Antenna Basics 4

Elementary electric dipole - power

Components of electric and magnetic fields generated by an elementary electric dipole

$$H_\phi(r,\theta) = \frac{jkIL}{4\pi}\sin\theta\left[\frac{1}{r} + \frac{1}{jkr^2}\right]e^{-jkr}$$

$$E_r(r,\theta) = \frac{Z_0 IL}{2\pi}\cos\theta\left[\frac{1}{r^2} + \frac{1}{jkr^3}\right]e^{-jkr}$$

$$E_\theta(r,\theta) = \frac{jkZ_0 IL}{4\pi}\sin\theta\left[\frac{1}{r} + \frac{1}{jkr^2} - \frac{1}{k^2r^3}\right]e^{-jkr}$$

$$H_r = H_\theta = 0$$

$$E_\phi = 0$$

General distance out of sources

Energy density – Poynting vector W/m^2

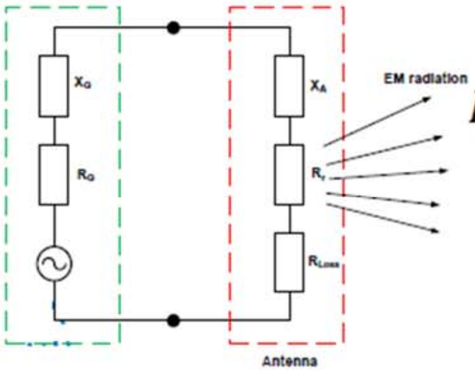
$$\boldsymbol{S} = \frac{1}{2}(\boldsymbol{E}\times\boldsymbol{H}^*) = \frac{1}{2}(\boldsymbol{r}_0 E_r + \boldsymbol{\theta}_0 E_\theta)\times(\boldsymbol{\phi}_0 H_\phi^*) = \frac{1}{2}\left(\boldsymbol{r}_0 E_\theta H_\phi^* - \underbrace{\boldsymbol{\theta}_0 E_r H_\phi^*}_{0}\right)$$

Complex power (active, reactive) flowing through closed surface S (sphere), enclosing the antenna

$$P = \oiint_S \boldsymbol{S}\cdot d\boldsymbol{S} = \frac{1}{2}\oiint_S (\boldsymbol{E}\times\boldsymbol{H}^*)\cdot d\boldsymbol{S} = P_r + j2\omega(W_m - W_e)$$

$$= 10k^2(IL)^2\left[1 - j\frac{1}{(kr)^3}\right]$$

Note that for $r\to\infty$, the sphere is at infinity S_∞ and the power is purely real



$$P_r = 10k^2(IL)^2 = \frac{1}{2}R_r|I|^2 \Rightarrow R_r = 20(kL)^2 = 80\pi^2\left(\frac{L}{\lambda}\right)^2$$

Radiation resistance of a short dipole (with constant current)

$$L = \lambda/20 \quad R_r = 80\pi^2\left(\frac{1}{20}\right)^2 \cong 2\Omega$$

Elementary electric dipole, field zones

Components of electric and magnetic fields generated by an elementary electric dipole

$$H_\phi(r,\theta) = \frac{jkIL}{4\pi}\sin\theta\left[\frac{1}{r}+\frac{1}{jkr^2}\right]e^{-jkr}$$

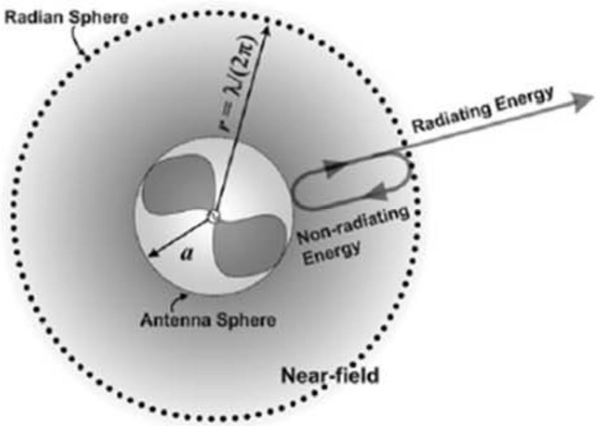
$$E_r(r,\theta) = \frac{Z_0 IL}{2\pi}\cos\theta\left[\frac{1}{r^2}+\frac{1}{jkr^3}\right]e^{-jkr}$$

$$E_\theta(r,\theta) = \frac{jkZ_0 IL}{4\pi}\sin\theta\left[\frac{1}{r}+\frac{1}{jkr^2}-\frac{1}{k^2r^3}\right]e^{-jkr}$$

$$H_r = H_\theta = 0$$

$$E_\phi = 0$$

General distance out of sources



Near (reactive) field region $kr \ll 1$ $(r \ll \lambda/2\pi)$

Fields are similar to those of a static electric dipole and to that of a static current element (quasistationary fields)

$$\boldsymbol{S}_{near} = \frac{1}{2}(\boldsymbol{E}_{near} \times \boldsymbol{H}^*_{near}) = 0$$

E_r and E_θ and **out-of phase** with H_ϕ. There is no time-average power flow, no radiated power, energy is stored in near-zone

Intermediate field region $kr > 1$ $(r > \lambda/2\pi)$

Fields are similar to those of a static electric dipole and to that of a static current element (quasistationary fields)

E_θ and H_ϕ approach time-phase → formation of time-average power flow in the outward (radial) direction).

Far field region $kr \gg 1$ $(r \gg \lambda/2\pi)$

Most important region of an antenna, E_r vanishes and only transversal (to r) field components (here E_θ and H_ϕ) remain

$$\boldsymbol{S} = \frac{1}{2}(\boldsymbol{E} \times \boldsymbol{H}^*) = real$$

Elementary electric dipole – FAR FIELD

Far field region $kr \gg 1 \; (r \gg \lambda/2\pi)$

$$H_r = H_\theta = E_\phi = E_r = 0$$

The E and H field components are perpendicular to each other, transverse to the radial direction of propagation, and the r variations are separable from those of θ and ϕ. The shape of the pattern is not a function of r, and the fields form a transverse (TEM) wave, E_θ and H_ϕ are in phase.

$$H_\phi(r,\theta) = \frac{jkIL}{4\pi}\frac{e^{-jkr}}{r}\sin\theta$$

$$E_\theta(r,\theta) = \frac{jkZ_0IL}{4\pi}\frac{e^{-jkr}}{r}\sin\theta$$

$$\frac{E_\theta}{H_\phi} = Z_0 = 120\pi$$

$$|\text{field intensity}| \sim \frac{1}{r}$$

$$\boldsymbol{E} = -j\omega\boldsymbol{A} - \cancel{\nabla\varphi} \qquad \mathbf{H} = r_0 \times \frac{\boldsymbol{E}}{Z_0}$$

Field structure of an arbitrary antenna in the far-field, i.e. fields are observed at sphere of very large radius:

$$\textbf{field intensity}(r,\theta,\phi) = C\,\frac{e^{-jkr}}{r}\,\boldsymbol{f}(\theta,\phi)$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

C: Constant involving current etc.

