# Antennas and Propagation

## Chapter 3:  Antenna Parameters

JACOBS UNIVERSITY

# Introduction

## Purpose

Introduce standard terms and definitions for antennas

Need a common language to specify performance

## Two types of parameters

1. Radiation parameters

   *What is the spatial selectivity of the element?*

   *Indicate where is power sent / collected from.*

2. Network parameters

   *What does the antenna present at its port(s)?*

   *Indicates requirements for system it connects to.*

# Outline: Radiation Parameters

## Goal

Precisely define the spatial selectivity of antennas

## Main Concepts

Radiation patterns, pattern cuts, beamwidth

Field regions: far-field, near-field

Power density of EM fields

Radiation Power Density

Directivity / Gain

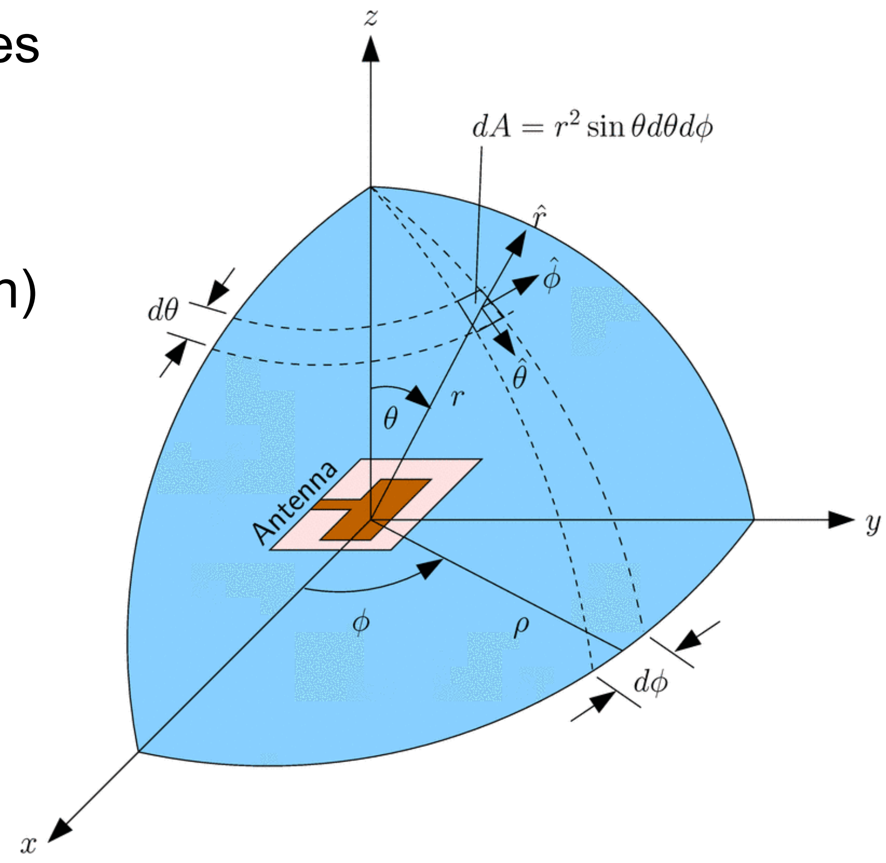
# Radiation Patterns

## Definition

- Graphical representation of radiation (or reception) properties
- Function of spatial coordinates

## Possible quantities

- Power density (most common)
- Field strength
- Directivity
- Gain
- Phase
- Polarization



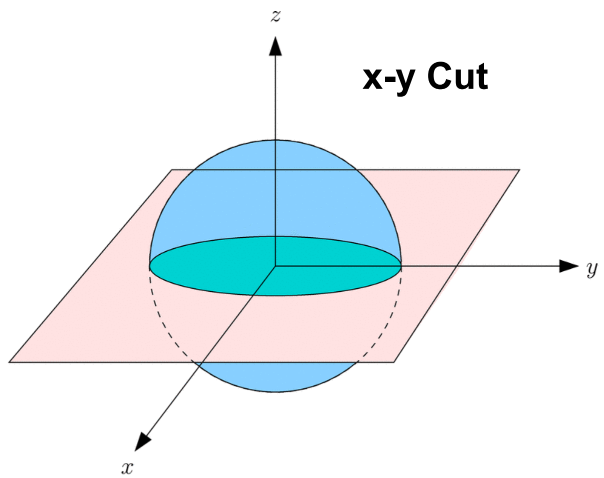
# Far-field / Cuts

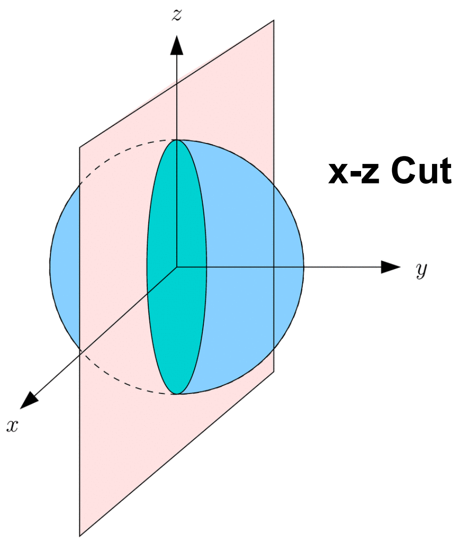
## Far-Field Patterns

- Usually more interesting than near fields
- Pattern only a function of angles ($\theta$, $\phi$)

