Mutual Coupling in Antenna Arrays

1 Introduction

The **electromagnetic interaction** between the antenna elements in an antenna array is called **mutual coupling**. By its nature, mutual coupling exhibits differently in transmitting and receiving antenna arrays and therefore has to be treated differently.

The effect of mutual coupling is serious if the element spacing is small. It will affect the antenna array mainly in the following ways:

1. change the array radiation pattern
2. change the array manifold (the received element voltages)
3. change the matching characteristic of the antenna elements (change the input impedances)

We will mainly study the first two effects in this chapter. We will consider the change of array radiation pattern in a transmitting antenna array while study the change of the array manifold in a receiving antenna array.
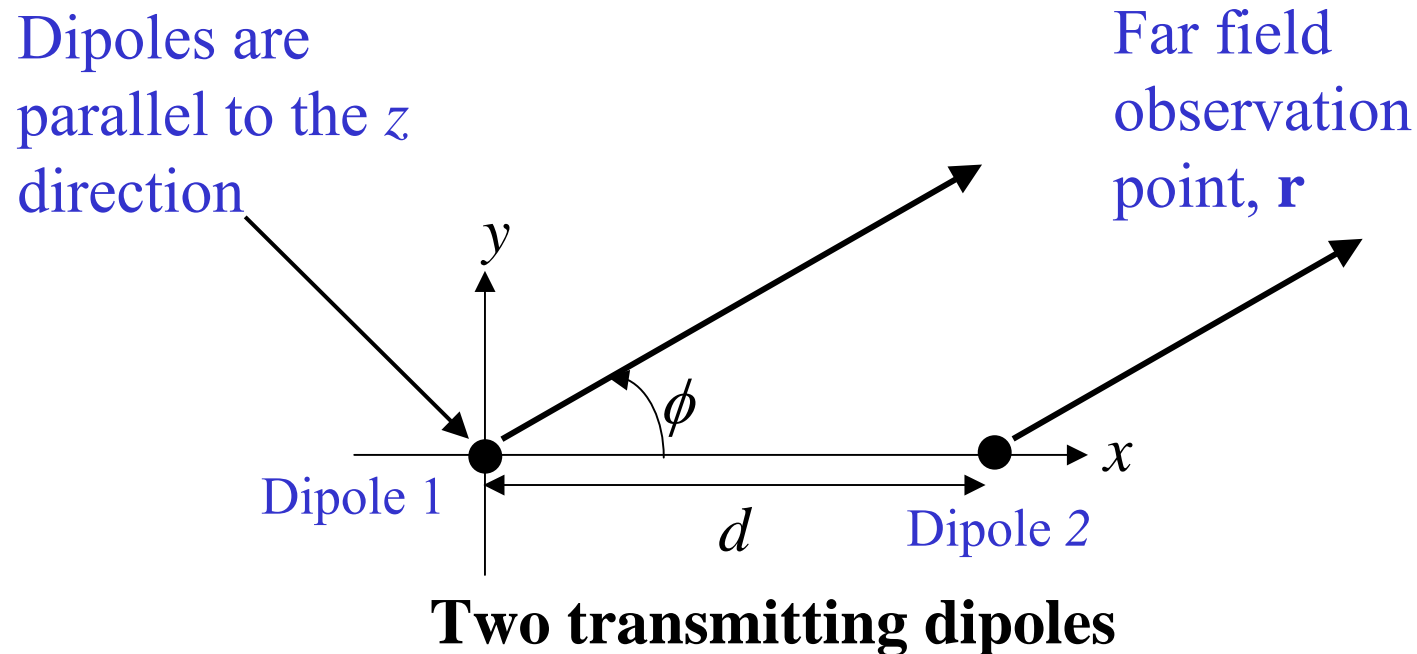
2   Mutual Coupling in Transmitting Antenna Arrays

In a transmitting antenna array, the mutual coupling effect renders the pattern multiplication principle being inapplicable for obtaining the array radiation pattern. This is because it results in the element patterns being not all the same.
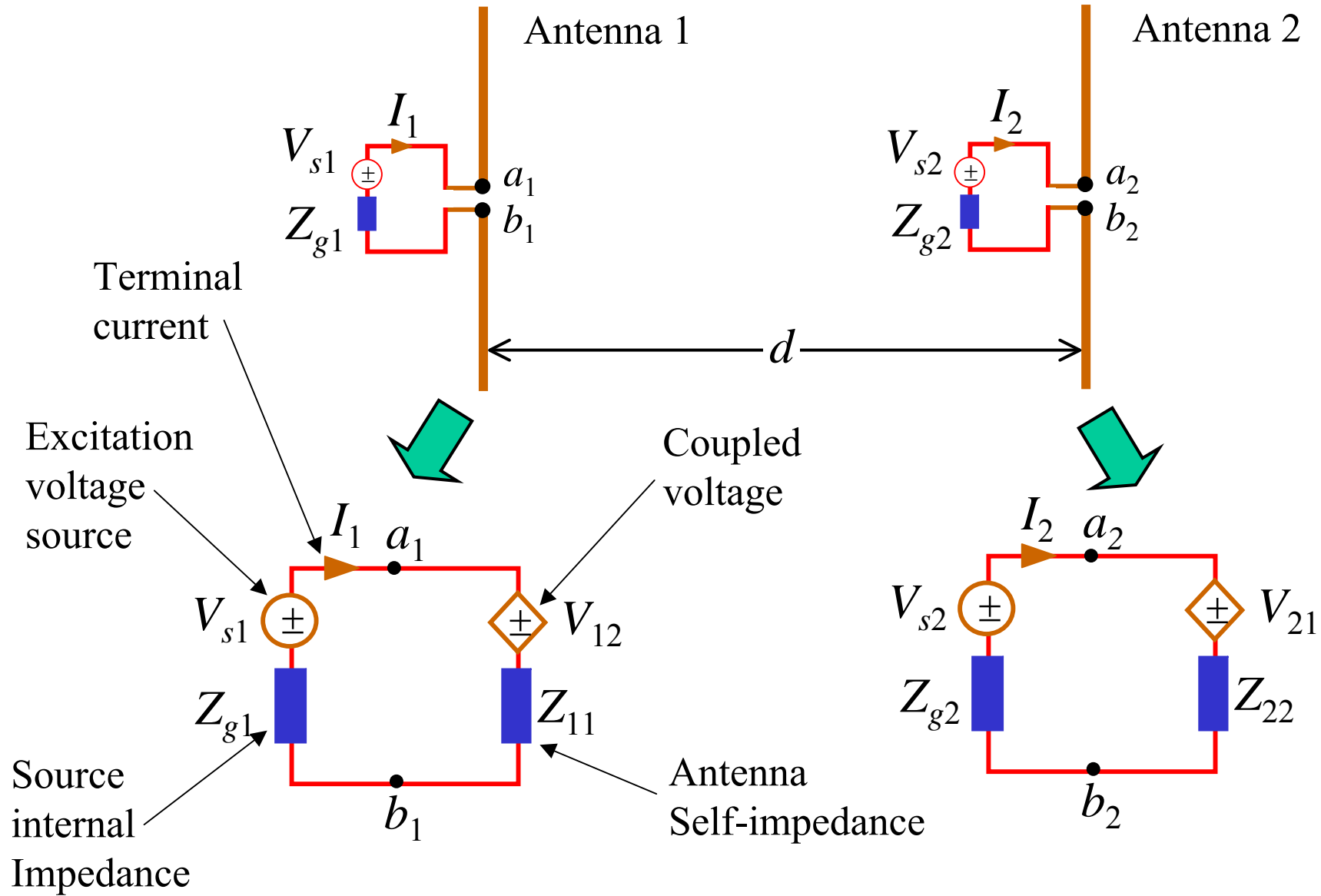
We study an example of a two-element dipole array. We characterize the mutual coupling effect using the **mutual impedance** (the conventional mutual impedance).