$\frac{e^{-jkr}}{r}$: Amplitude and phase representing point source (spherical wave)

$\boldsymbol{f}(\theta,\phi)$: Farfield pattern, function involving angular dependence of the field, $\sin\theta$ in our case of infinitely small current element. The dependence comes from transformation of unit vectors between cartesian and spherical coordinates

Elementary electric dipole – FAR FIELD

Let's concentrate on the electric field only, because we know that in farfield $H_\phi = E_\theta / Z_0$

$$E_\theta(r,\theta) = \frac{jkZ_0 IL}{4\pi}\frac{e^{-jkr}}{r}\sin\theta = C\frac{e^{-jkr}}{r}\boldsymbol{f}(\theta,\phi)$$

Farfield radiation pattern

Representation of the radiation properties of the antenna as a function of space coordinates

$\boldsymbol{f}(\theta,\phi)$ field pattern

$\boldsymbol{f}^2(\theta,\phi)$ power pattern $\sim \left|E_\theta(r,\theta)\right|^2 + \left|E_\phi(r,\theta)\right|^2$

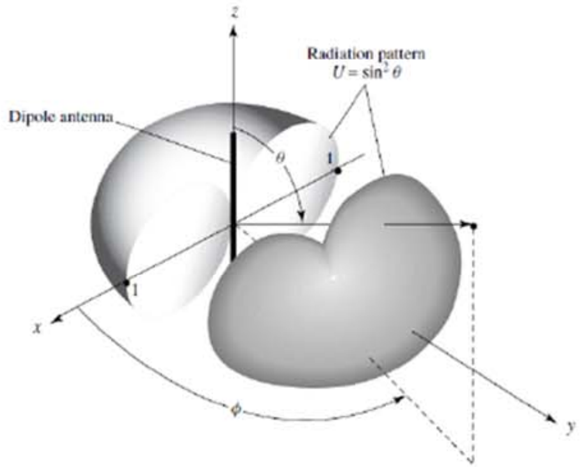
$$\boldsymbol{S} = S_r = \frac{1}{2}\Re(\boldsymbol{E}\times\boldsymbol{H}^*) = \boldsymbol{r}_0\frac{1}{2}\frac{|E_\theta(r,\theta)|^2}{Z_0} = \boldsymbol{r}_0\frac{15}{4\pi}(kIL)^2\frac{\sin^2\theta}{r^2}$$

$$P_{rad} = \oiint_S \boldsymbol{S}\cdot d\boldsymbol{S} = \int_0^{2\pi}\int_0^{\pi} S_r r^2 \sin\theta\, d\theta d\phi = 10(kIL)^2$$

Note that we are able to obtain the radiated power from the far-field only

$$dS = r^2\sin\theta\, d\theta d\phi \qquad d\Omega = \frac{dA}{r^2} = \sin\theta\, d\theta d\phi$$

steradian (space angle)



normalized field pattern

$$f_n(\theta,\phi) = |\boldsymbol{E}(\theta,\phi)|_n = \frac{|\boldsymbol{E}(\theta,\phi)|}{|\boldsymbol{E}(\theta,\phi)|_{\max}} = \sin\theta$$

normalized power pattern

$$f_n^2(\theta,\phi) = P_n(\theta,\phi) = \frac{S(\theta,\phi)}{S(\theta,\phi)_{\max}} = \sin^2\theta$$

normalized power pattern in dB

$$10\log \boldsymbol{f}_n^2(\theta,\phi) = 20\log \boldsymbol{f}_n(\theta,\phi)$$

Radiation pattern, directivity

Isotropic (point) source, power density $S = const. = S_0$

$$P_r = \oiint_S S_0 dS = 4\pi r^2 S_0 \qquad S_0 = \frac{P_r}{4\pi r^2} \qquad U_0 = r^2 S_0 = \frac{P_r}{4\pi}$$

Isotropic antenna has input power $P_{in} = 1$W. Radiation intensity is not function of direction, is constant $U_0 = \frac{1}{4\pi} W/sr$

Radiation intensity U is defined as "the power radiated from an antenna per unit space angle (steradian)" and is related to the far zone E field of an antenna: $U(\theta,\phi) = r^2 S(\theta,\phi) = \frac{r^2}{2Z_0}|\boldsymbol{E}(\theta,\phi)|^2$.

Directivity

Our source (antenna)

$$D(\theta,\phi) = \frac{U(\theta,\phi)}{U_0} = \frac{U(\theta,\phi)}{P_r/4\pi} = \frac{4\pi U(\theta,\phi)}{P_r}$$

Isotropic source

Maximum directivity

$$D_{max} = D = \frac{U_{max}}{U_0} = \frac{4\pi U_{max}}{P_r} = \frac{4\pi}{\oiint_S f_n^2(\theta,\phi)dS}$$

Directivity = radio of radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions (isotropic source)

Elementary electric dipole $f_n^2(\theta,\phi) = \sin^2\theta$

$$D_{max} = \frac{4\pi}{\oiint_S f_n^2(\theta,\phi)dS} = \frac{4\pi}{2\pi\int_0^\pi \sin^2\theta \sin\theta d\theta} = \frac{2}{4/3} = \frac{3}{2}$$

$$D_{dBi} = 10\log 3/2 = 1.76 \text{ dBi}$$

dBi ... decibels over isotropic radiatior

Effective isotropic radiated power (EIRP)

$$EIRP = D \cdot P_{in}$$

Antenna with $D = 30$ dBi and $P_{in} = 1$W

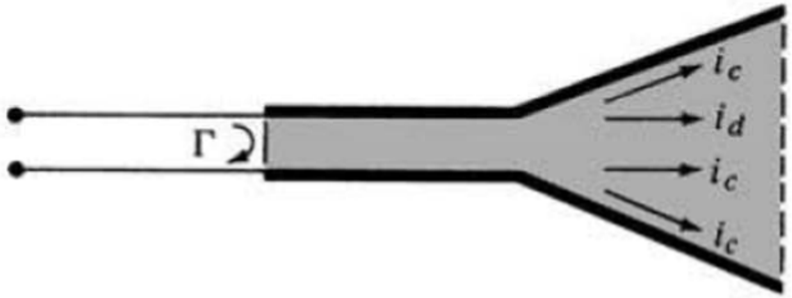
$EIRP = D \cdot P_{in} = 10^{\frac{30}{10}} \cdot 1 = 1000$ W

(equivalent to isotropic source with $P_{in} = EIRP$)

Antenna is the best amplifier!!