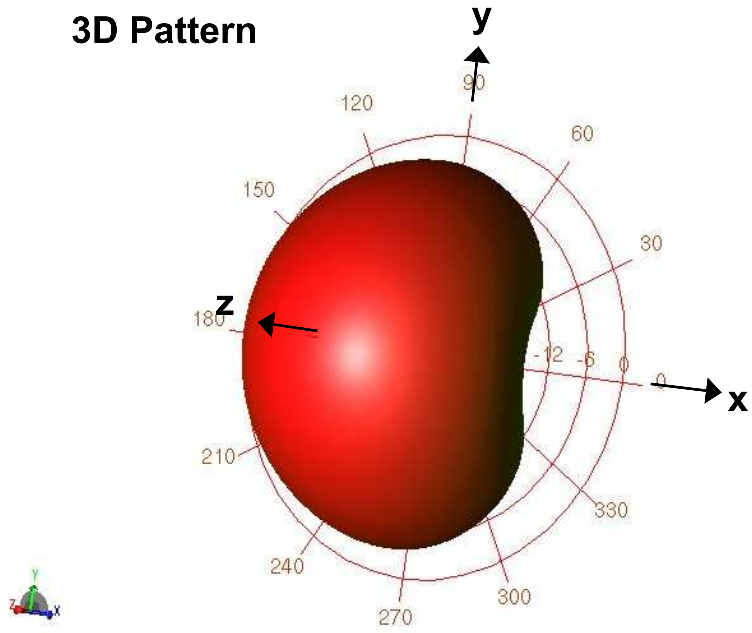
## Field Cuts

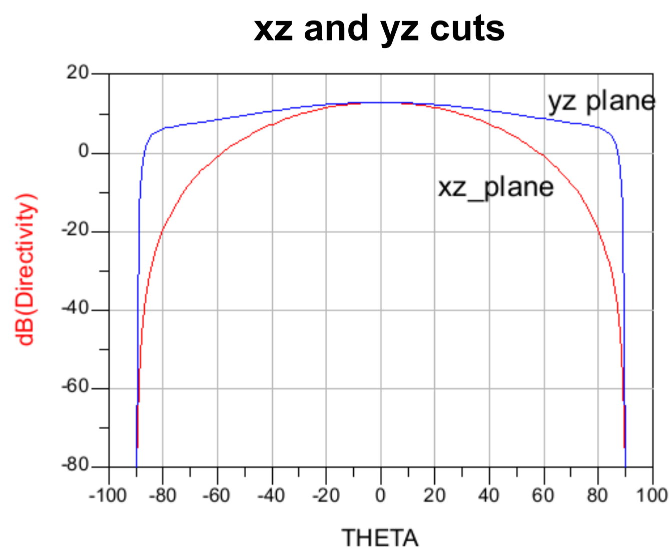
- Complete 3D pattern difficult to visualize (and plot!)
- More precise to look at cuts of the pattern:

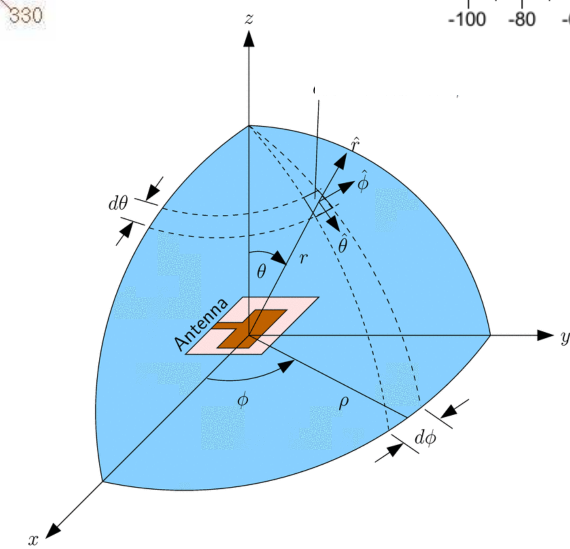


# Far-field / Cuts: Patch Antenna



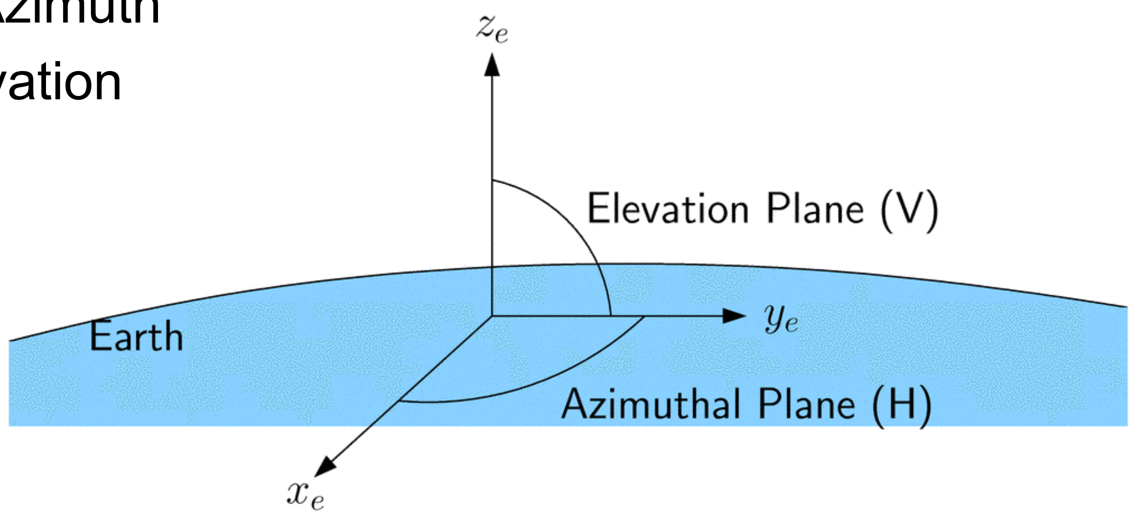



# Earth Coordinate System

## Global Coordinate System

Horizontal (H) / Azimuth

Vertical (V) / Elevation




## Caution

Depends on how antenna is mounted

Natural coordinates for analyzing antenna (x, y, z)

May be different from way mounted relative to Earth

Need to rotate axes

# General Pattern Types

## Isotropic Pattern

- Power (or field) equally radiated in all directions
- In practice, does not exist!
- Used as a reference

## Omnidirectional Pattern

- Radiated field constant in azimuth ($\phi$)
- May vary with elevation ($\theta$)
- Examples: dipole or small loop

## Directional Pattern

- Radiates significantly more power in some directions than others
- "Directional in the ____________ plane"
- Significantly more directional than a half-wave dipole

