Antenna Directivity and Gain Measurements

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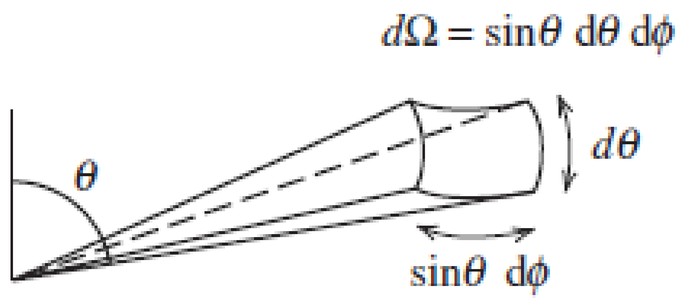
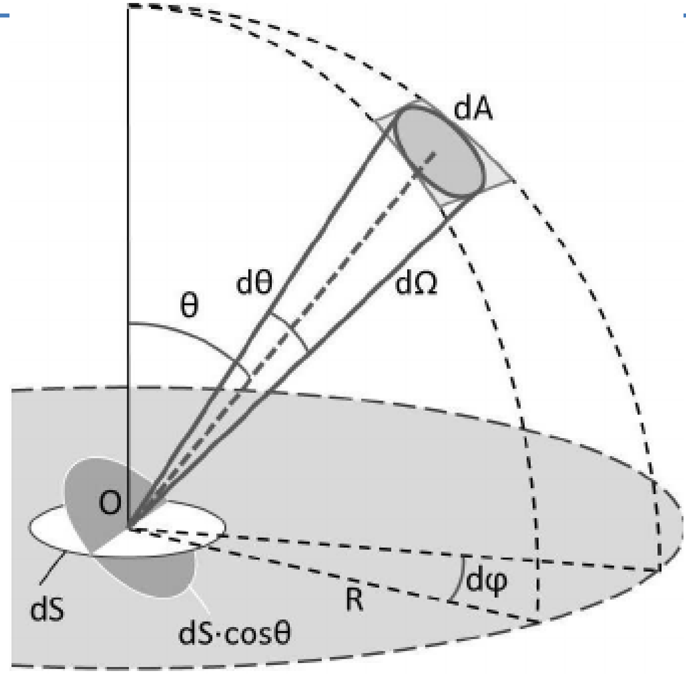
1. Antenna Directivity

- Derivation of the Directivity





$$P_r = \frac{1}{2}\mathrm{Re}\oint_S \mathbf{E}\times\mathbf{H}^* \cdot d\mathbf{S} = \frac{1}{2\eta}\oint_S (|E_\theta|^2 + |E_\varphi|^2) r^2 d\Omega \text{ (total radiated power)}$$

$|E_\theta|, |E_\varphi|$ (magnitude of the theta and phi components of the far-zone electric field)

$d\Omega = \sin\theta\, d\theta d\varphi$ (solid angle)

$$U(\theta,\varphi) = \frac{1}{2\eta}(|E_\theta|^2 + |E_\varphi|^2) r^2 = U_\theta + U_\varphi \quad \text{(power density per steradian)}$$

$$U_{\text{ave}} = \frac{P_r}{4\pi} \quad \text{(average power density per steradian)}$$

$$D(\theta,\varphi) \equiv \frac{U(\theta,\varphi)}{U_{\text{ave}}} = \frac{U_\theta}{U_{\text{ave}}} + \frac{U_\varphi}{U_{\text{ave}}} = D_\theta + D_\varphi \text{ (directional directivity)}$$

D_θ, D_φ (direcitivity of the theta and phi components)

$$U(\theta,\varphi) \equiv U_m \,|\, F(\theta,\varphi)\,|^2 = \frac{1}{2\eta}(|\,E_\theta\,|^2 + |\,E_\varphi\,|^2)\,r^2$$

$$\max[U(\theta,\varphi)] = U_m$$

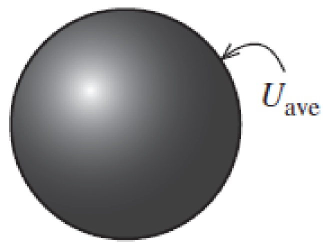
$|\,F(\theta,\varphi)\,|^2$ (normalized pattern factor)