2.1 Definition of the Mutual Impedance

Consider two *transmitting* antennas (such as dipoles) as shown on next page. They are separated by a distance of d and the excitation voltage sources, V_{s1} and V_{s2}, have a phase difference of β but an equal magnitude. Hence if there is no mutual coupling effect, the excitation currents also differ by a phase difference of β and have an equal magnitude. When the mutual coupling effect is taken into account, the two coupled antennas can be modelled as two equivalent circuits as shown on page 6.

Dipoles are
parallel to the z
direction

Far field
observation
point, $\mathbf{r}$

y

ϕ

Dipole 1

d Dipole 2

x

Two transmitting dipoles

Now because of the mutual coupling effect, there are two additional excitation sources (the controlled voltage sources) in the equivalent circuits. These controlled voltage sources are to model the coupled voltages induced by the currents on the other antennas (see next page).
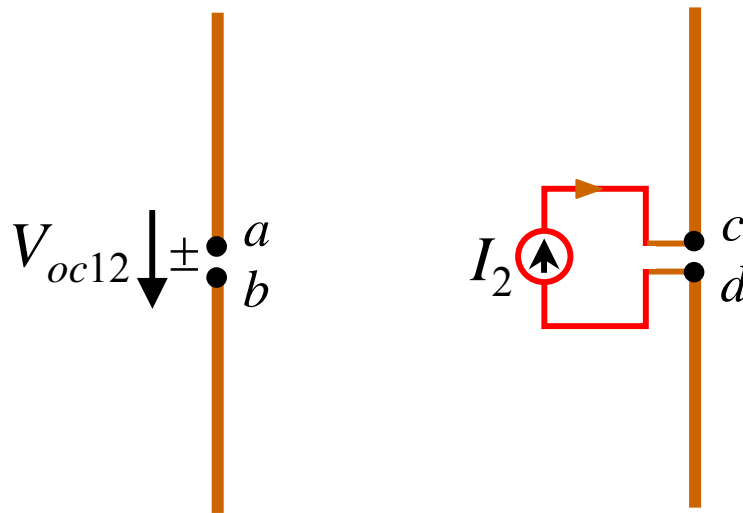
Antenna 1   Antenna 2

I_1 I_2

V_{s1} $\pm$ V_{s2} $\pm$

Z_{g1} a_1 Z_{g2} a_2
                         b_1 b_2

Terminal
current

$\longleftarrow$ d $\longrightarrow$

Excitation
voltage
source   Coupled
                                          voltage

I_1 a_1 I_2 a_2

V_{s1} $\pm$ $\pm$ V_{12} V_{s2} $\pm$ $\pm$ V_{21}

Z_{g1} Z_{11} Z_{g2} Z_{22}

Source
internal
Impedance   Antenna
                                          Self-impedance

b_1 b_2

Z_{12} = mutual impedance with antenna 2 excited

$$= \frac{V_{12}}{I_2}$$

$$= \frac{\text{coupled voltage across antenna 1's open-circuit terminal}}{\text{excitation current at antenna 2's shorted terminal}}$$

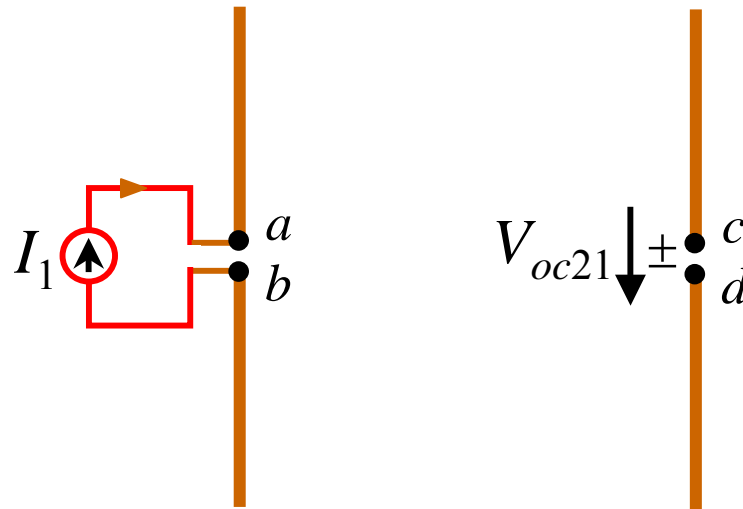$$= \left. \frac{V_{oc12}}{I_2} \right|_{I_1=0, V_{s1}=0} \tag{1}$$

Z_{21} = mutual impedance with antenna 1 excited

$$= \frac{V_{21}}{I_1}$$

$$= \frac{\text{coupled voltage across antenna 2's open-circuit terminal}}{\text{excitation current at antenna 1's shorted terminal}}$$

$$= \frac{V_{oc21}}{I_1}\bigg|_{I_2=0,V_{s2}=0} \tag{2}$$

I_1 a b V_{oc21} $\pm$ c d

Note that for passive antennas, $Z_{12} = Z_{21}$

The mutual impedance can be either measured or theoretically calculated. For measurement, usually the two antennas in the array are treated as a two-port network and its s parameters, S_{11}, S_{12}, S_{21}, and S_{22}, are measured. The z parameters, Z_{11}, Z_{12}, Z_{21}, and Z_{22}, are then obtained from the s parameters as follows:

$$Z_{11} = Z_0 \frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}} \quad Z_{12} = Z_0 \frac{2S_{12}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$$

$$Z_{21} = Z_0 \frac{2S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}} \quad Z_{22} = Z_0 \frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}} \quad (3)$$

where Z_0 is the system impedance. The Z_{12} and Z_{21} of the z parameters are then the mutual impedances.

Alternatively, the mutual impedance can be measured directly using the method described in ref. [1]. A rigorous theoretical method for the calculation of the mutual impedance can be found in ref. [2].

For an N-element antenna array, the mutual impedances can be obtained by considering two antennas at a time. The total mutual impedances of the array, Z_{ij} (i,j=1,2,…,N) will then be the set of two-antenna mutual impedances for all possible pair of antennas in the array.

The following figure is an example which shows the **conventional mutual impedance** Z_{12} between two monopole antennas against their separation at a frequency of 2.4 GHz. The dimensions of the monopole antennas are: monopole length = 3 cm (0.24 λ at 2.4 GHz), radius of the monopole wires = 0.3 mm. Antenna 1 is open-circuited while antenna 2 is connected to a current source (with no terminal load).
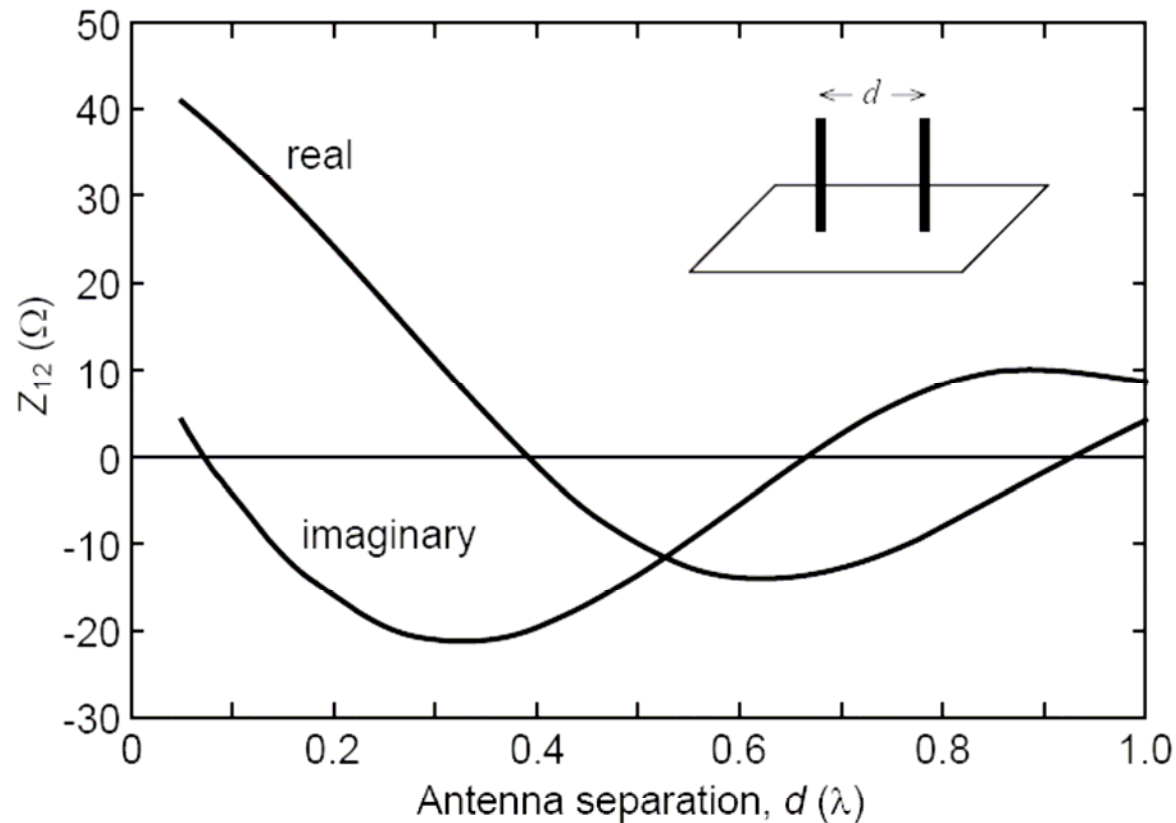
2.2 Effect on the Array Radiation Pattern