Antenna efficiency (gain)

Antenna gain $G(\theta, \phi) = \eta D(\theta, \phi)$



$(1 - |\Gamma|^2)$

$$\eta = \eta_R \eta_c \eta_d$$

Joule losses in metal

Joule losses in dielectrics

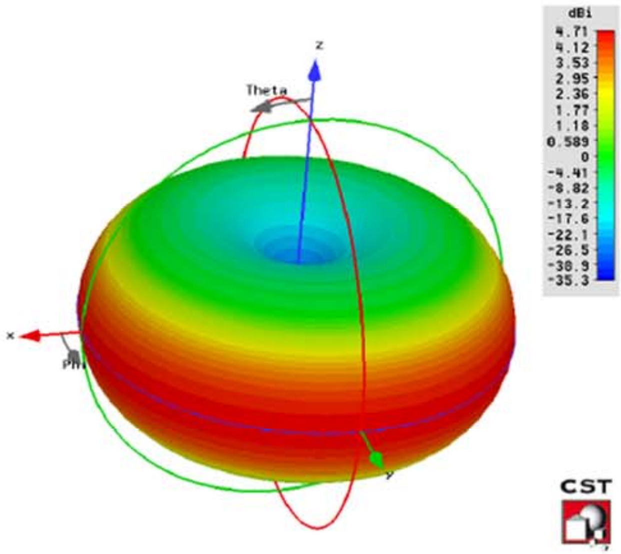
Mostly only conductive losses are considered, $\eta_c = \frac{R_r}{R_r + R_{Loss}}$

Radiation pattern

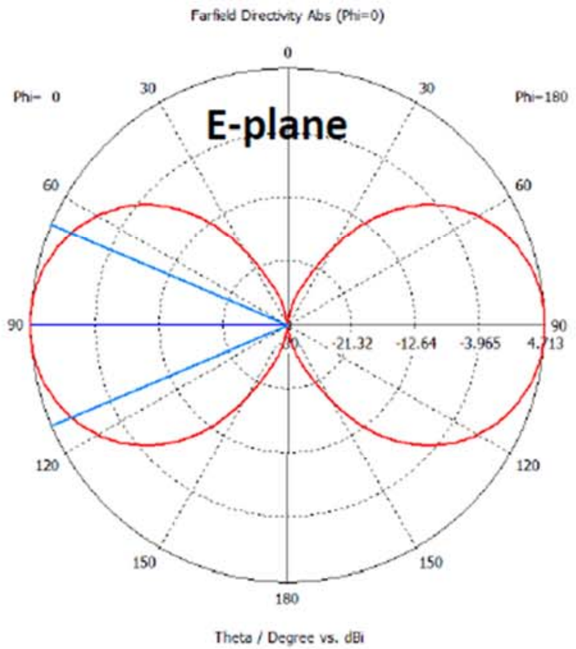
Full 3D radiation pattern

z-directed dipole → only θ component of directivity

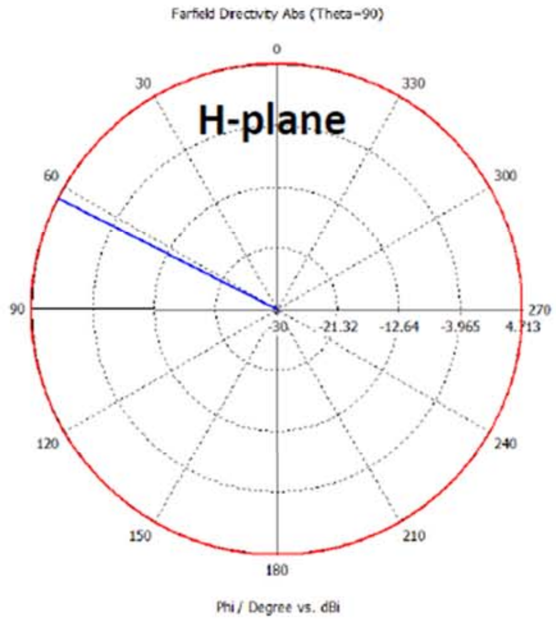
$$D_\theta(\theta, \phi)$$



$\phi = 0°, var.\ \theta$



$\theta = 90°, var.\ \phi$



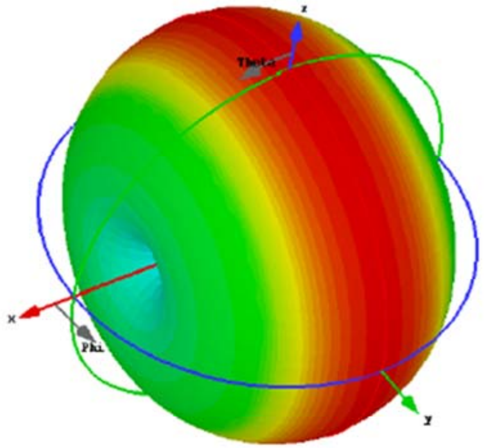
Radiation pattern

Full 3D radiation pattern

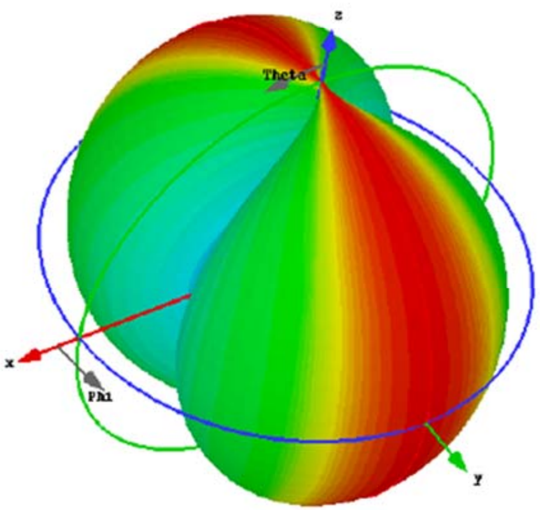
x-directed dipole has both components $D_\theta(\theta,\phi)$ and $D_\phi(\theta,\phi)$.

$$|D(\theta,\phi)| = \sqrt{[D_\theta(\theta,\phi)]^2 + [D_\phi(\theta,\phi)]^2}$$