$$P_r = U_m \oint_S |\,F(\theta,\varphi)\,|^2 \, d\Omega \equiv U_m \Omega_A$$

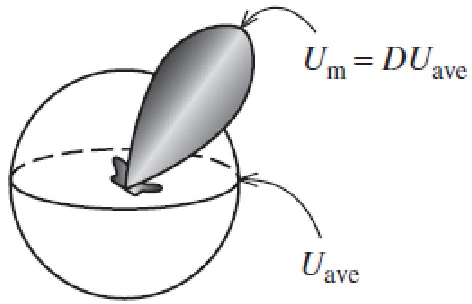
$$\Omega_A = \oint_S |\,F(\theta,\varphi)\,|^2 \, d\Omega \text{ (beam solid angle)}$$

$$D(\theta,\varphi) = \frac{4\pi}{\Omega_A} |\,F(\theta,\varphi)\,|^2 \text{ (directional directivity)}$$

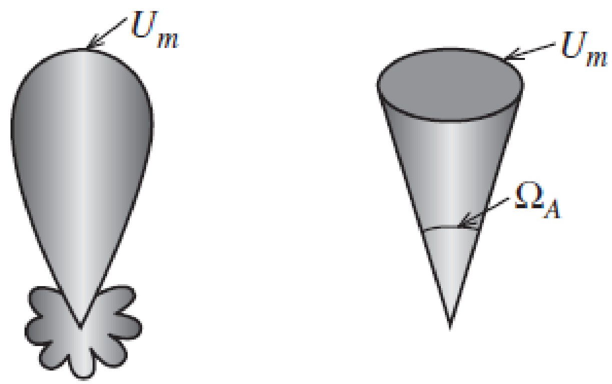
$$D = \max[D(\theta,\varphi)] = \frac{4\pi}{\Omega_A} \text{ (antenna's directivity or maximu}$$



(*a*) Radiation intensity distributed isotropically.



(*b*) Radiation intensity from an actual antenna.



Actual pattern

$$U(\theta,\varphi) \equiv U_m \, | F(\theta,\varphi) |^2 = U_{\theta m} \, | F_\theta(\theta,\varphi) |^2 + U_{\varphi m} \, | F_\varphi(\theta,\varphi) |^2$$

$$U_{\theta m} \, | F_\theta(\theta,\varphi) |^2 = \frac{1}{2\eta} | E_\theta |^2 r^2 \quad \text{(power density of the theta component)}$$

$$U_{\varphi m} \, | F_\varphi(\theta,\varphi) |^2 = \frac{1}{2\eta} | E_\varphi |^2 r^2 \quad \text{(power density of the phi component)}$$

$$P_r = U_{\theta m} \oint_S | F_\theta(\theta,\varphi) |^2 \, d\Omega + U_{\varphi m} \oint_S | F_\varphi(\theta,\varphi) |^2 \, d\Omega = U_{\theta m} \Omega_{\theta A} + U_{\varphi m} \Omega_{\varphi A}$$

$$D(\theta,\varphi) = 4\pi \frac{U_{\theta m} \, | F_\theta(\theta,\varphi) |^2 + U_{\varphi m} \, | F_\varphi(\theta,\varphi) |^2}{U_{\theta m} \Omega_{\theta A} + U_{\varphi m} \Omega_{\varphi A}} \quad \text{(antenna's directional directivity)}$$

$$D_\theta(\theta,\varphi) = 4\pi \frac{U_{\theta m} \, | F_\theta(\theta,\varphi) |^2}{U_{\theta m} \Omega_{\theta A} + U_{\varphi m} \Omega_{\varphi A}} \quad \text{(directivity of the theta component)}$$

$$D_\varphi(\theta,\varphi) = 4\pi \frac{U_{\varphi m} \, | F_\varphi(\theta,\varphi) |^2}{U_{\theta m} \Omega_{\theta A} + U_{\varphi m} \Omega_{\varphi A}} \quad \text{(directivity of the phi component)}$$

Directivity of a Circularly Polarized Antenna

- Replace E_θ, E_φ with E_R and E_L

- Use the same formulas.