Using the mutual impedance, the **coupled voltages** V_{12} and V_{21} can be expressed as follows:

$$V_{12} = Z_{12}I_2 \qquad\qquad V_{21} = Z_{21}I_1 \qquad (4)$$

I_1 and I_2 are the **actual terminal currents** at the antennas when there is mutual coupling effect. From the antenna equivalent circuits,

$$I_1 = \frac{V_{s1} - V_{12}}{Z_{g1} + Z_{11}} \qquad\qquad I_2 = \frac{V_{s2} - V_{21}}{Z_{g2} + Z_{22}} \qquad (5)$$

I_{s1} and I_{s2} are the terminal currents at the antennas when there is no mutual coupling effect.

$$I_{s1} = \frac{V_{s1}}{Z_{g1} + Z_{11}} \qquad I_{s2} = \frac{V_{s2}}{Z_{g2} + Z_{11}} \qquad (6)$$

Our aim is to express I_1 and I_2 in terms of I_{s1} and I_{s2}.

$$I_1 = \frac{V_{s1} - V_{12}}{Z_{g1} + Z_{11}} \qquad I_2 = \frac{V_{s2} - V_{21}}{Z_{g2} + Z_{22}}$$

$$= I_{s1} - \frac{I_2 Z_{12}}{Z_{g1} + Z_{11}} \qquad = I_{s2} - \frac{I_1 Z_{21}}{Z_{g2} + Z_{22}} \qquad (7)$$

From these two relations, we can find:

$$I_1 = \frac{I_{s1} - \dfrac{Z_{12}}{Z_{11} + Z_{g1}} I_{s2}}{\left[1 - \dfrac{Z_{12}Z_{21}}{\left(Z_{11} + Z_{g1}\right)\left(Z_{22} + Z_{g2}\right)}\right]}, \quad I_2 = \frac{I_{s2} - \dfrac{Z_{21}}{Z_{22} + Z_{g2}} I_{s1}}{\left[1 - \dfrac{Z_{12}Z_{21}}{\left(Z_{11} + Z_{g1}\right)\left(Z_{22} + Z_{g2}\right)}\right]}$$

$$(8)$$

That is:

$$I_1 = \frac{1}{D}\left(I_{s1} - Z'_{12}I_{s2}\right) \qquad I_2 = \frac{1}{D}\left(I_{s2} - Z'_{21}I_{s1}\right) \qquad (9)$$

where

$$D = 1 - \frac{Z_{12}Z_{21}}{\left(Z_{11} + Z_{g1}\right)\left(Z_{22} + Z_{g2}\right)} \tag{10}$$

$$Z'_{12} = \frac{Z_{12}}{Z_{11} + Z_{g1}} \tag{11}$$

$$Z'_{21} = \frac{Z_{21}}{Z_{22} + Z_{g2}} \tag{12}$$

Now if we want to find the array pattern **E** on the horizontal plane ($\theta = \pi/2$) with mutual coupling effect, then **E** is just equal to the array factor.

Vector magnitude, not absolute value

$$\|\mathbf{E}\| = AF = \frac{1}{I_1}\left[I_1 + I_2 e^{jkd\cos\phi}\right] \tag{13}$$

$$\|\mathbf{E}\| = \frac{1}{I_1}\left[I_1 + I_2 e^{jkd\cos\phi} \right]$$

$$= \frac{1}{I_1 D}\left[\left(I_{s1} - Z'_{12}I_{s2} \right) + \left(I_{s2} - Z'_{21}I_{s1} \right) e^{jkd\cos\phi} \right]$$

$$= \frac{1}{I_1 D}\left[\left(I_{s1} + I_{s2}e^{jkd\cos\phi} \right) - Z'_{12}\left(I_{s2} + I_{s1}e^{jkd\cos\phi} \right) \right] \ \left(\text{with } Z'_{12} = Z'_{21} \right)$$

$$= \frac{I_{s1}}{I_1 D}\left[\left(1 + e^{j\beta}e^{jkd\cos\phi} \right) - Z'_{12}\left(e^{j\beta} + e^{jkd\cos\phi} \right) \right] \ \left(\text{with } \frac{I_{s2}}{I_{s1}} = e^{j\beta} \right)$$

$$= \frac{I_{s1}}{I_1 D}\left\{ \underbrace{\left[1 + e^{j(kd\cos\phi+\beta)} \right]}_{\text{original pattern}} - Z'_{12}e^{j\beta}\underbrace{\left[1 + e^{j(kd\cos\phi-\beta)} \right]}_{\text{additional pattern}} \right\} \qquad (14)$$

We see that the array pattern now consists of two parts: the original array pattern plus an additional pattern:

$$Z'_{12} e^{j\beta} \left[1 + e^{j(kd\cos\phi - \beta)} \right] \qquad (15)$$

which has a reverse current phase $-\beta$ and a modified amplitude with a multiplication of a complex number $Z'_{12} e^{j\beta}$. Note that all parameters in the above formula can be calculated except I_1 which will be removed after normalization. Normalization of the above formula can only be done when its maximum value is known, for example by numerical calculation.