# Principal Patterns

## Motivation

Defines patterns independent of coordinate system

Useful for antennas with linear polarization

## E-Plane Pattern

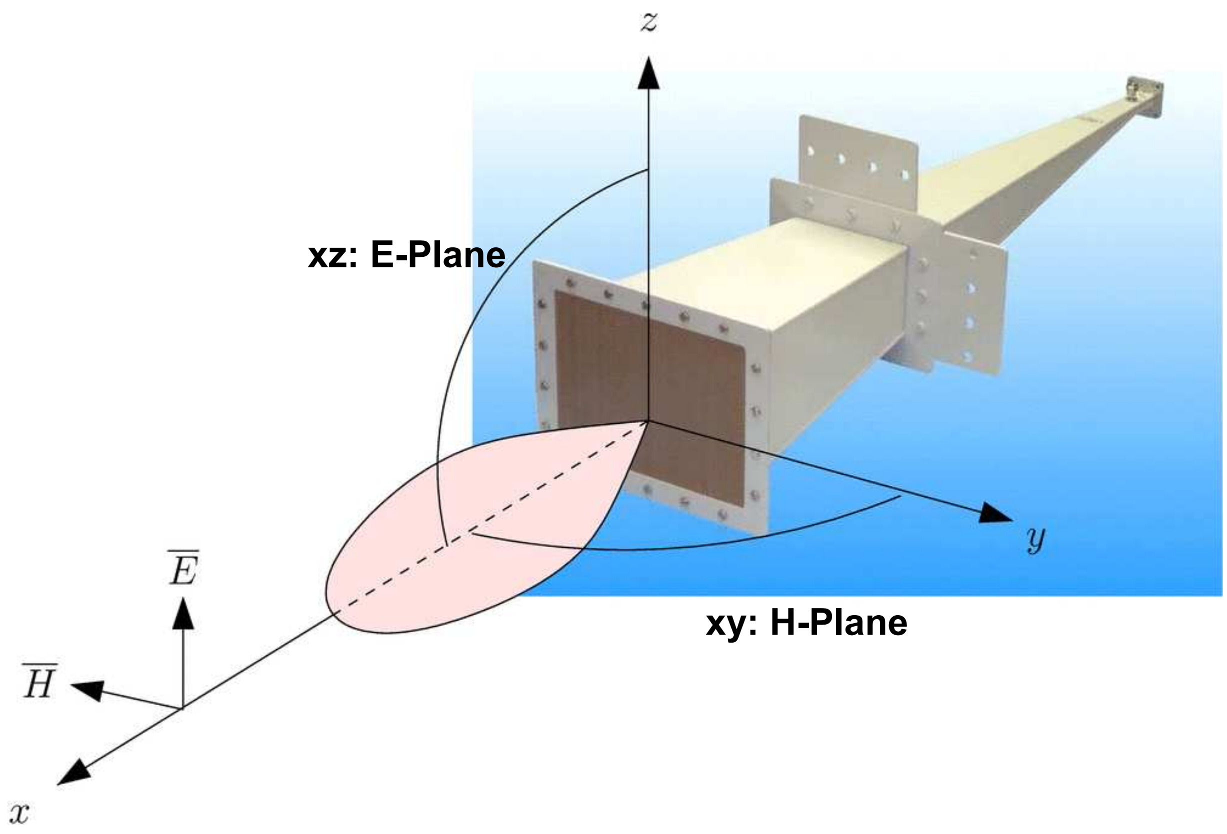
Cut of the pattern containing $\overline{E}$ and the direction of max radiation

## H-Plane Pattern

Cut of the pattern containing $\overline{H}$ and the direction of max radiation

# Principal Patterns

Example: Horn Antenna



# Beamwidth

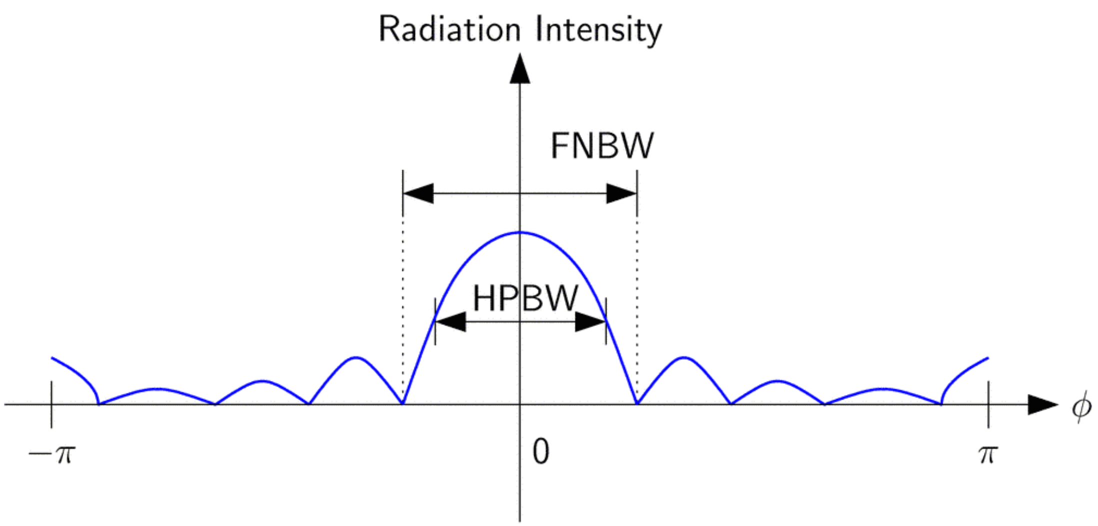
## Definition

Angular extent of the main beam

Critera

*HPBW: Half-power beamwidth*

*FNBW: First null beamwidth*



# Field Regions

## Reactive Near-field

Region immediately surrounding antenna

Convention: $R < 0.62\sqrt{D^2/\lambda}$

Fields can be very intense

Mostly reactive (stored energy, not propagating)

**Note:**

D = largest antenna dimension

$\lambda$ = wavelength

## Caution

Expressions do not work for electrically small antennas

Maximum dimension must be comparable or larger than $\lambda$

# Field Regions (2)

## Radiating Near-Field (Fresnel) Region

Fields are radiating

But, radiation pattern is a strong function of distance *r*

Convention: $0.62\sqrt{D^3/\lambda} \leq R < 2D^2/\lambda$

## Far-Field (Fraunhofer) Region

Angular field distribution nearly independent of distance

Fields are transverse to direction of propagation

Convention: $R \geq 2D^2/\lambda$

# Power Flow

## Power Flow of EM Field

Instantaneous Poynting Vector

$$\overline{W}(t) = \overline{E}(t) \times \overline{H}(t)$$

W/m$^2$ V/m A/m

## Time-average power

$$< \overline{W}(t) >_t = < \overline{E}(t) \times \overline{H}(t) >_t$$

In frequency domain, this becomes

$$\overline{W} = \frac{1}{2}\mathrm{Re}\left\{\overline{E} \times \overline{H}^*\right\}$$