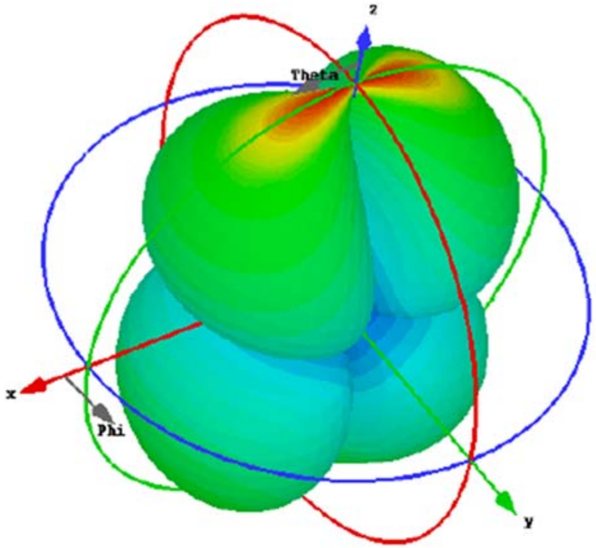
$|D(\theta,\phi)|$



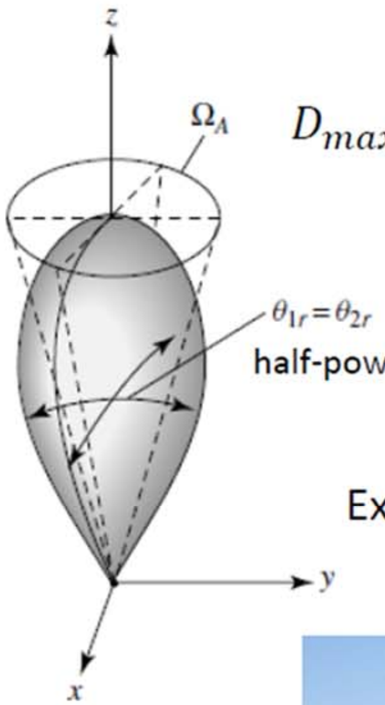
$D_\phi(\theta,\phi)$



$D_\theta(\theta,\phi)$



Radiation pattern – directional antenna

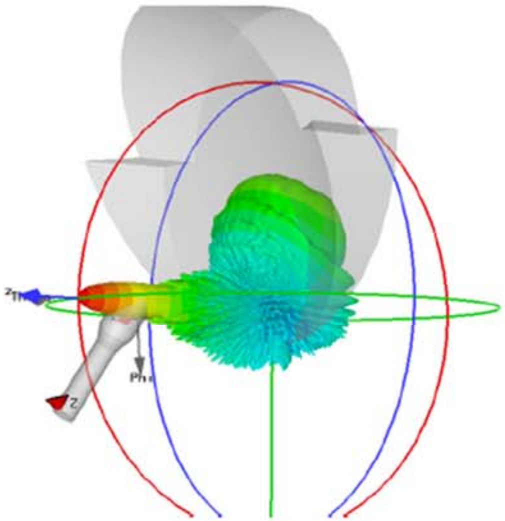


half-power beamwidths

$$D_{max} = D = \frac{4\pi U_{max}}{P_r} = \frac{4\pi}{P_r}\frac{P_r}{\Omega_A} = \frac{4\pi}{\Omega_A} \cong \frac{4\pi}{\theta_{1r}\theta_{2r}} = \frac{41253}{\theta_{1deg}\theta_{2deg}}$$

beam solid angle

Example $\theta_{1deg}\theta_{2deg} = 10 \cdot 10$ → $D_{max} = 412.5$, $D_{dBi} = 26$ dBi



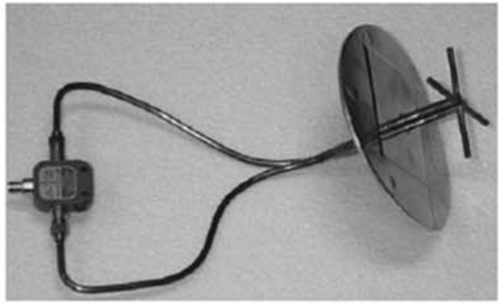
Polarization of the radiated field

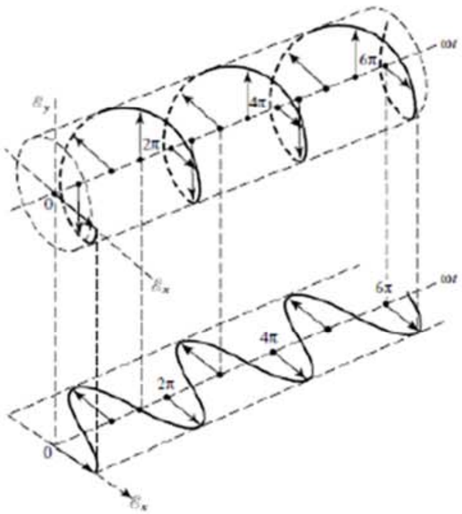
- property of an electromagnetic wave describing the time-varying direction and relative magnitude of the **E**.

Linear polarization - fields
- only one component or
- two orthogonal linear components in-phase or out-of-phase

Circular polarization (LHC/RHC) - fields
- must have two orthogonal linear components, and
- the two components must have the same magnitude, and
- the two components must have a time-phase difference of $90°$ (+ odd multiples)

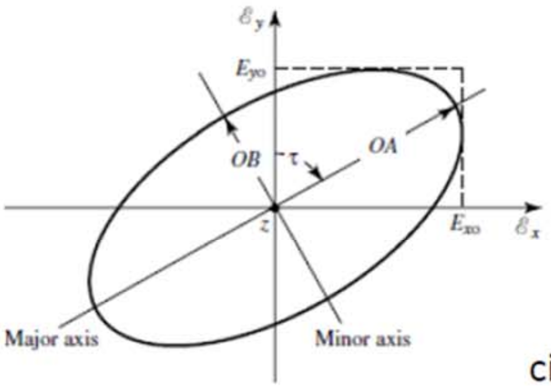
RHC: Wave travels away from observer, rotation is clockwise
LHC: Wave travels away from observer, rotation is counterclockwise