Example 1

Absolute value

Find the normalized array pattern $|\mathbf{E}_n|$ on the horizontal plane $(\theta=\pi/2)$ of a two-monopole array with the following parameters with mutual coupling taken into account:
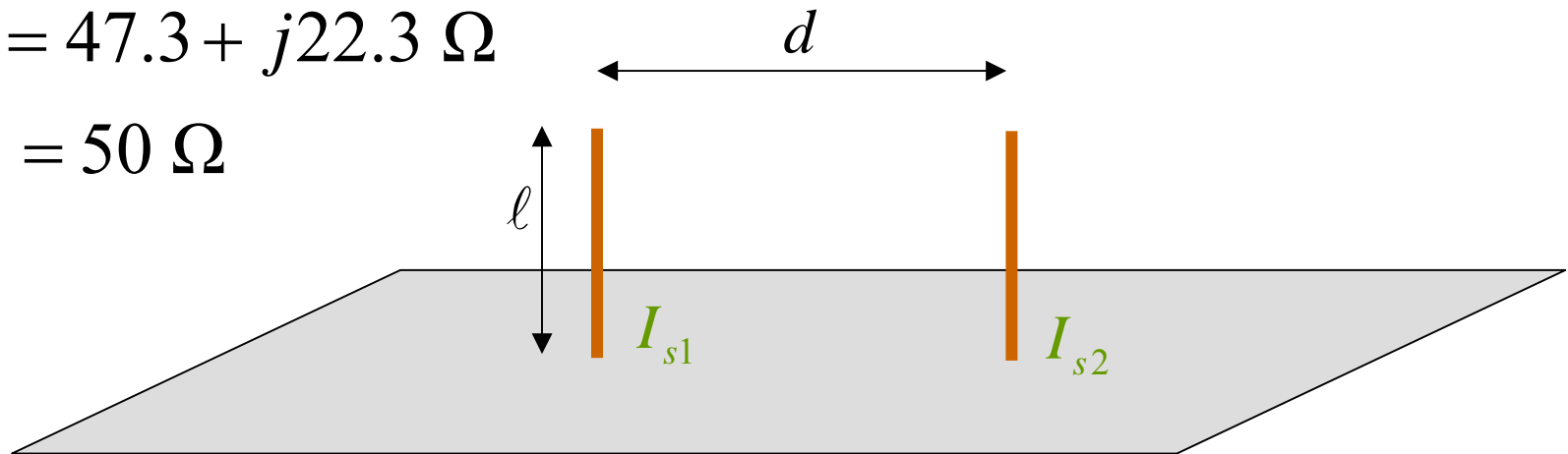
$$I_{s1} = 1, \quad I_{s2} = e^{j\beta}, \quad |I_{s1}| = |I_{s2}| = 1, \quad \beta = 150°$$

$$d = \lambda/4, \quad \ell = \lambda/4$$

$$Z_{12} = Z_{21} = 21.8 - j21.9 \ \Omega$$

$$Z_{11} = Z_{22} = 47.3 + j22.3 \ \Omega$$

$$Z_{g1} = Z_{g2} = 50 \ \Omega$$

Solution

$$I_{s1} = 1, \quad I_{s2} = e^{j\beta}$$

$$\left| I_{s1} \right| = \left| I_{s2} \right| = 1$$

$$\angle I_{s1} = 0°, \quad \angle I_{s2} = \beta = 150° = 2.62 \text{ rad}$$

> **As the required array pattern $|E_n|$ is on the horizontal plane, it is equal to the normalized array factor $|AF_n|$.**

$$kd = \frac{2\pi}{\lambda} \frac{\lambda}{4} = \frac{\pi}{2}$$

$$Z_{12}' = \frac{Z_{12}}{Z_{11} + Z_{g1}} = \frac{Z_{21}}{Z_{22} + Z_{g2}} = 0.16 - j0.26$$

$$D = 1 - \frac{Z_{12} Z_{21}}{\left(Z_{11} + Z_{g1} \right) \left(Z_{22} + Z_{g2} \right)} = 1.042 + j0.09$$

$$\left|\mathbf{E}\right| = \left|\mathrm{AF}\right| = \left|\frac{I_{s1}}{I_1 D}\left\{\left[1 + e^{j(kd\cos\phi+\beta)}\right] - Z_{12}'e^{j\beta}\left[1 + e^{j(kd\cos\phi-\beta)}\right]\right\}\right|$$

$$= \left|\frac{0.95 - j0.08}{I_1}\left[1 + e^{j2.62}e^{j(\pi/2)\cos\phi}\right.\right.$$

$$\left.\left. -(0.16 - j0.26)e^{j2.62}\left(1 + e^{-j2.62}e^{j(\pi/2)\cos\phi}\right)\right]\right|$$

$$= \left|\frac{0.94 - j0.37}{I_1}\right|\left|1 + (-1.14 + j0.40)e^{j(\pi/2)\cos\phi}\right|$$

The pattern of $f = \left|1 + (-1.14 + j0.40)e^{j(\pi/2)\cos\phi}\right|$ is shown on next page.
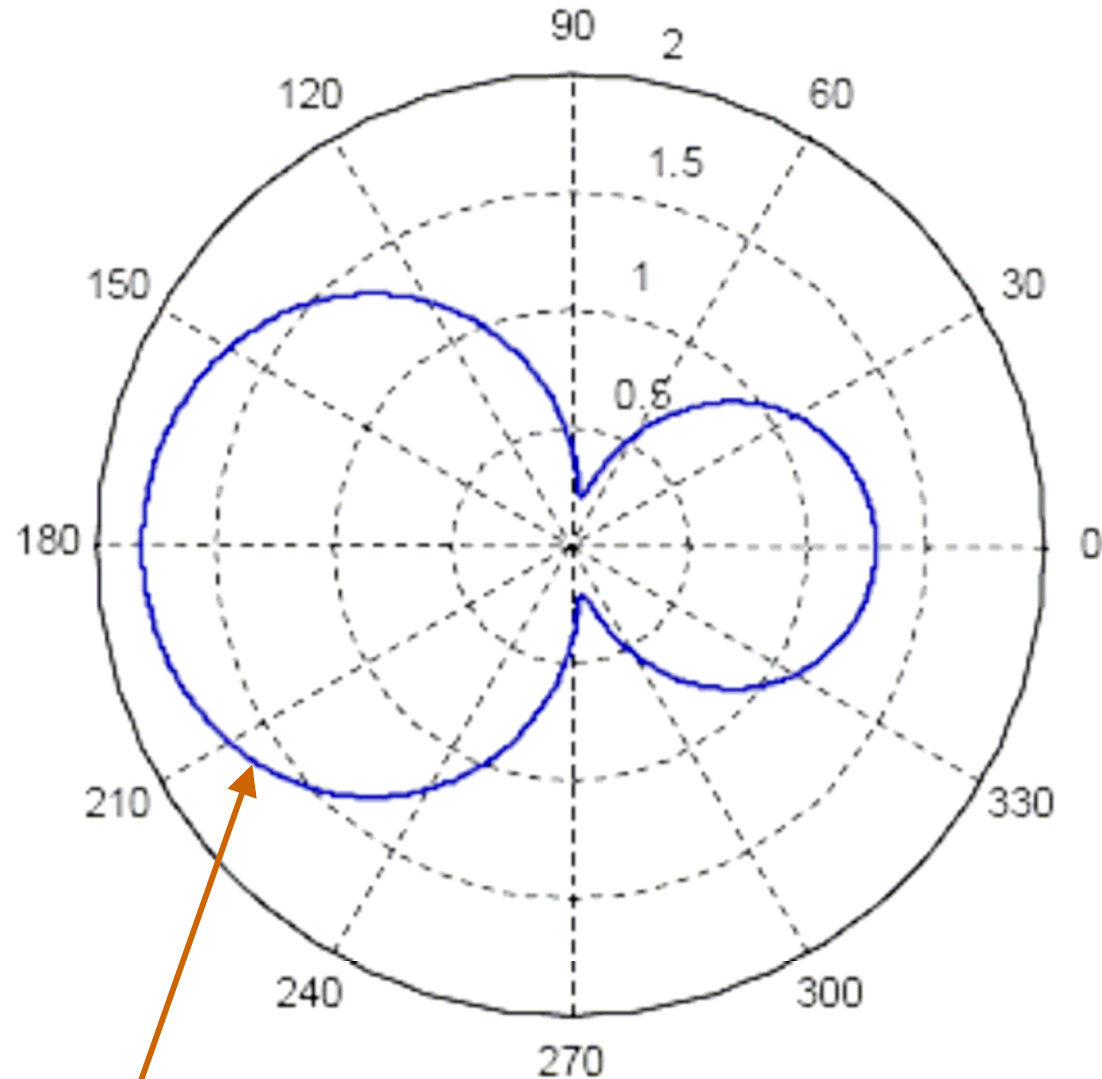
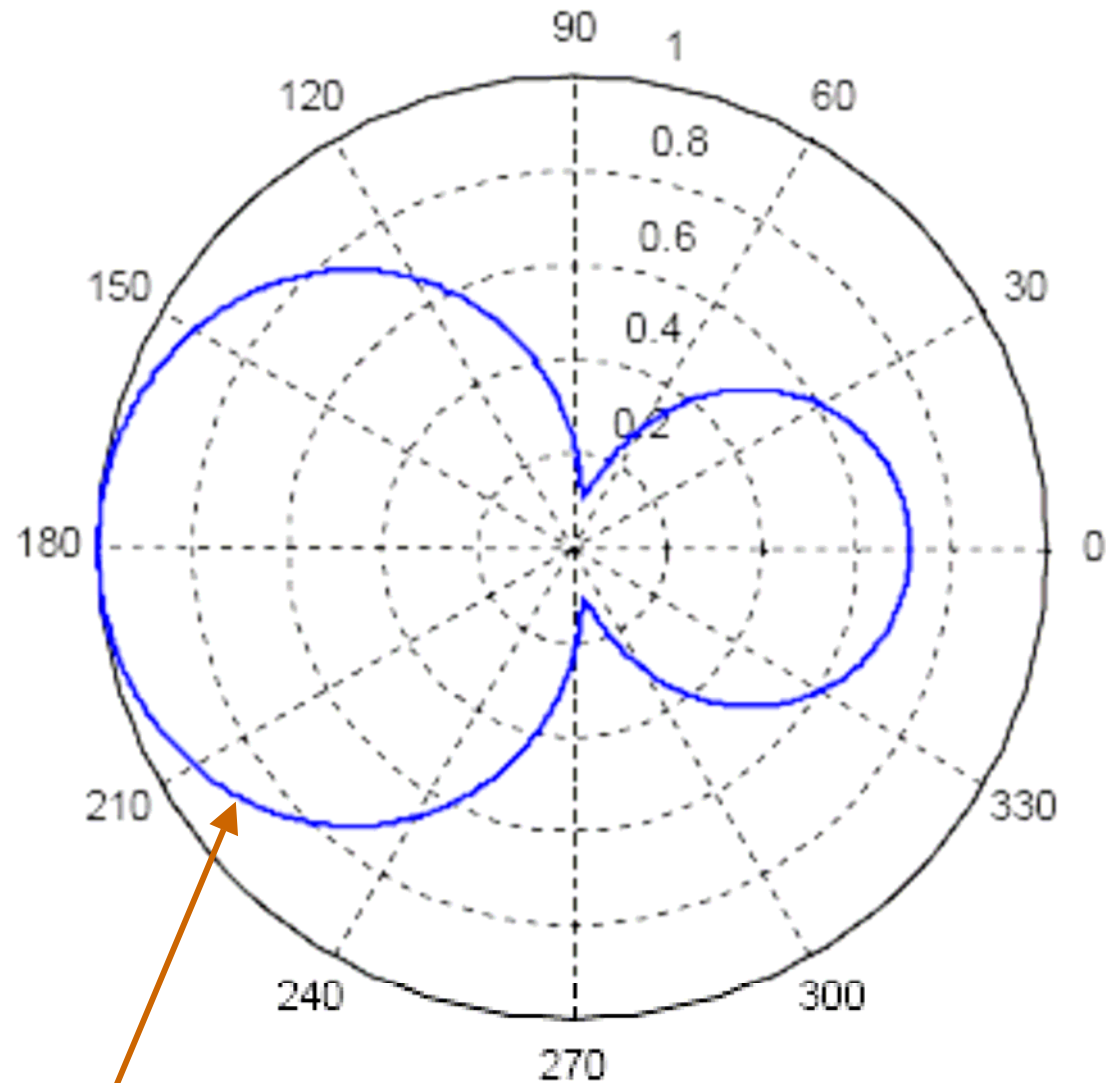$$f = \left| 1 + \left(-1.14 + j0.40 \right) e^{j(\pi/2)\cos\phi} \right|$$