Directivity Units

D (dB) = 10 $\log_{10} D$

D (dBi) : relative to an isotropic antenna

D (dBic) : relative to an isotropic circularly polarized antenna

D = 0 dBi : isotropic linearly polarized antenna

D = 0 dBic : isotropic circularly polarized antenna

Normalized Pattern Factor Examples

- Small dipole antenna & small loop antenna

$$\mathbf{F}(\theta,\varphi) = \sin\theta\,\hat{\boldsymbol{\theta}} \quad ; \quad D = 1.5 = 1.8 \text{ dBi}$$

- Half-wave Diopole Antenna (z-directed current)

$$\mathbf{F}(\theta,\varphi) = \frac{\cos(0.5\pi\cos\theta)}{\sin\theta}\,\hat{\boldsymbol{\theta}} \; ; \; D = 1.64 = 2.15 \text{ dBi}$$

- Dipole with a sinusoidal current distribution (z-directed current)

$$\mathbf{F}(\theta,\varphi) = \frac{\cos[(\beta L/2)\cos\theta] - \cos(\beta L/2)}{[1-\cos(\beta L/2)]\sin\theta}\hat{\boldsymbol{\theta}}$$

- Uniform line source (z-directed current)

$$\mathbf{F}(\theta,\varphi) = \sin\theta\frac{\sin[(\beta L/2)\cos\theta]}{(\beta L/2)\cos\theta}\hat{\boldsymbol{\theta}}$$

- Uniformly excited and equally spaced linear array (on z-axis)

$$\mathbf{F}(\theta,\varphi) = \frac{\sin[(N/2)(\beta d\cos\theta + \alpha)]}{N\sin[(N/2)(\beta d\cos\theta + \alpha)]}\mathbf{e}(\theta,\varphi) \quad (\mathbf{e} : \text{element factor})$$

- Uniform rectangular aperture (aperture on the *xy*-plane, electric field in *y* direction)

$$\mathbf{F}(\theta,\varphi)=\frac{1+\cos\theta}{2}P_y(\sin\varphi\,\hat{\boldsymbol{\theta}}+\cos\varphi\,\hat{\boldsymbol{\varphi}})$$

$$P_y=\frac{\sin[(\beta L_x/2)u]\,\sin[(\beta L_y/2)v]}{[(\beta L_x/2)u]\,[(\beta L_y/2)v]}$$

$$u=\sin\theta\cos\varphi,\;\; v=\sin\theta\sin\varphi$$

- Uniform rectangular aperture (aperture on the *xy*-plane, electric field in *x* direction)

$$\mathbf{F}(\theta,\varphi)=\frac{1+\cos\theta}{2}P_x(\cos\varphi\,\hat{\boldsymbol{\theta}}-\sin\varphi\,\hat{\boldsymbol{\varphi}})$$

$$P_x=\frac{\sin[(\beta L_x/2)u]\,\sin[(\beta L_y/2)v]}{[(\beta L_x/2)u]\,[(\beta L_y/2)v]}$$

$$u=\sin\theta\cos\varphi,\;\; v=\sin\theta\sin\varphi$$

Rectangular Aperture of Separable Aperture Distribution

- Directivity

$$D = \pi D_x D_y$$

$$D_x = c_x \frac{2L_x}{\lambda}, \quad D_y = c_y \frac{2L_y}{\lambda}$$

c_x, c_y: factor determined by aperture distribution shape

$c_x = c_y = 1, 0.933, 0.863, 0.801, 0.751, 0.709, 0.674, 0.641$ for

SLR (sidelobe ratio) = 13.3, 20, 25, 30, 35, 40, 45, 50 dB respectively with Taylor one-parameter distribution

- Beamwidth

$$HP_x = k_x \frac{\lambda}{L_x}, \quad HP_y = k_y \frac{\lambda}{L_y}$$

k_x, k_y: factor determined by aperture distribution shape

$k_x = k_y = 0.886, 1.001, 1.091, 1.173, 1.250, 1.320, 1.386, 1.449$ for

SLR (sidelobe ratio) = 13.3, 20, 25, 30, 35, 40, 45, 50 dB respectively with Taylor one-parameter distribution

- Rectangular aperture with cosine and cosine squared distribution

$c = 0.810,\ k = 1.19,\ \text{SLR} = 23.0\ \text{dB}$ (cosine distribution)

$c = 0.667,\ k = 1.44,\ \text{SLR} = 31.7\ \text{dB}$ (cosine-squared distribution)

- Directivity-beamwidth Product

$$DB = 4\pi(c_x c_y)(k_x k_y)$$

$DB = 32{,}383 \deg^2$ (uniform rectangular aperture)

$DB = 35{,}230 \deg^2$ (cosine-uniform rectangular aperture)

$DB = 33{,}709 \deg^2$ (uniform circular aperture)

$(c_x = c_y = \sqrt{\pi/2} = 1.253,\ k_x = k_y = 1.02,\ L_x = L_y = 2a)$

$DB = 38{,}933 \deg^2$ (circular aperture with -12dB pedestal distribution)