(a) Rotation of wave

General elliptical polarization



(b) Polarization ellipse

Axial ratio

$$AR = \frac{\text{major axis}}{\text{minor axis}} = \frac{OA}{OB}$$

$$1 \leq AR \leq \infty$$

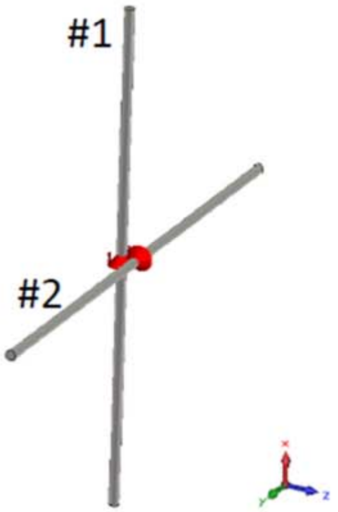
$$0 \leq AR_{dB} \leq \infty$$

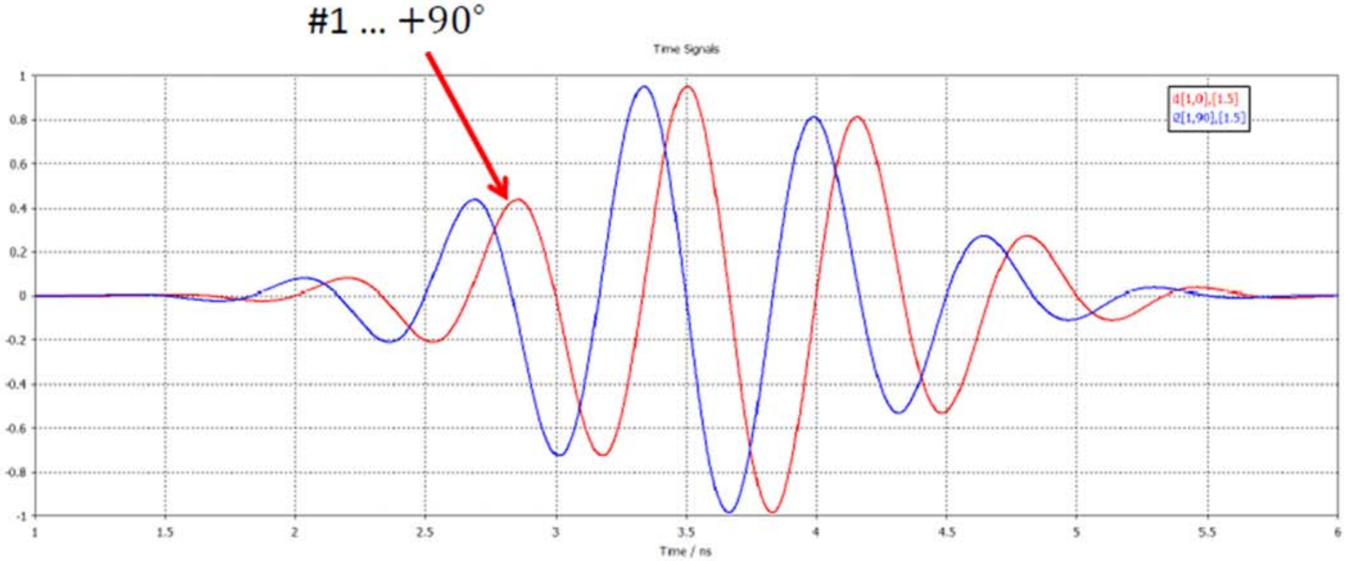
circular

elliptical

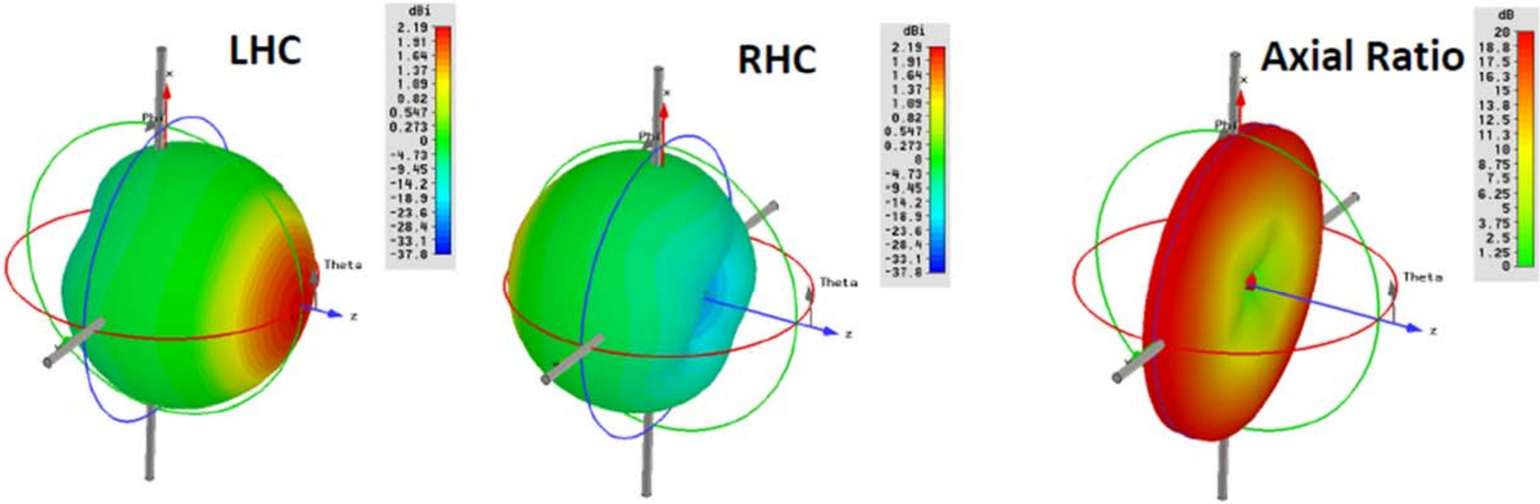
linear

Polarization of the radiated field

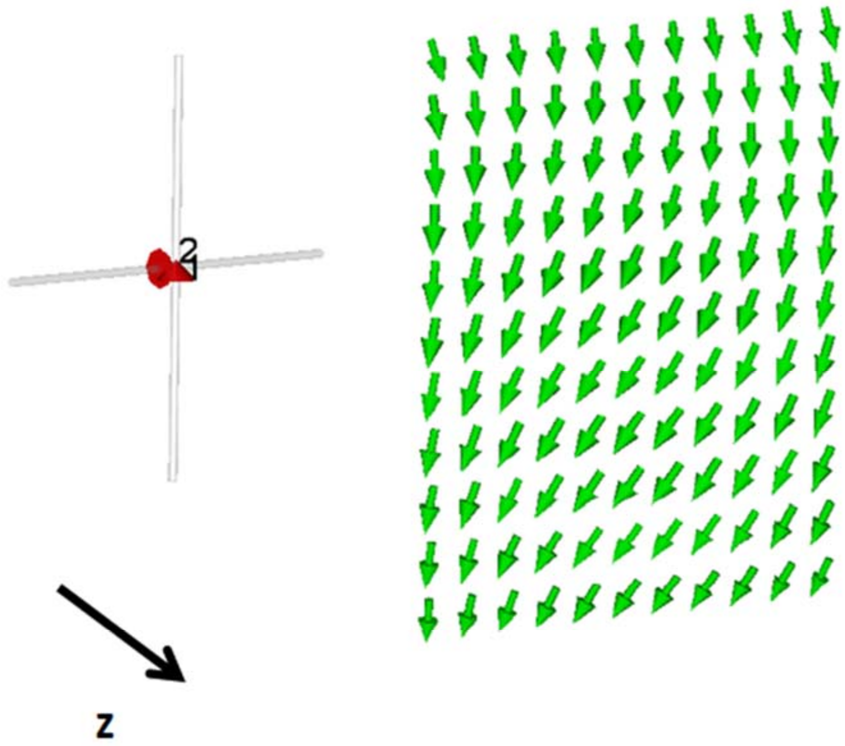




Two crossed dipoles – equal amplitudes, 90° phase shift



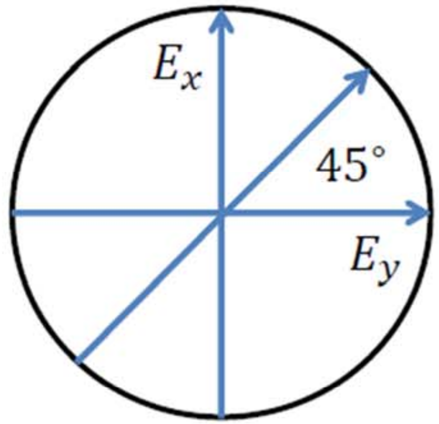
Polarization of the radiated field



LHC – CCW rotation

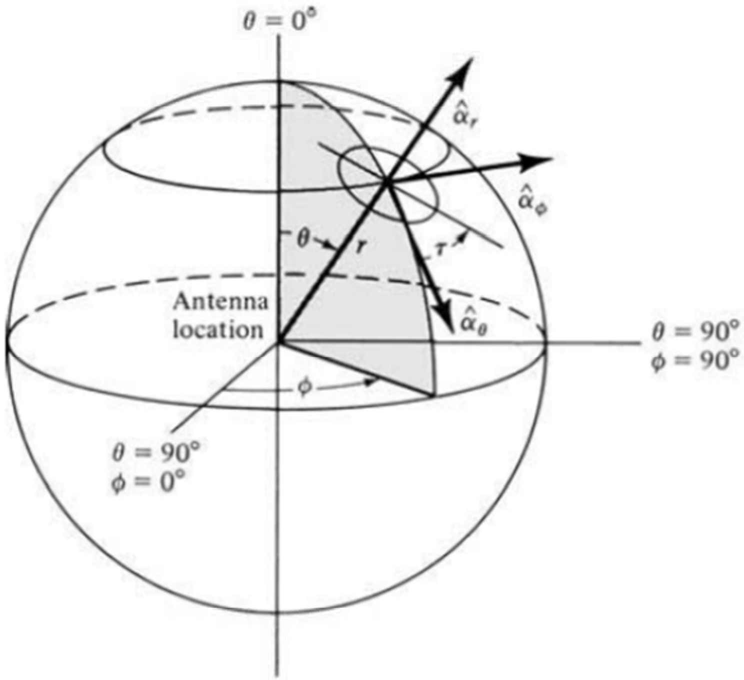
Polarization of the radiated field

Slanted linear polarization

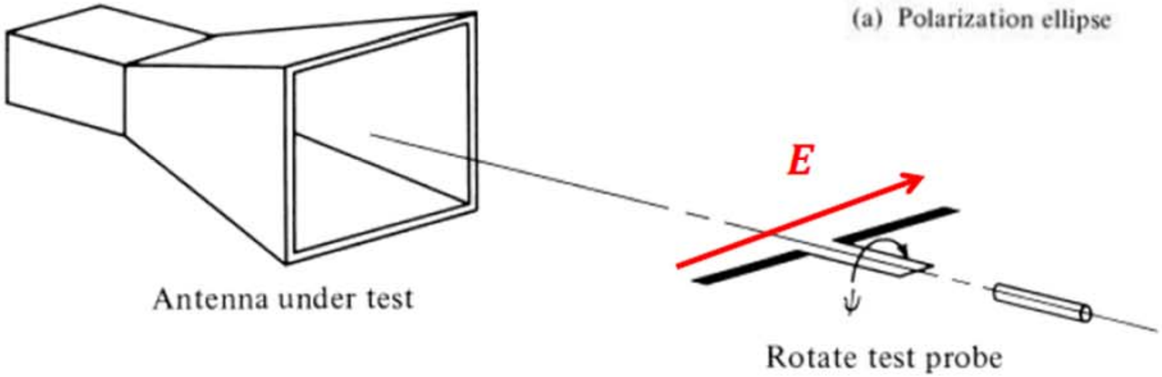