Normalization

The pattern of f attains the maximum value when $\phi = 180°$. When $\phi = 180°$,

$$\left| \mathbf{E} \right|_{\phi=180°}$$

$$= \left| \frac{0.94 - j0.37}{I_1} \right| \left| 1 + \left(-1.14 + j0.40 \right) e^{j(\pi/2)\cos\phi} \right|_{\phi=180°}$$

$$= \frac{1.83}{\left| I_1 \right|}$$

Hence we normalize |$\mathbf{E}$| by this factor $(1.83/|I_1|)$ to get:

$$\left| \mathbf{E}_{n} \right| = \frac{\left| \dfrac{0.94 - j0.37}{I_{1}} \left[1 + \left(-1.14 + j0.40 \right) e^{j(\pi/2)\cos\phi} \right] \right|}{\dfrac{1.83}{\left| I_{1} \right|}}$$

$$= 0.52 \left| 1 + \left(-1.14 + j0.40 \right) e^{j(\pi/2)\cos\phi} \right|$$

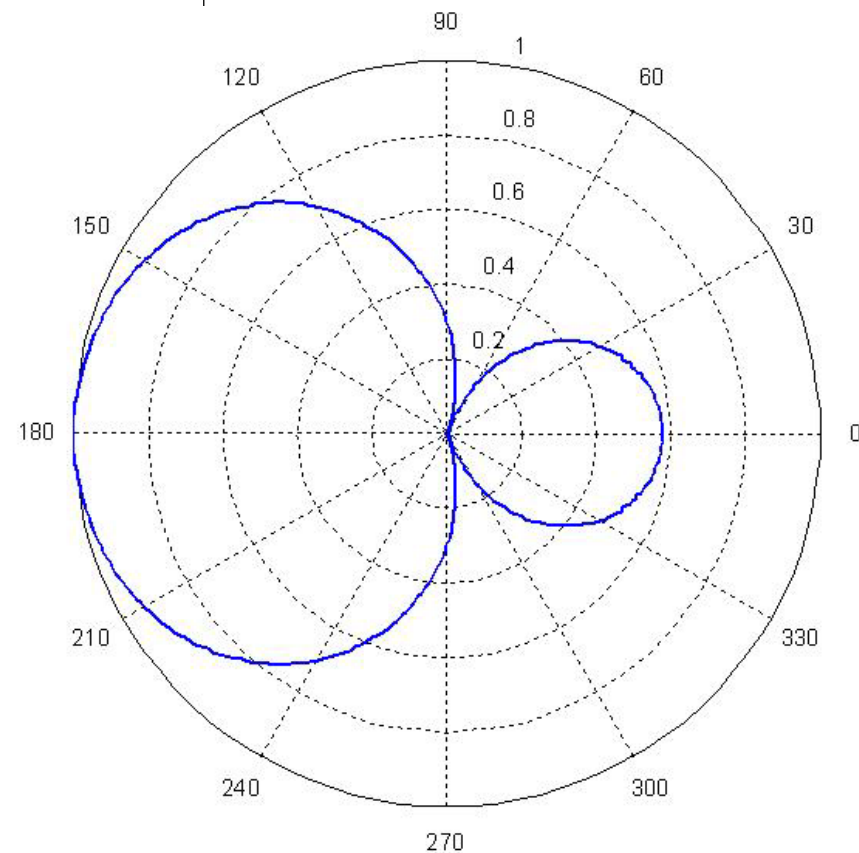The polar plot of $|\mathbf{E}_{n}|$ is shown on next page.

$$\left|\mathbf{E}_n\right| = 0.52\left|1+\left(-1.14+j0.40\right)e^{j(\pi/2)\cos\phi}\right|$$

The case when there is no mutual coupling is shown below for comparison.

$$\left. \mathbf{E}_n \right|_{\text{no mutual coupling effect}} = \frac{1}{\Gamma}\left| 1 + e^{j\beta}e^{jkd\cos\phi} \right|$$

where Γ is a constant to make the largest value of $|AF_n|$ equal to one ($\Gamma = 1.73$).

