applications

- Oscillators
- Filters
- Sensors

2. Types

- Transmission line resonator: strip, ring, patch (circula, rectangular)
- Cavity resonator
- Dielectric resonator

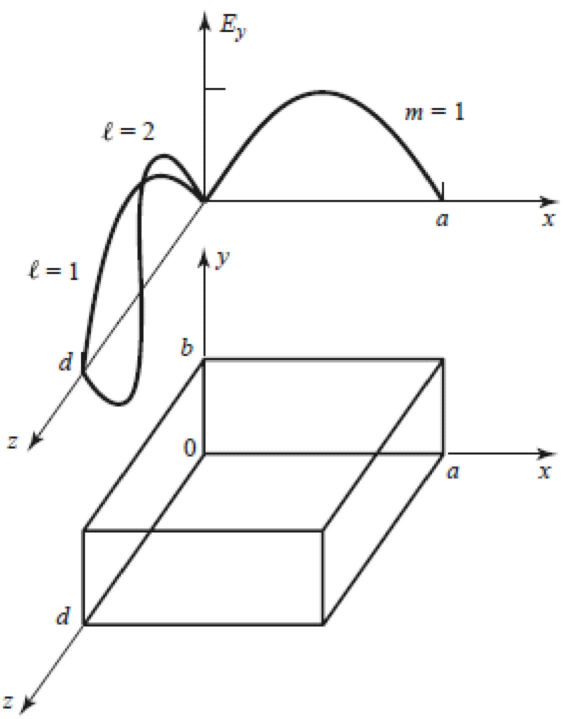
3. Resonator specifications and measurements

- Resonant frequency
- *Q* factor: unloaded, loaded
- Coupling coefficient

4. Resonator design

- Theory
- Simulation

5. Rectangular cavity resonator



Field expansion:

$$E_x = E_{x0}\cos(k_x x)\sin(k_y y)\sin(k_z z)$$
$$E_y = E_{y0}\sin(k_x x)\cos(k_y y)\sin(k_z z)$$
$$E_z = E_{z0}\sin(k_x x)\sin(k_y y)\cos(k_z z)$$

$$(\nabla^2 + k^2)\mathbf{E} = 0 \rightarrow -k_x^2 - k_y^2 - k_z^2 + k^2 = 0 \ (k^2 = \omega^2\mu\varepsilon)$$

$$\nabla\cdot\mathbf{E} = 0 \rightarrow k_x E_{x0} + k_y E_{y0} + k_z E_{z0} = 0$$

$$\mathbf{H} = -\frac{1}{j\omega\mu}\nabla\times\mathbf{E}$$

Boundary conditions:

$$k_x = \frac{m\pi}{a}, k_y = \frac{n\pi}{b}, k_z = \frac{p\pi}{d} (m, n, p = 0,1,2,...)$$

$$k_{mnp}{}^2 = \omega_{mnp}^2\mu\varepsilon = k_x^2 + k_y^2 + k_z^2 \text{ (resonant wavenumber)}$$

Lowest-order resonance:

$(m,n,p):(1,0,0)$ or $(0,1,0)$ or $(0,0,1)$ all impossible solutions

$m = 0$ or $n = 0$ or $p = 0$

Select 0 for the smallest dimension among a, b, and d.

TE_{101} mode:

$$E_y = E_0\sin\frac{\pi x}{a}\sin\frac{\pi z}{d},\ E_x = 0,\ E_z = 0$$
$$H_y = 0$$
$$H_y = \frac{-jk_z}{\omega\mu}E_0\sin\frac{\pi x}{a}\cos\frac{\pi z}{d}$$
$$H_z = \frac{jk_x}{\omega\mu}E_0\cos\frac{\pi x}{a}\sin\frac{\pi z}{d}$$

Stored energy

$$W_e = \int_V \frac{1}{4}\varepsilon' E^2 dV = \frac{1}{16}\varepsilon' E_0^2 abd$$
$$W_m = \int_V \frac{1}{4}\mu' H^2 dV = \frac{1}{16}\varepsilon' E_0^2 abd$$
$$W = W_e + W_m = \frac{1}{8}\varepsilon E_0^2 abd$$