No 90° phase shift (in-phase) → linear polarization

Polarization of the radiated field

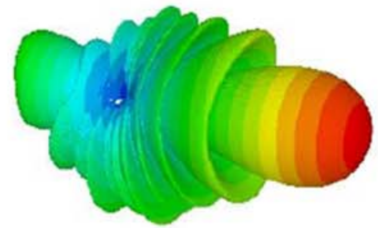


(a) Polarization ellipse

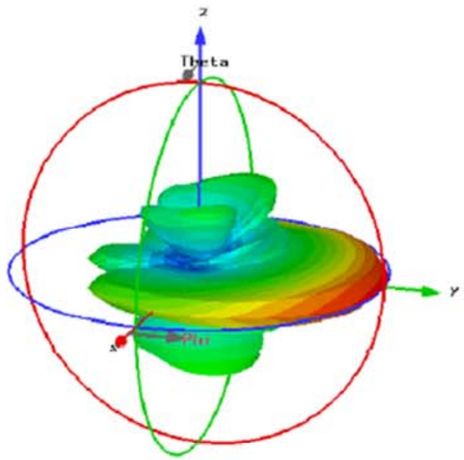


Antennas

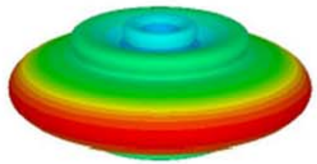
- **Directive**
- radiated power is concentrated into narrow space angle
- Radiowave P2P links, space communication antennas
- $D \sim 10 - 50$ dBi
- Arecibo radiotelescope 70 dBi (10 million linear gain)



- **Sector**
- radiated power is concentrated into given sector
- Base station (access point) antennas, satellite antennas
- $D \sim 10 - 20$ dBi



- **“Omnidirectional”**
- 360 degrees in horizontal plane, vertical plane could be narrower
- Mobile phone antennas, receiving antennas
- $D \sim 1{,}5 - 6$ dBi