2.3 Effect on the Array Manifold

The total voltages (due to the excitation sources and the coupled voltages) V_{tot1} and V_{tot2} across the *whole lengths* of the two antenna elements can be expressed as the summation of the excitation voltage V_{s1} and V_{s2} due to the excitation sources alone and the coupled voltages V_{12} and V_{21} as follows:

$$V_{tot1} = V_{s1} - V_{12} \qquad (16)$$

$$V_{tot2} = V_{s2} - V_{21} \qquad (17)$$

The total voltages V_{tot1} and V_{tot2} can be expressed in terms of the terminal currents I_1 and I_2.

$$V_{tot1} = I_1 \left(Z_{g1} + Z_{11} \right) \qquad (18)$$

$$V_{tot2} = I_2 \left(Z_{g2} + Z_{22} \right) \qquad (19)$$

From (16) & (17),

$$V_{s1} = V_{tot1} + V_{12} = V_{tot1} + I_2 Z_{12} \qquad (20)$$

$$V_{s2} = V_{tot2} + V_{21} = V_{tot2} + I_1 Z_{21} \qquad (21)$$

Putting (18) & (19) into (20) & (21), we have

$$V_{s1} = V_{tot1} + \frac{Z_{12}}{\left(Z_{g2} + Z_{22} \right)} V_{tot2} \qquad (22)$$

$$V_{s2} = V_{tot2} + \frac{Z_{21}}{\left(Z_{g1} + Z_{11} \right)} V_{tot1} \qquad (23)$$

Hence when there is mutual coupling, the total excitation voltages for the two antennas, V_{tot1} and V_{tot2}, are related to the excitation voltages without mutual coupling, V_{s1} and V_{s2}, by the following matrix formula:

$$\begin{bmatrix} V_{s1} \\ V_{s2} \end{bmatrix} = \begin{bmatrix} 1 & \dfrac{Z_{12}}{Z_{g2}+Z_{22}} \\ \dfrac{Z_{21}}{Z_{g1}+Z_{11}} & 1 \end{bmatrix} \begin{bmatrix} V_{tot1} \\ V_{tot2} \end{bmatrix} \qquad (24)$$

For an N-element transmitting antenna array, the uncoupled array manifold V_{s1}, V_{s2}, …, and V_{sN} (the uncoupled excitation voltages) is related to the coupled array manifold V_{tot1}, V_{tot2}, …, and V_{totN} (contaminated with the coupled voltages) by the following formula:

$$
\begin{bmatrix} V_{s1} \\ V_{s2} \\ \vdots \\ V_{sN} \end{bmatrix} = \begin{bmatrix} 1 & \dfrac{Z_{12}}{Z_{g2}+Z_{22}} & \cdots & \dfrac{Z_{1N}}{Z_{gN}+Z_{NN}} \\ \dfrac{Z_{21}}{Z_{g1}+Z_{11}} & 1 & \cdots & \dfrac{Z_{2N}}{Z_{gN}+Z_{NN}} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{Z_{N1}}{Z_{g1}+Z_{11}} & \dfrac{Z_{N2}}{Z_{g2}+Z_{22}} & \cdots & 1 \end{bmatrix} \begin{bmatrix} V_{tot1} \\ V_{tot2} \\ \vdots \\ V_{totN} \end{bmatrix} \quad (25)
$$

3 Mutual Coupling in Receiving Antenna Arrays

In a receiving antenna array, strictly speaking, we cannot use the conventional mutual impedances to measure the mutual coupling effect (see refs. [3]-[7]). This is because the excitation source for a receiving antenna array is at the far-field zone and not at the terminals of the antenna elements. Usually the current distributions on the antenna elements (which cause the mutual coupling) in a receiving antenna array are different from those on the antenna elements in a transmitting antenna array.

Hence we need to re-define the mutual impedances in a receiving antenna array as follows:

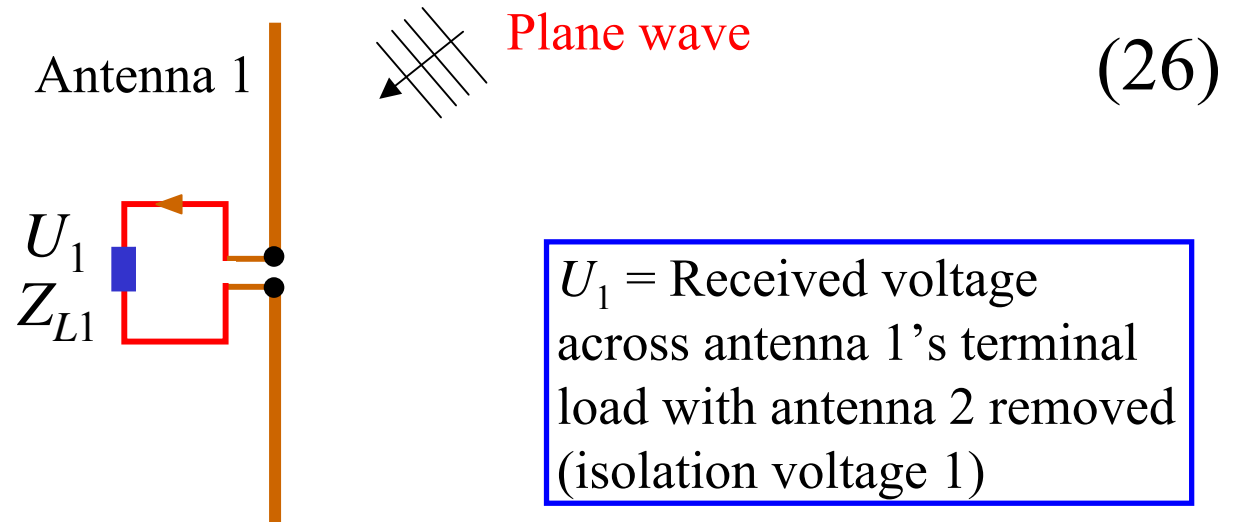# 3.1 Definition of the Receiving Mutual Impedance
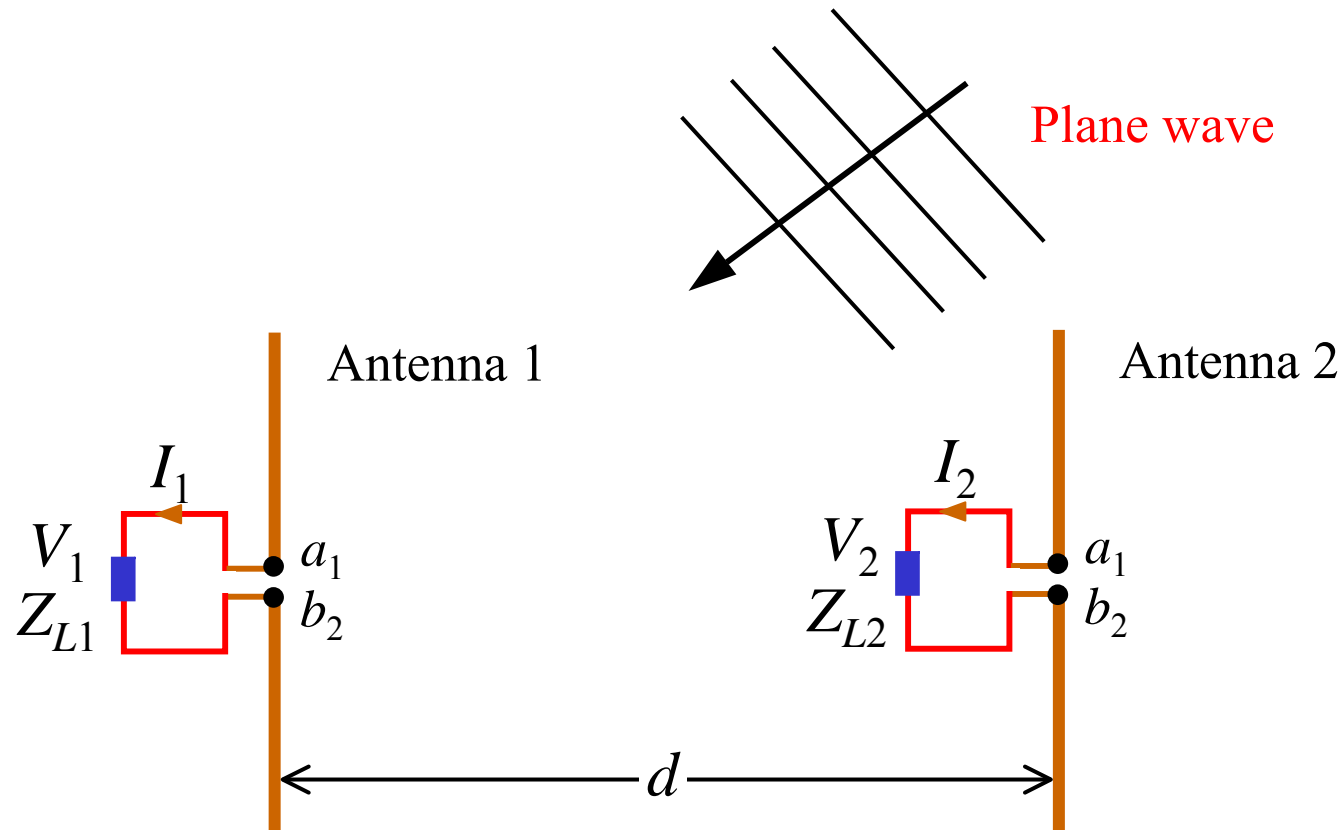
The scenario for defining the receiving mutual impedance