## Interpretation

Power per unit area $\Rightarrow$ power density

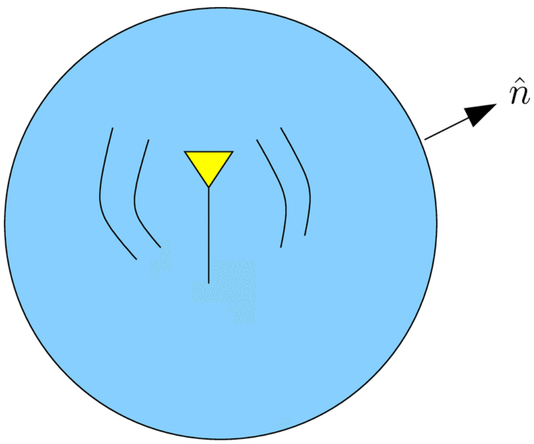
Direction is direction of power flow

# Power Radiated by Antenna

## Total radiated power

Integrate over surface enclosing antenna

$$P_{\text{rad}} = \oiint_S \overline{W}(\bar{r}) \cdot \hat{n} \; dS$$




## $\overline{w}$ on surface of $S$

Power *radiated* per unit area

<u>Radiation power density</u>

## Visualization

Generally fix $r$ and plot $W(\theta, \phi)$

# Normalization

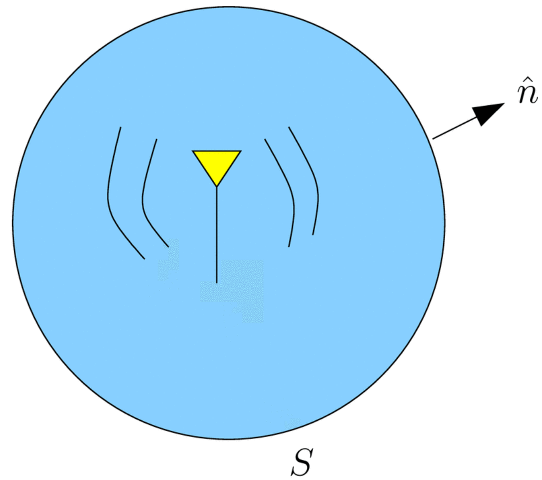
## Can normalize *W*

$$U(\theta,\phi) = r^2 W(r,\theta,\phi)$$

Obtain power per unit solid angle

Independent of distance from antenna

*U* is called <u>radiation intensity</u>




## In Far-Field Region

$$\overline{H} = \frac{1}{\eta}\hat{r} \times \overline{E}$$

$$\overline{W}(\overline{r}) = \frac{1}{2}\mathrm{Re}\left\{\overline{E} \times \overline{H}^*\right\} = \frac{1}{2}\mathrm{Re}\left\{\overline{E} \times \eta^{-1}(\hat{r} \times \overline{E}^*)\right\}$$

$$= \frac{|\overline{E}(r,\theta,\phi)|^2}{2\eta}\hat{r}$$

$$U(\theta,\phi) = \frac{r^2|\overline{E}(r,\theta,\phi)|^2}{2\eta}$$

$$dA = r^2 \underbrace{\sin\theta d\theta d\phi}_{\text{Solid Angle}}$$

# Directivity

## Definition

Sometimes called “directive gain”, given by

$$\frac{\text{Radiation intensity of given antenna}}{\text{Radiation intensity of a reference antenna}}$$

Note: Total radiated power same for two antennas

Reference Antenna: Standard is to choose isotropic radiator

$$D(\theta,\phi) = \frac{U(\theta,\phi)}{U_0} = \frac{U(\theta,\phi)}{P_{\text{rad}}/(4\pi)}$$

**Radiation intensity of isotropic radiator**

In terms of radiation density

$$D(\theta,\phi) = \frac{|\overline{W}(r,\theta,\phi)|r^2}{P_{\text{rad}}/(4\pi)} = \frac{|\overline{W}(r,\theta,\phi)|}{P_{\text{rad}}/(4\pi r^2)}$$

# Directivity (2)

## Maximum Directivity

$$D = \max_{(\theta,\phi)} D(\theta,\phi) = \frac{U_{\text{max}}}{P_{\text{rad}}/(4\pi)}$$

When directivity given as a single number $\Rightarrow$ Maximum directivity

## Notes

Directivity of an isotropic radiator is 1

Therefore, D > 1 in practice

D usually expressed in dB

# Directivity (3)

## Explicit Computation

Given far E-fields,

$$D(\theta, \phi) = 4\pi \frac{|\overline{E}(\theta, \phi, \boldsymbol{r})|^2}{\int_0^{2\pi} \int_0^{\pi} |\overline{E}(\theta', \phi', r)|^2 \sin\theta' \; d\theta' d\phi'}$$

Observation:

*Directivity is the radiation density divided by the average radiation intensity (over solid angle)*

# Gain

## Comparison with Directivity

Directivity/Gain

$$\frac{\text{Radiation intensity of given antenna}}{\text{Radiation intensity of a reference antenna}}$$

**Directivity:**

Total radiated power of two antennas kept the same

**Gain:**

*Input* power of two antennas kept the same

What is the difference?

<u>Losses</u>

# Gain (2)

## Computation

$$G(\theta,\phi) = \frac{U(\theta,\phi)}{P_{\text{in}}/(4\pi)} = e_t D(\theta,\phi)$$

## Efficiency

$e_t$ is the total efficiency of the antenna

$e_t = e_r\, e_c\, e_d$

where

$e_r$ = Reflection efficiency $1-|\Gamma|^2$

$e_c$ = Conduction efficiency

$e_d$ = Dielectric efficiency

## Radiation Efficiency

$e_t = P_{\text{rad}} / P_{\text{in}} = R_{\text{rad}} / (R_{\text{rad}} + R_{\text{L}})$

# Antenna Polarization

## Definition

TX: Polarization of the radiated wave produced by the antenna

RX: Polarization of incident plane wave yielding maximum available output power at the antenna terminals