$DB = 41{,}253 \deg^2$ (low-gain unequal-beamwidth antenna with no sidelobe)

$DB = 52{,}525 \deg^2$ (equal-beamwidth antenna with no sidelobe)

$DB = 26{,}000 \deg^2$ (general use for real antennas)

2. Antenna Patten Modeling

- Cosine-q pattern for antenna with y-directed current or y-directed E-field

$$\mathbf{F}(\theta,\varphi) = \cos^{q_e}\theta\cos\varphi\,\hat{\boldsymbol{\theta}} - \cos^{q_h}\theta\sin\varphi\,\hat{\boldsymbol{\varphi}}$$

q_e and q_h are determined by matching the pattern at -3dB or -10dB angle.

$$q_e = \frac{\log|F_\theta(\theta,\varphi=0)|}{\log(\cos\theta)},\ q_h == \frac{\log|F_\varphi(\theta,\varphi=90°)|}{\log(\cos\theta)}$$

Directivity can be obtained in a closed form.

$$D = \frac{2(2q_e+1)(2q_h+1)}{q_e+q_h+1}$$

Rotationally symmetric pattern:

$$q_e = q_h$$

$$|\mathbf{F}(\theta,\varphi)| = \cos^q\theta$$

$$D = 2(2q+1)$$

3. Antenna Directivity Measurements

- Measure $|E_\theta(\theta, \varphi)|$ and $|E_\varphi(\theta, \varphi)|$ for $0 \le \theta \le \pi$, $0 \le \varphi \le 2\pi$.

- Find the directivity of AUT using the following relations.

$$\Omega_A = \oiint_S |\mathbf{E}(\theta,\varphi)|^2 / \max\{|\mathbf{E}(\theta,\varphi)|^2\} \sin\theta d\theta d\varphi : \text{ beam solid angle (sr)}$$

$$\mathbf{E}(\theta,\phi) = \hat{\boldsymbol{\theta}} E_\theta(\theta,\varphi) + \hat{\boldsymbol{\varphi}} E_\varphi(\theta,\varphi)$$

$$D(\theta,\phi) = \frac{4\pi}{\Omega_A} \frac{|\mathbf{E}(\theta,\varphi)|^2}{\max\{|\mathbf{E}(\theta,\varphi)|^2\}}$$

$$D = \max\{D(\theta,\phi)\} = \frac{4\pi}{\Omega_A}$$

4. Antenna Gain

- Definition of the Antenna Gain

$$G = eD \quad \text{(antenna gain)}$$

$$e = \frac{P_r}{P_i} = e_L e_m \quad \text{(antenna efficiency)}$$

$$e_L = \frac{P_r}{P_a} = \frac{P_r}{P_L + P_r} \quad \text{(antenna efficiency due to internal loss)}$$

$$e_m = \frac{P_a}{P_i} = 1 - |S_{11}|^2 \quad \text{(antenna impedance match efficiency)}$$

P_r : power radiated from antenna

$P_a = P_L + P_r$: power transferred to antenna

P_L : power dissipated inside the antenna

$$P_a = P_i(1 - |S_{11}|^2)$$

P_i : power input to antenna

$$S_{11} = \frac{Z_{\text{in}} - Z_0}{Z_{\text{in}} + Z_0}$$

Z_{in} : antenna input impedance (ohm)