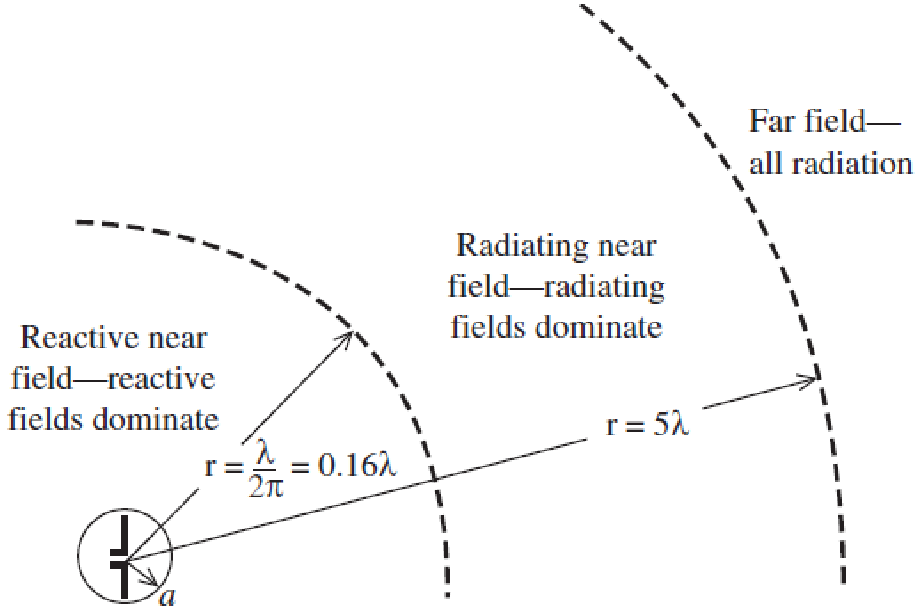


- Anenna Field Regions

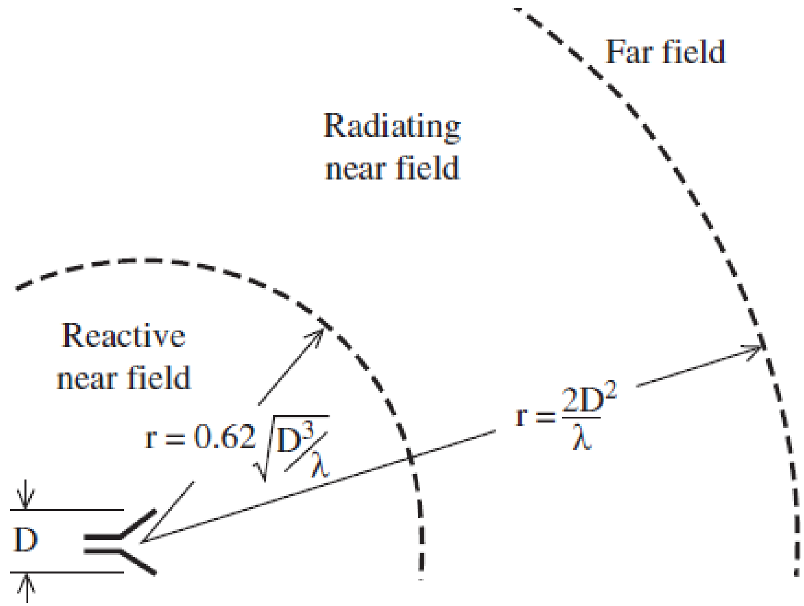
- Reactive NF region: Stored energy dominant

- Radiatinig NF region (= Fresnel region): Stored and radiated energy

- FF region (= Fraunhofer region): Radiated energy dominant.
  Radiation pattern independent of distance



(a) Small antennas ($a << \lambda$).

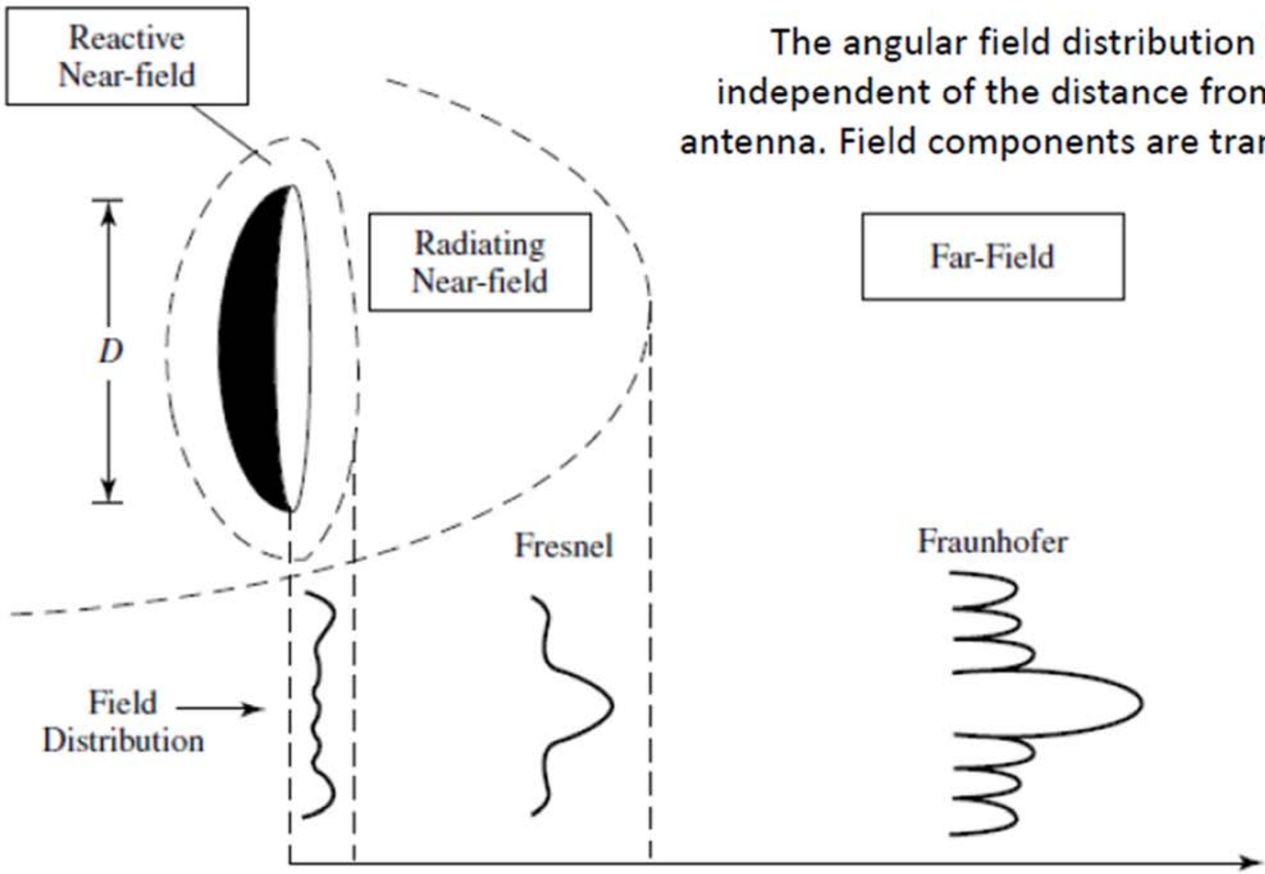


(b) Large antennas ($D > 2.5\lambda$).

Radiation zones

Field pattern = function of the radial distance, radial field component may be appreciable.

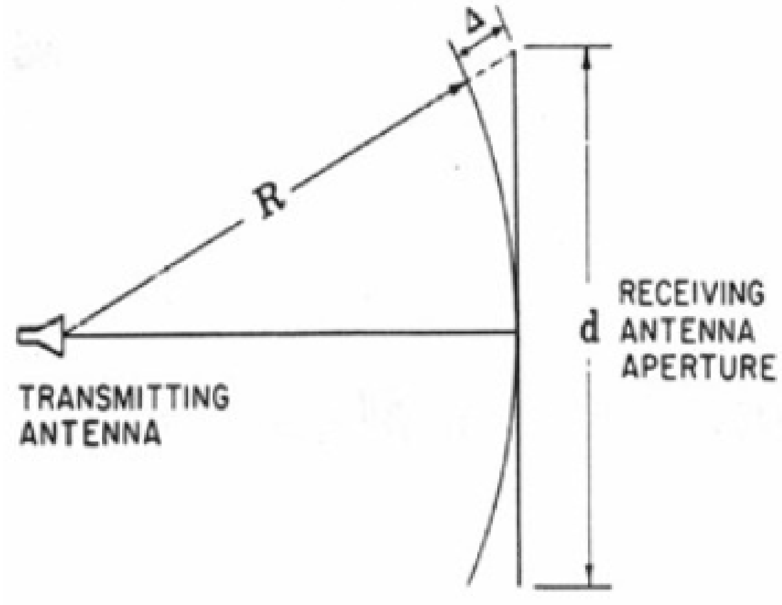
The angular field distribution is independent of the distance from the antenna. Field components are transverse