## Directional Dependence

Polarization can be defined

1. As a function of direction
2. For direction of maximum gain (assumed if no direction specified)

# Review of EM Polarization

## Definition

For a plane wave propagating in the - $\hat{z}$ direction

Instantaneous field is

$$\overline{E}(z,t) = \hat{x}E_x(z,t) + \hat{y}E_y(z,t)$$

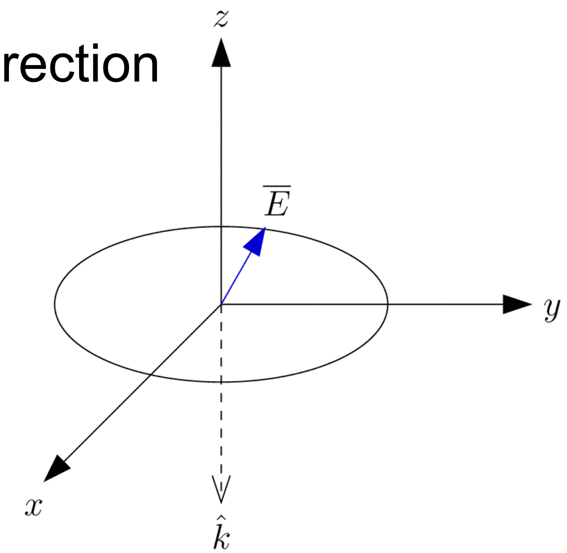
where

$$E_x(z,t) = \mathrm{Re}\left\{E_{x0}e^{j(\omega t + kz + \phi_x)}\right\}$$

$$E_y(z,t) = \mathrm{Re}\left\{E_{y0}e^{j(\omega t + kz + \phi_y)}\right\}$$

**Maximum Amplitude of x,y Components**

**Phase of x,y Components**



Direction of Propagation

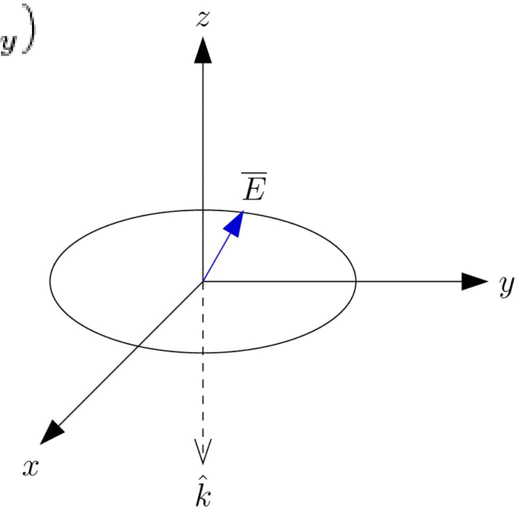
Polarization = Shape of curve traced by tip of E vector in xy plane

# Review of EM Polarization (2)

## In xy plane

$$\overline{E}(t, z=0) = \hat{x}E_{x0}\cos(\omega t + \phi_x) + \hat{y}E_{y0}\cos(\omega t + \phi_y)$$

Traces out an ellipse in general



Direction of Propagation

## Special Cases

Linear polarization

$$\Delta\phi = \phi_y - \phi_x = 0, \pi$$

$$\overline{E}(t) = (\hat{x}E_{x0} \pm \hat{y}E_{y0})\cos(\omega t)$$

Circular Polarization

$$E_{x0} = E_{y0}$$

$$\Delta\phi = \phi_y - \phi_x = \begin{cases} +\frac{\pi}{2}, & \text{CW (right hand circular)} \\ -\frac{\pi}{2}, & \text{CCW (left hand circular)} \end{cases}$$

# Outline: Network Parameters

## Goal

- Precisely define the “input/output interface” of the antenna

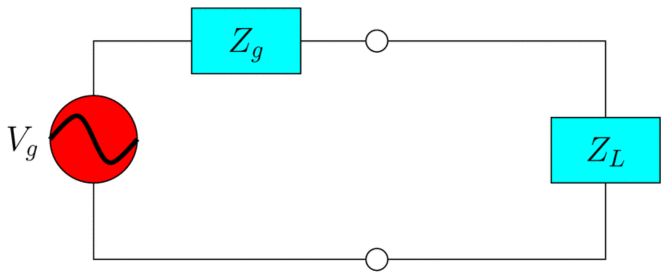
## Main Concepts

- Input impedance
- Reflection coefficient / VSWR
- Mutual Coupling, Z-parameters, S-parameters

# Antenna Input Characteristics

## Input Impedance

Have seen that mismatch reduces efficiency of antenna system




To ensure maximum transmission,

Conjugate match condition: $Z_L = Z_g^*$

## In Practice

Antennas designed to have *convenient* input impedance (50 Ohms)

<u>Matching network</u> integrated in antenna

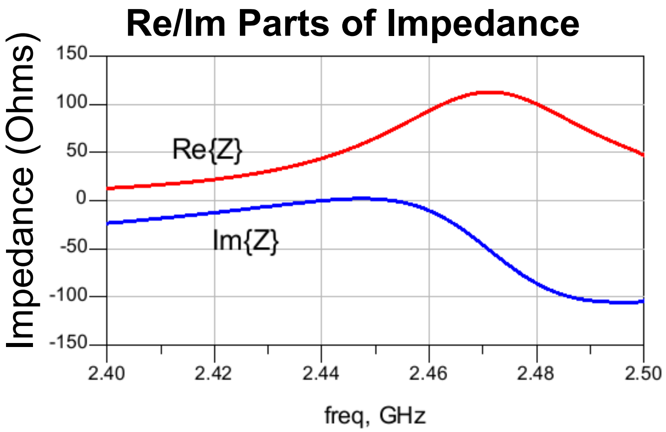
Transforms raw antenna impedance to $Z_0$

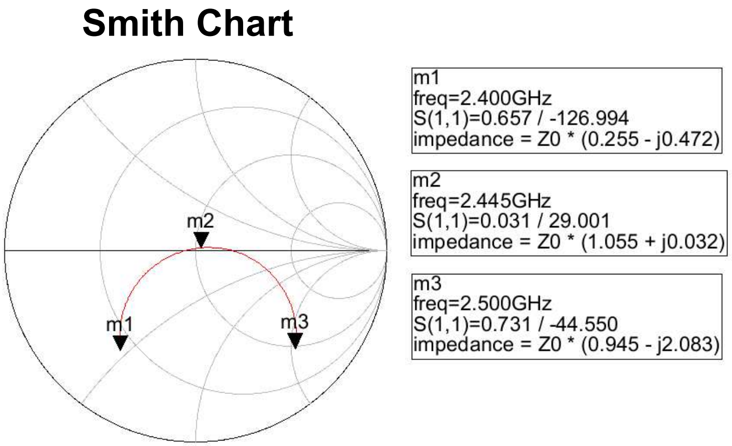
# Antenna Input Characteristics (2)

- Nominal input impedance is $Z_0$
- Actual impedance varies slightly with frequency
- Also, no fabrication process is perfect
- Variations in impedance from one antenna to next

## Characterizing input Impedance

- Graphical representations



# Antenna Input Characteristics (3)

## Problem with Providing Input Impedance

- Impedance varies from one device to the next (fabrication variations)
- Every antenna must be measured

## More common approach

- Antenna design assumes system impedance of $Z_0$
- Specify:
  1. Worst case reflection, or
  2. Voltage standing wave ratio (VSWR)

# Reflection Coefficient

## Definition

$\Gamma = V^{-}/V^{+}$ on the feeding line

$|\Gamma|^2$ indicates what fraction of power is reflected

Power lost, because not delivered to antenna

## Return Loss

Related to $\Gamma$:

Return Loss = $-20 \log_{10} |\Gamma|$

IEEE definition: Return Loss as a positive value (hence – sign)