$$Z_t^{12} = \text{receiving mutual impedance with a receiving}$$

$$\text{current (} current\ distribution \text{) at antenna 2}$$

$$= -\frac{V_{12}}{I_2}$$

$$= \frac{\text{coupled voltage across antenna 1's terminal load}}{\text{receiving current through antenna 2's terminal load}}$$
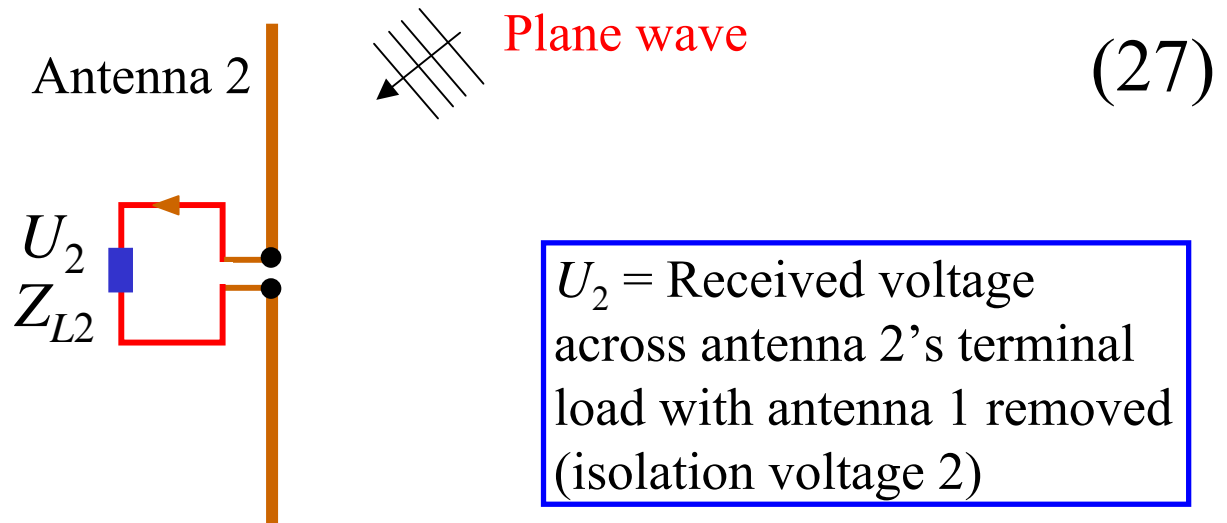
$$= -\frac{V_1 - U_1}{I_2} \tag{26}$$

Antenna 1

Plane wave

U_1

Z_{L1}

U_1 = Received voltage across antenna 1's terminal load with antenna 2 removed (isolation voltage 1)

$$Z_t^{21} = \text{receiving mutual impedance with a receiving}$$

$$\text{current (} current\ distribution \text{) at antenna 1}$$

$$= -\frac{V_{21}}{I_1}$$

$$= \frac{\text{coupled voltage across antenna 2's terminal load}}{\text{receiving current through antenna 1's terminal load}}$$

$$= -\frac{V_2 - U_2}{I_1} \qquad (27)$$

Antenna 2

Plane wave

U_2
Z_{L2}

U_2 = Received voltage across antenna 2's terminal load with antenna 1 removed (isolation voltage 2)

Note that in general, the definition of the receiving mutual impedance implies that the receiving mutual impedance is dependent on the direction of the plane wave, which is used as the excitation source. But for omni-directional antennas such as dipole and monopole antennas, the receiving mutual impedance is independent of the azimuth angle of the excitation plane wave.

Furthermore, for a general antenna array, as the excitation source is not applied to the feeding ports of the antennas directly (the antennas being excited by an external plane wave source), reciprocity of the receiving mutual impedances does not hold and hence $Z_t^{ij} \neq Z_t^{ji}$ in general.

3.2 Measurement of the Receiving Mutual Impedance

The receiving mutual impedance can be either measured or theoretically calculated. For measurement, the antenna array has to be placed inside an anechoic chamber and its received power wave (the S_{12} parameter) is measured by a VNA machine. As an example, for a typical two-element monopole antenna array, the following procedure is used:

1. Use the transmitting antenna in the anechoic chamber as the plane wave source and place the monopole array at a fixed position (not rotating). (See the setup on next page.)

transmitting antenna

monopole antennas

ground plane

Z_L

Network analyser
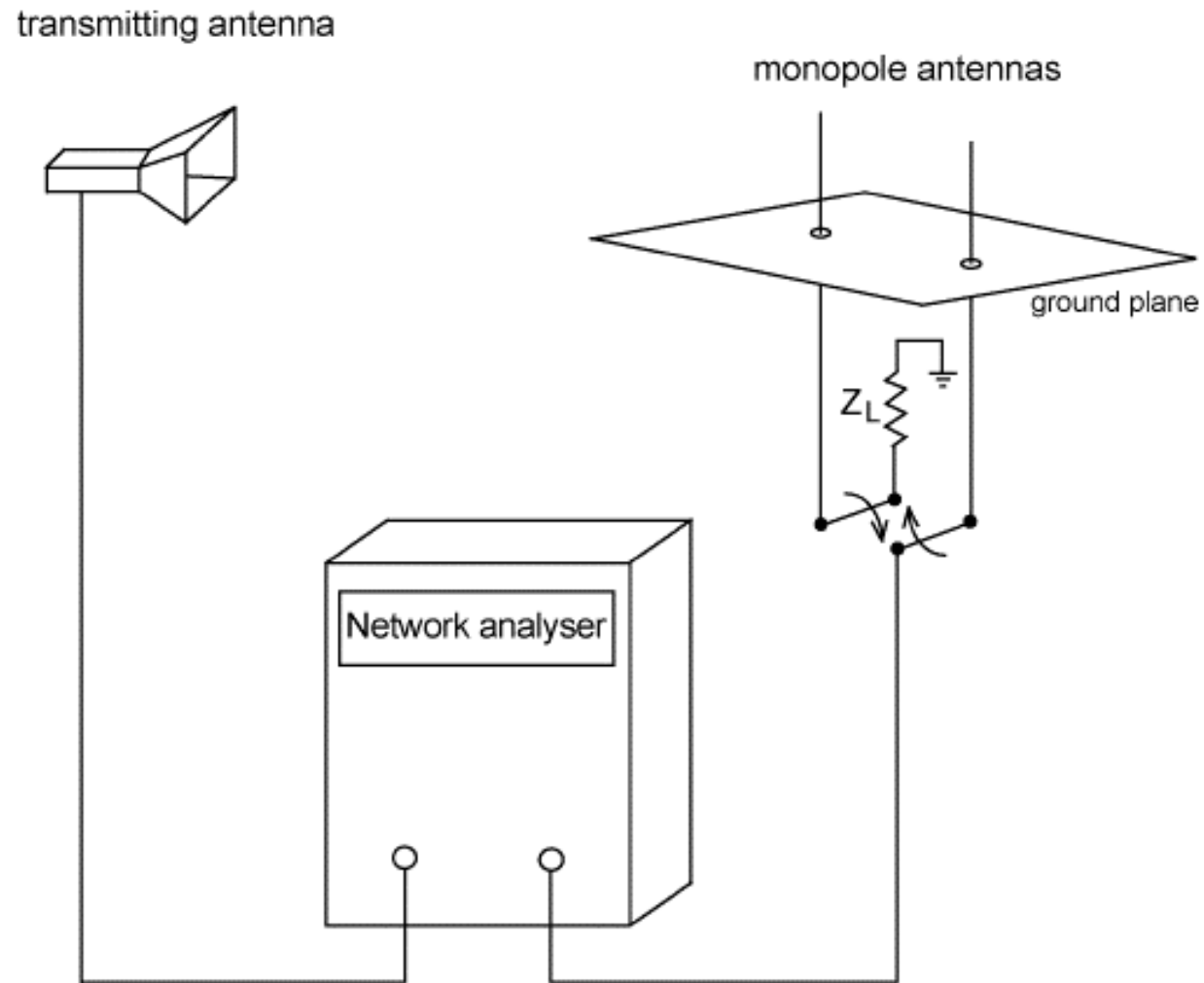
The measurement of the receiving mutual impedances