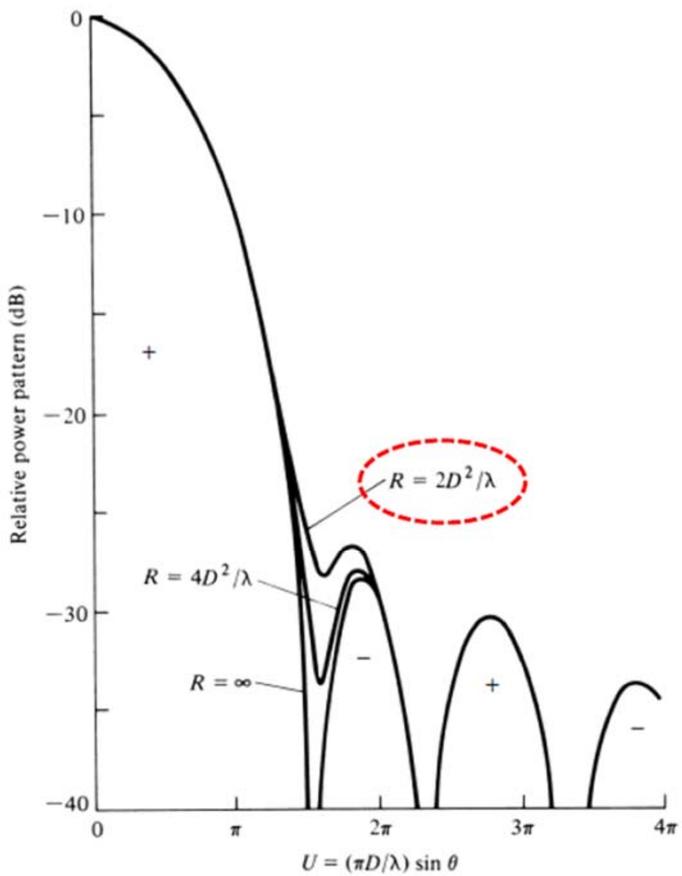
- Rayleigh distance

$D_R = 2D^2/\lambda$

- Phase error of $\pi/8$ relative perfectly parallel rays.



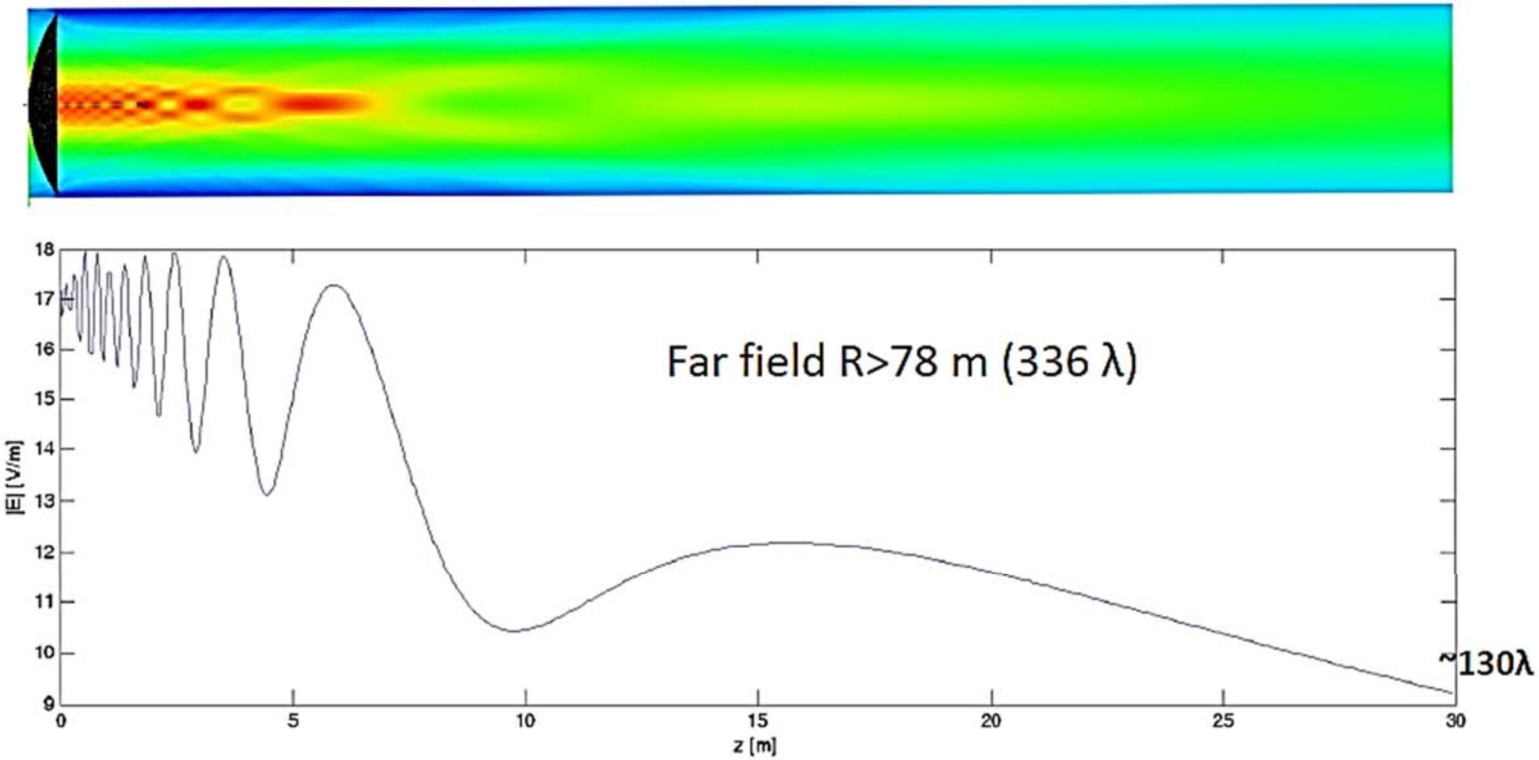
Radiation zones



Radiation zones

f=1.296 GHz, λ=232 mm

Ø 3m



The small (elementary) dipole and loop

Duality

Elementary dipole	**Elementary loop**
$E_\theta(r,\theta) = \frac{jkZ_0IL}{4\pi}\frac{e^{-jkr}}{r}\sin\theta$	$E_\phi(r,\theta) = \frac{Z_0k^2IS}{4\pi}\frac{e^{-jkr}}{r}\sin\theta$
$H_\phi(r,\theta) = \frac{jkIL}{4\pi}\frac{e^{-jkr}}{r}\sin\theta = \frac{E_\theta(r,\theta)}{Z_0}$	$H_\theta(r,\theta) = \frac{-k^2IS}{4\pi}\frac{e^{-jkr}}{r}\sin\theta = \frac{-E_\phi(r,\theta)}{Z_0}$

The loop antenna and the electric dipole are said to be duals, because the magnetic field radiated by the electric dipole has the same form as the electric field radiated by the loop antenna. A small electric loop antenna is also sometimes called a magnetic dipole.