Worst case return loss

Return Loss $_{min} = -20 \log_{10} |\Gamma|_{max}$

# VSWR

## Definition

Voltage standing wave ratio

(max voltage to min voltage on feed line)

$$\mathrm{VSWR} = \frac{|V(z)|_{\max}}{|V(z)|_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

Reason:

Wave ratio was easy to measure with old slotted waveguides

Still used in many specifications of RF parts / antennas

Expressed as a ratio

i.e. 1.2:1 or 2:1

If a single number, indicates worst-case value

# Mutual Coupling

## Where important

- Antenna arrays
- Multimode or multipolarization antennas $\Rightarrow$ Multiple ports

## Basic problem

- Antenna elements close together
- Signals on one element $\Rightarrow$ create signal on other element
- Usually want to receive signals on antennas independently

## SP Algorithms

- Typically are degraded by the effect

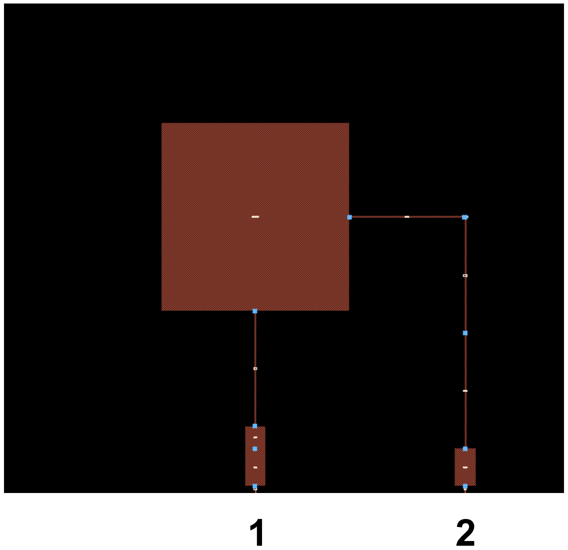
# Characterizations: Z-Parameters

## Example: Dual Polarization Patch Antenna

Square patch at 2.44 GHz

Feed 1 $\Rightarrow$ Vertical Pol.

Feed 2 $\Rightarrow$ Horizontal Pol.



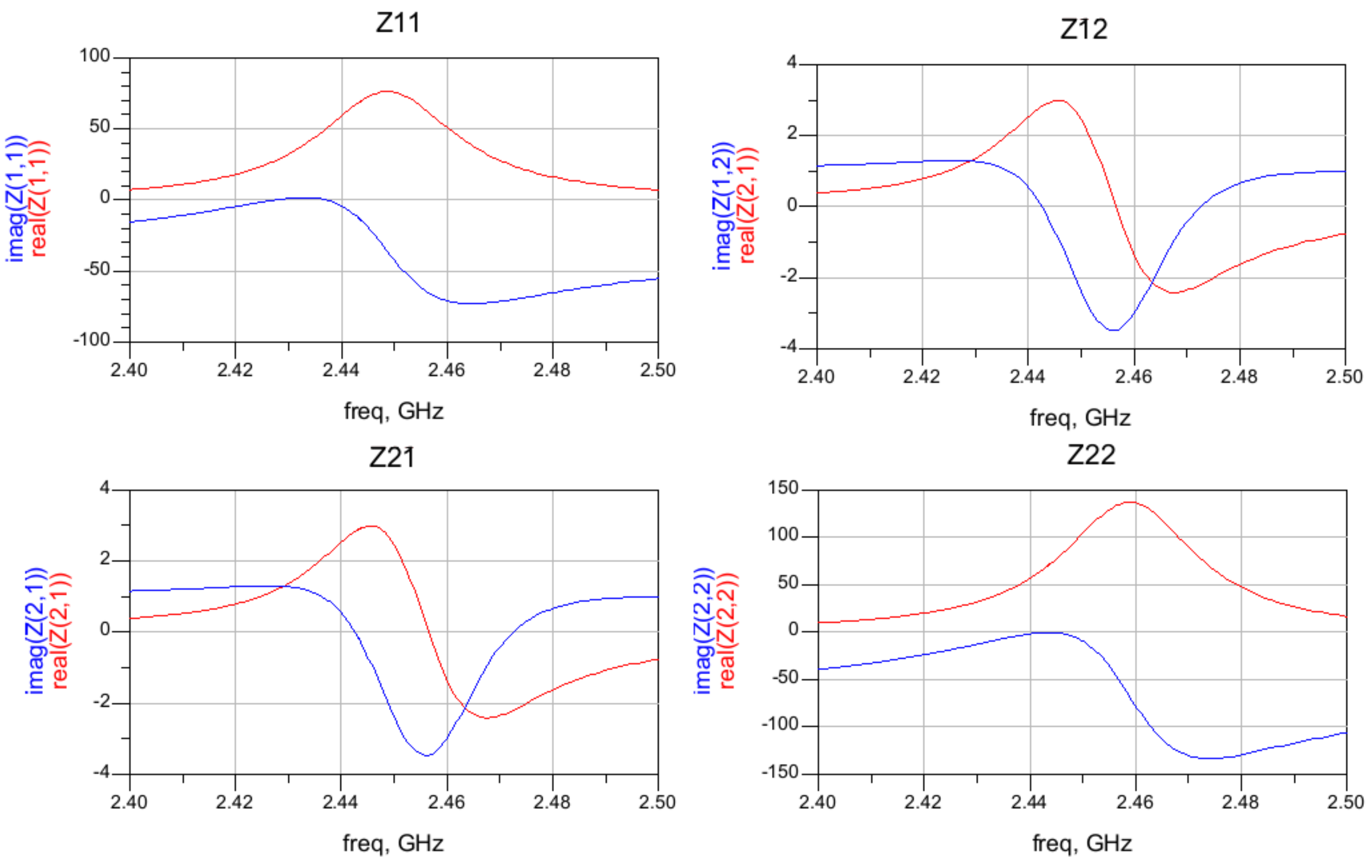

## Network Characterizations

1. Z-Parameters

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \qquad \overline{v} = \overline{\overline{Z}}\,\overline{i}$$