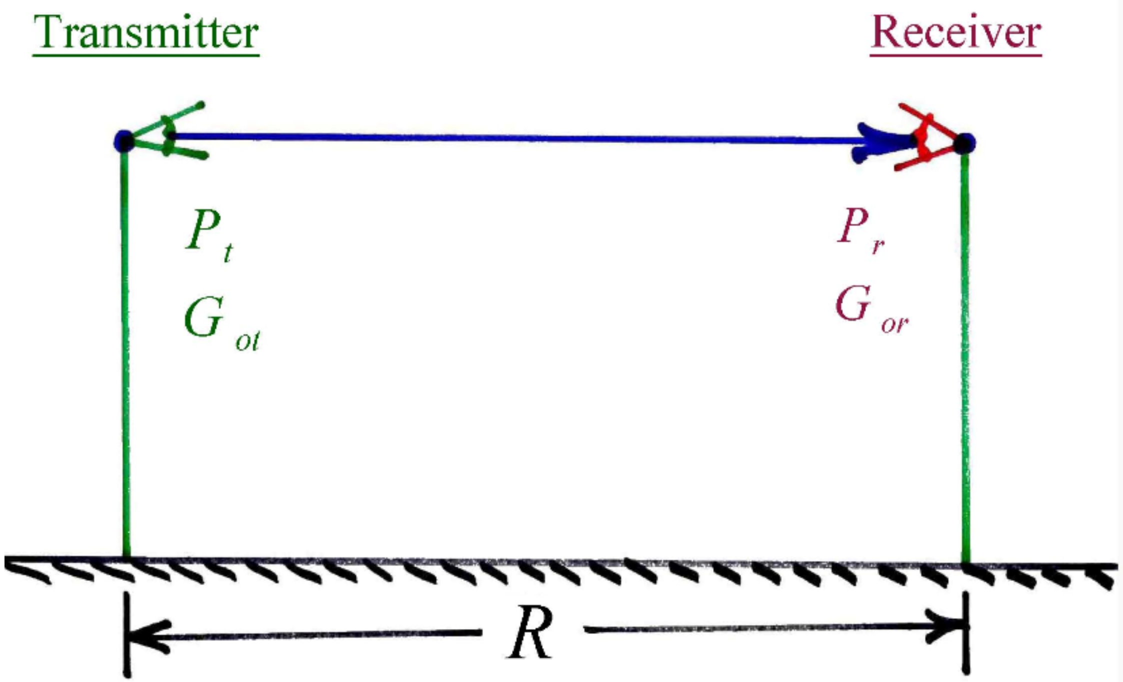
Z_0 : antenna input line characteristic impedance (ohm)

5. Antenna Gain Measurements

- Two Identical Polarization-matched Antennas

$$\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R}\right)^2 G_{ot} G_{or} = \left(\frac{\lambda}{4\pi R}\right)^2 G^2$$

$$G = \frac{4\pi R}{\lambda}\sqrt{\frac{P_r}{P_t}}$$



- ## Gain Tranfer Method

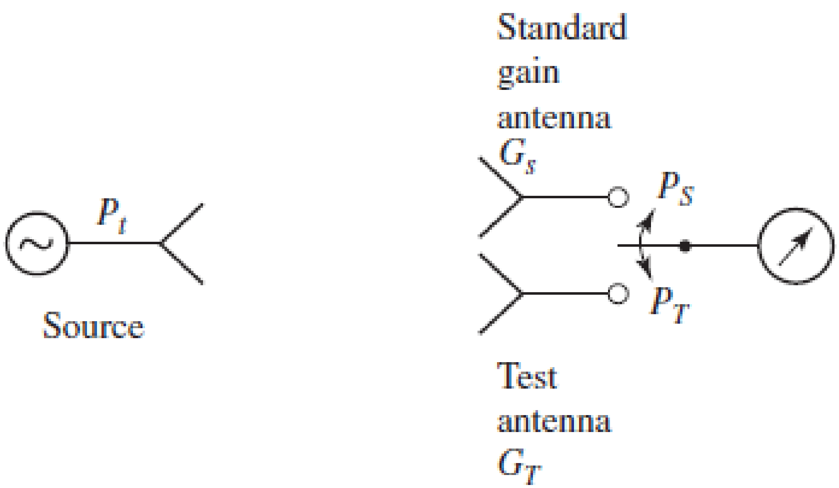
$$G_T = \frac{P_T}{P_S} G_S$$

which is expressed in dB as

$$G_T(\text{dB}) = P_T(\text{dBm}) - P_S(\text{dBm}) + G_S(\text{dB})$$

where

$G_T(\text{dB})$ = gain of the test antenna in dB—calculated
$G_S(\text{dB})$ = gain of the standard gain antenna in dB—known
$P_T(\text{dBm})$ = power received by the test antenna in dBm—measured
$P_S(\text{dBm})$ = power received by the standard gain antenna in dBm—measured



Three-Antenna Method

- Three antenna: *a*, *b*, *c*

- Three measurements: 3 unknows and 3 equations

a (Tx) - *b* (Rx)

b (Tx) - *c* (Rx)

c (Tx) - *a* (Rx)

$$G_a + G_b = 20\log_{10}\left(\frac{4\pi R}{\lambda}\right) + 10\log_{10}\frac{P_{rb}}{P_{ta}}$$

$$G_b + G_c = 20\log_{10}\left(\frac{4\pi R}{\lambda}\right) + 10\log_{10}\frac{P_{rc}}{P_{tb}}$$

$$G_c + G_a = 20\log_{10}\left(\frac{4\pi R}{\lambda}\right) + 10\log_{10}\frac{P_{ra}}{P_{tc}}$$

Fin

(End)