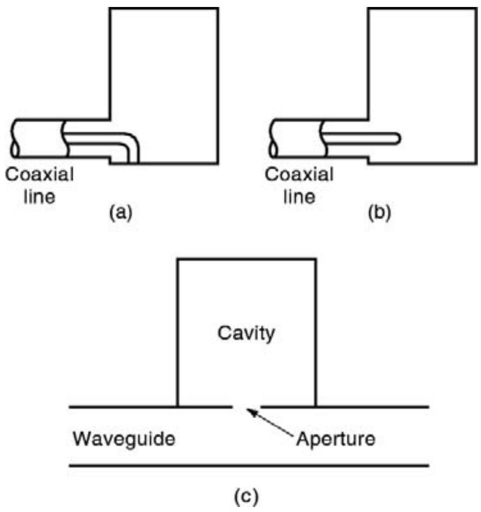
Quality factors:

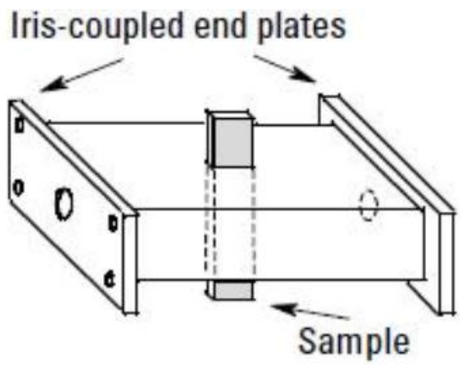
$$P_c = \frac{R_s}{2}\int_{\text{walls}}|H_t|^2\, dS = \frac{R_s E_0^2\lambda^2}{8(\mu/\varepsilon)}\left(\frac{ab}{d^2} + \frac{bd}{a^2} + \frac{a}{2d} + \frac{d}{2a}\right) \text{ (conductor loss)}$$

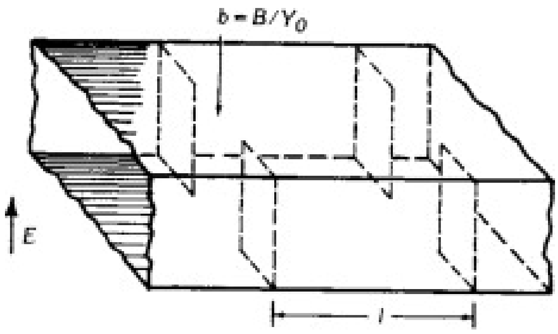
$$Q_c = \frac{2\omega W_e}{P_c}$$

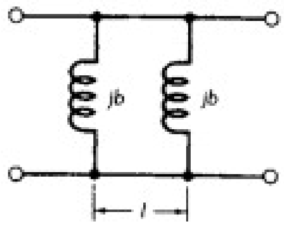
$$P_d = \frac{1}{2}\int_V \sigma E^2 dV = \frac{\omega\varepsilon''}{2}\int_V E^2 dV$$

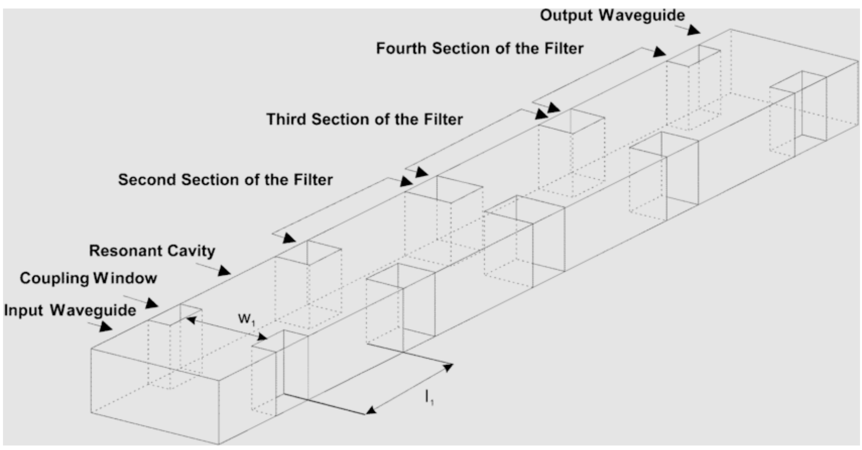
$$Q_d = \frac{2\omega W_e}{P_d} = \frac{\varepsilon'}{\varepsilon''} = \frac{1}{\tan\delta}$$











Lab

1. Design a rectangular cavity (TE_{101} mode) resonator using WR-90 waveguide resonating at 10 GHz.
2. Design a circular coupling hole between a WR-90 rectangular waveguide and the above cavity resonator.
3. Simulate S_{11}. Plot S_{11} (dB). Find the resonant frequency. Plot S_{11} on the Smith chart.
4. Modify 2 into a through structure and simulate S_{21}. Plot S_{21} (dB)