Coupling means $Z_{21}$ or $Z_{12} \neq 0$

# Simulated Z-Parameters



# Characterizations: S-Parameters

## Network Characterizations

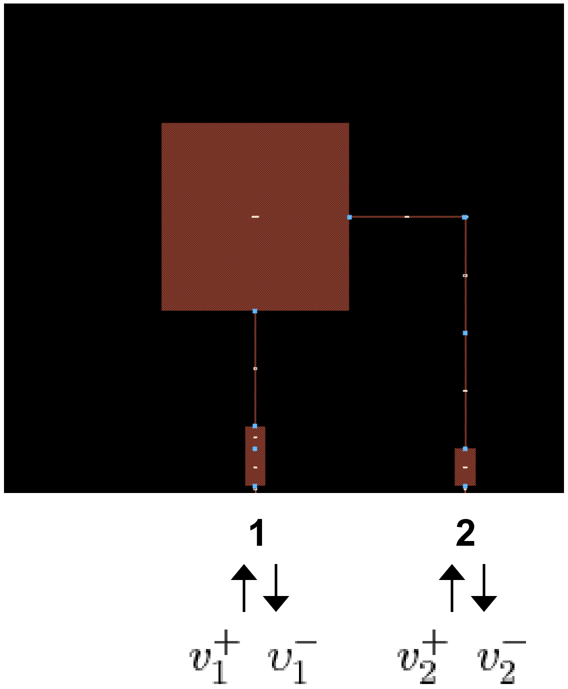
2. S-Parameters (S = “scattering”)

More useful for high-freq. analysis

$$\begin{bmatrix} v_1^- \\ v_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} v_1^+ \\ v_2^+ \end{bmatrix}$$

$$S_{ij} = \left. \frac{v_j^-}{v_i^+} \right|_{v_k^+ = 0, k \neq i}$$

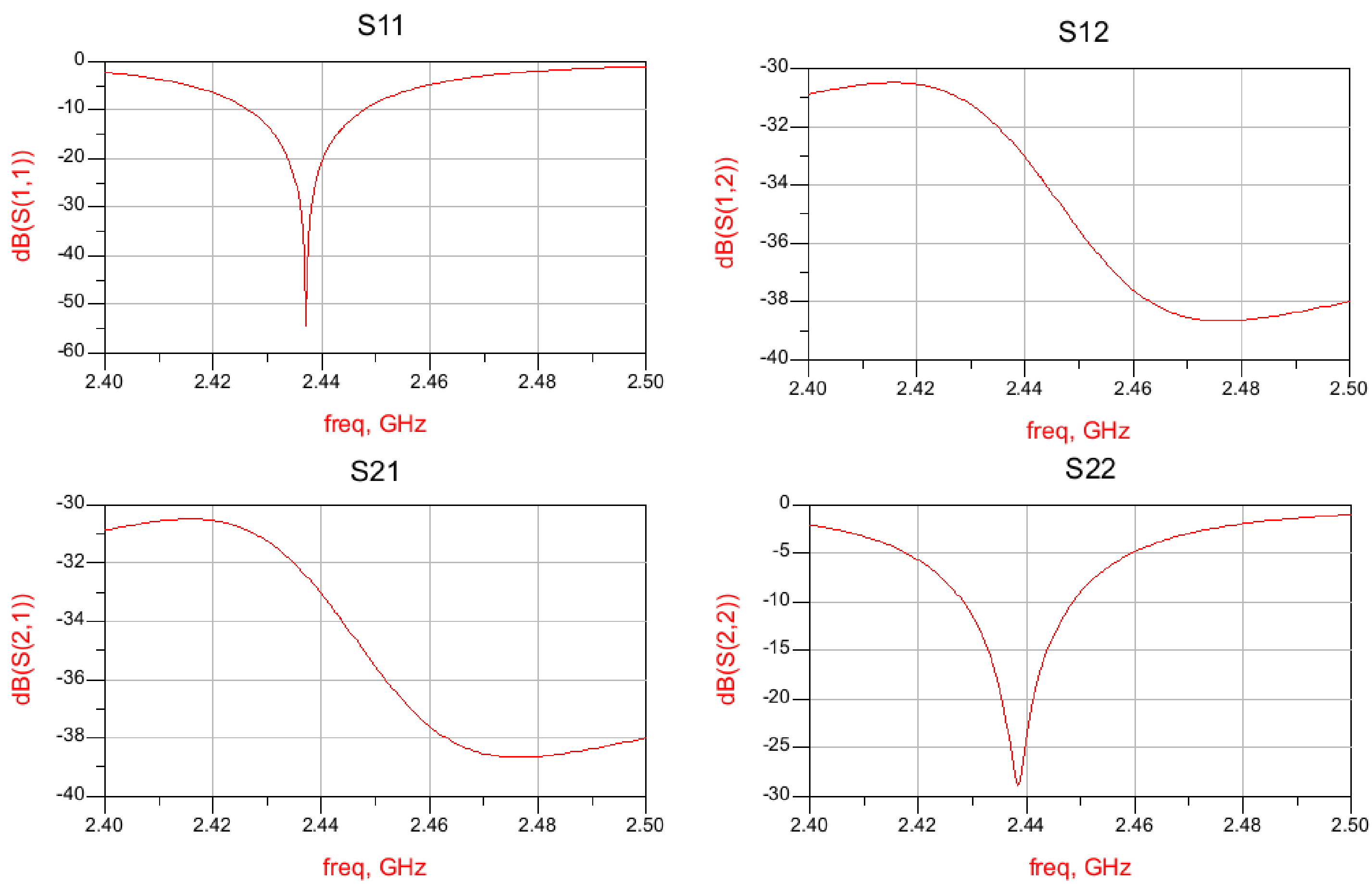
You should see relation to Γ




Worst-case coupling: 20 $\log_{10} |S_{21}|_{max}$

Often quoted as minimum isolation: -20 $\log_{10} |S_{21}|_{max}$

# Simulated S-Parameters



# Antenna Bandwidth

## Definition

Range of frequencies over which the antenna conforms to some specified standard

"Specified standard" includes any performance metrics so far:

*Patterns*

*Gain*

*Efficiency*

*Side lobe levels*

*Beamwidth*

*Input Impedance*

*Isolation*

*Etc.*

# Antenna Bandwidth (2)

## Wideband antennas

Bandwidth expressed as a ratio

$f_{max}/f_{min}$ : 1

E.g. 10:1 $\Rightarrow$ Maximum frequency ten times greater than minimum frequency

## Narrowband Antennas

Usually express as "fractional bandwidth," or

$(f_{max} - f_c)/f_c \times 100$ where $f_c \approx (f_{max} + f_{min})/2$.

E.g. 5% fractional bandwidth $\Rightarrow$
5% deviation from center frequency can be tolerated

# Summarizing

## So far ...

Have characterized a single antenna (patterns, port characteristics)

## But,

How do TX/RX antennas work together?

How do we use parameters to estimate gain of whole link?

## Simplest Case: Free Space Propagation

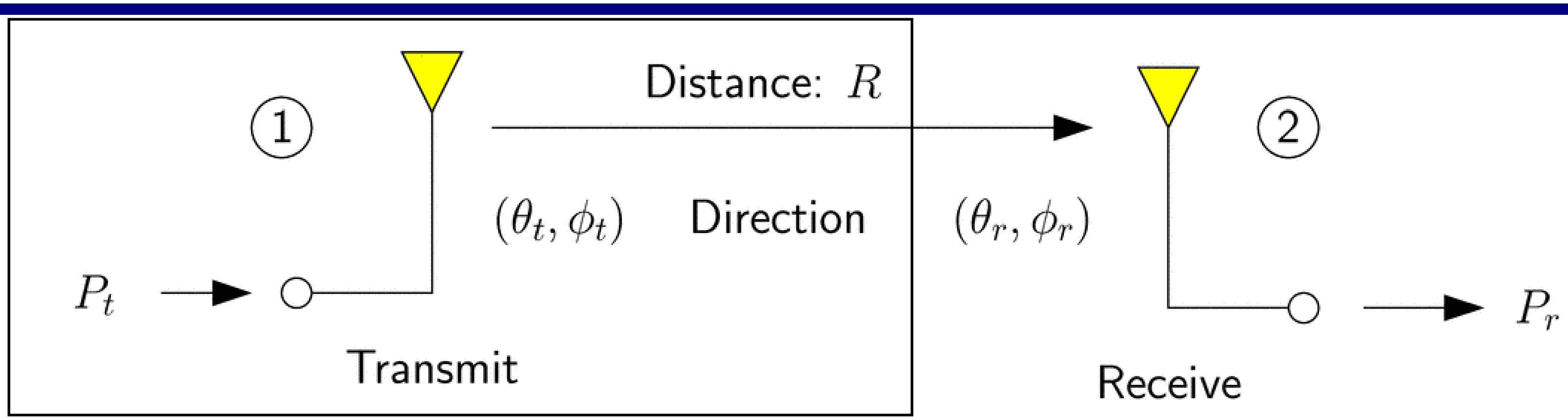
Governed by Friis Transmission Equation

## More complicated cases

Multipath, shadowing

Consider later in course (*Propagation* part of class!)

# Friis Transmission Equation

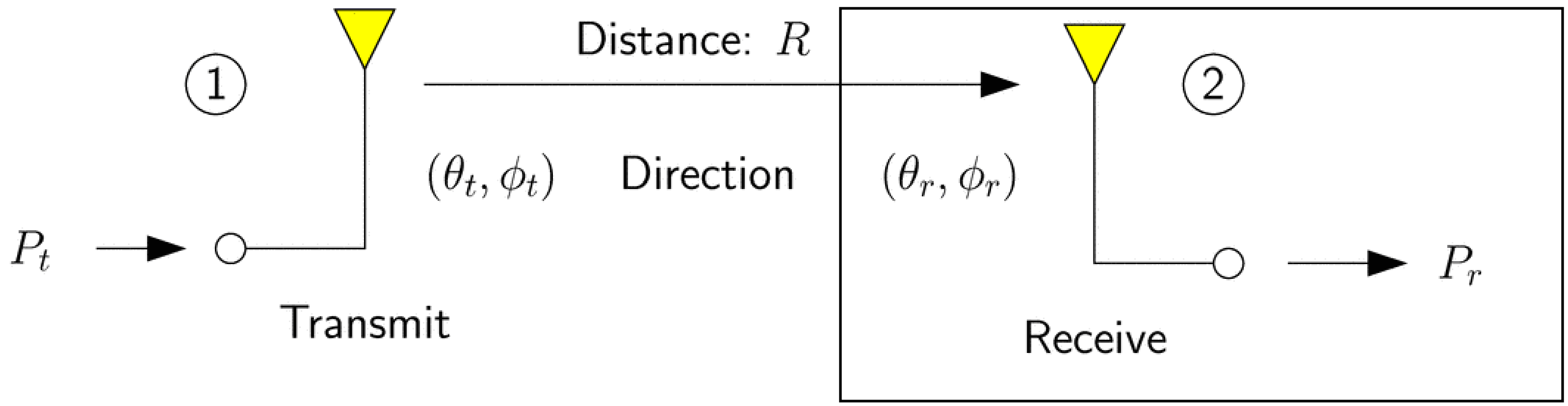



Radiation power density from transmitter

$$W_t = \frac{P_t G_t(\theta_t, \phi_t)}{4\pi R^2}$$

| | |
|---|---|
| $P_t$ | Transmit power |
| $R$ | Transmit/Receive separation |
| $(\theta_t, \phi_t)$ | Transmit direction |
| $G_t$ | Transmit gain |

# Friis Transmission Equation (2)




Next, power collected by receive antenna

Need to derive notion of receiving aperture or "effective area"

Assuming antenna captures power from area $A_r$

$$P_r = W_t A_r = \frac{P_t G_t A_r}{4\pi R^2}$$

Turns out that $A_r$ is a directional quantity related to $G_r$

# Relation of Effective Area and Gain

Consider again (assumed)

$$P_r = W_t A_r = \frac{P_t G_t A_r}{4\pi R^2}$$

Now consider making 1=RX and 2=TX (switch roles)

Know by reciprocity that *received power same*

$$P_r = \frac{P_t G_r A_t}{4\pi R^2}$$ **(switched TX/RX)**

Comparing, this means that

$$G_t A_r = G_r A_t \quad \text{or} \quad \frac{G_t}{A_t} = \frac{G_r}{A_r}$$

Since we have specified nothing about antennas,

$$\frac{G}{A} = \text{constant}$$ **(ANY reciprocal antenna!)**

# Friis Transmission Equation (3)

$$\frac{G}{A} = \text{constant}$$

How do we find the constant?

Analyze a convenient antenna and find both *G* and $A_r$

Later we will do this for an infinitesimal (Hertzian) dipole:

$A_r = 3\lambda^2/(8\pi)$ and $G_r = 1.5$

Which means for any antenna

$G_r/A_r = 4\pi/\lambda^2$

$$A_r(\theta, \phi) = \frac{G_r(\theta, \phi)\lambda^2}{4\pi}$$

$$\boxed{P_r = \frac{P_t G_t(\theta_t, \phi_t) G_r(\theta_r, \phi_r)\lambda^2}{(4\pi R)^2}}$$

# Summary

Standard terms and definitions for antennas

Radiation Parameters

Network (port) Parameters

Friis Transmission Equation

Next time: Start analyzing specific antenna types