2. Measure S_{21} at monopole 1's terminal with monopole 2's terminal connected to a terminal load. Denote this as S_{21_1}.

3. Measure S_{21} at monopole 2's terminal with monopole 1's terminal connected to a terminal load. Denote this as S_{21_2}.

4. Measure S_{21} at monopole 1's terminal with monopole 2 removed (taken away from the array). Denote this as S'_{21_1}.

5. Measure S_{21} at monopole 2's terminal with monopole 1 removed (taken away from the array). Denote this as S'_{21_2}.

Throughout the measurement, the relative positions of the two monopoles must not be changed with respect to the transmitting antenna as this change will alter the phase of S_{12}. By definition,

$$S_{21} = \frac{\beta}{\alpha} \tag{28}$$

where α is the square root of the power input to the transmitting antenna and $\beta = V / \sqrt{Z_0}$ is the square root of the power received by a monopole with V being the terminal voltage of the monopole and Z_0 being the system impedance. Both α and β are complex values with magnitudes and phases.

Now convert the measured S_{21}s into relative voltages as:

V_1 = total received terminal voltage on monopole 1

$$= S_{21\_1}\alpha\sqrt{Z_0} \tag{29}$$

V_2 = total received terminal voltage on monopole 2

$$= S_{21\_2}\alpha\sqrt{Z_0} \tag{30}$$

U_1 = isolation voltage on monopole 1

$$= S'_{21\_1}\alpha\sqrt{Z_0} \tag{31}$$

U_2 = isolation voltage on monopole 2

$$= S'_{21\_2}\alpha\sqrt{Z_0} \tag{32}$$

Then the measured receiving mutual impedances are obtained as follows:

$$Z_t^{12} = -\frac{V_1 - U_1}{I_2} = \frac{V_1 - U_1}{V_2/Z_0} = \frac{S_{21\_1} - S'_{21\_1}}{S_{21\_2}} Z_0 \quad (33)$$

$$Z_t^{21} = -\frac{V_2 - U_2}{I_1} = \frac{V_2 - U_2}{V_1/Z_0} = \frac{S_{21\_2} - S'_{21\_2}}{S_{21\_1}} Z_0 \quad (34)$$

For an N-element antenna array, the mutual impedances can be measured by considering two antennas at a time using the above procedure and repeating it for all the possible pairs of elements in the array.

The following figure is an example which shows the **receiving mutual impedance** Z_t^{12} between two monopole antennas against their separation at a frequency of 2.4 GHz. The dimensions of the monopole antennas are: monopole length = 3 cm (0.24 λ at 2.4 GHz), radius of the monopole wires = 0.3 mm. Each monopole antenna is connected to a terminal load of Z_L = 50 Ω. The external plane wave used to excite the array comes from the horizontal direction with $\theta = 90°$ and an arbitrary value of ϕ as the received currents on the two monopoles are independent of ϕ.
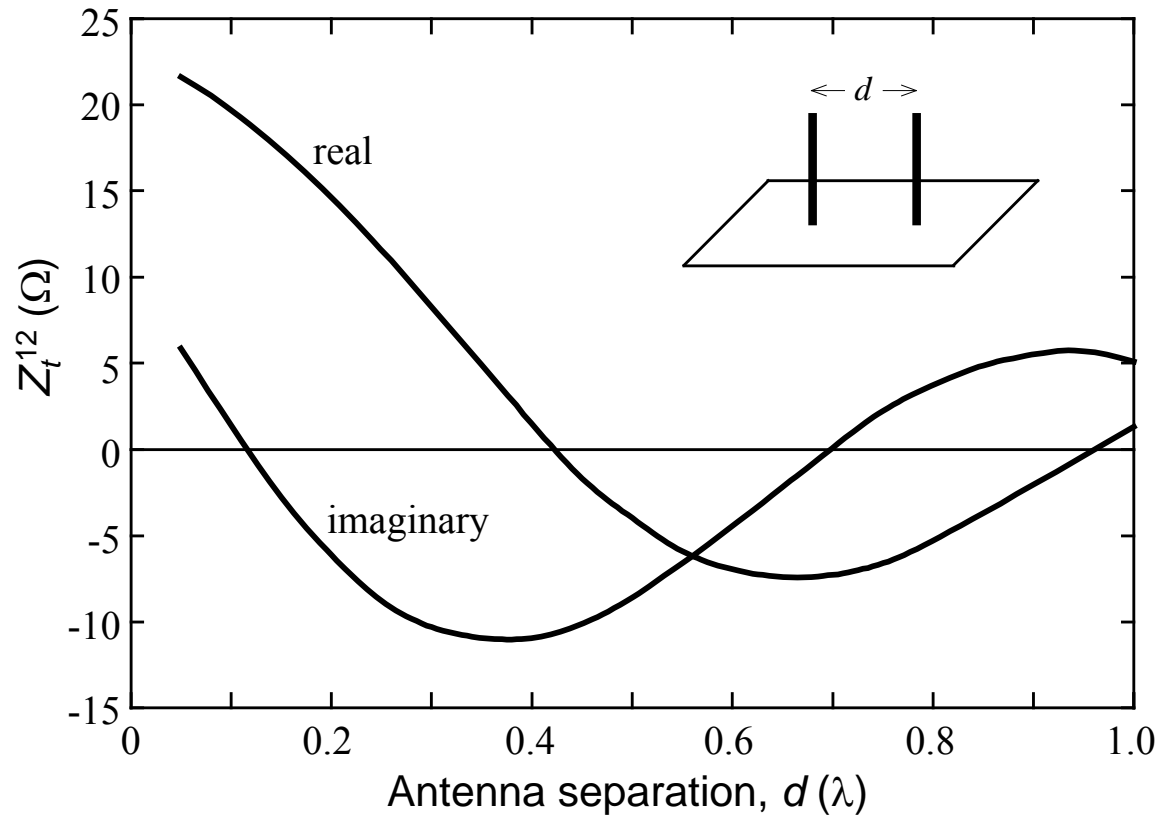
3.3 Effect on the Array Manifold

Using the receiving mutual impedances, the total received voltages V_1 and V_2 (contaminated with the coupled voltages) on the antenna elements can be expressed as the summation of the isolated terminal voltages U_1 and U_2 (without coupled voltages) and the coupled voltages V_{12} and V_{21} as follows:

$$V_1 = U_1 + V_{12} = U_1 - Z_t^{12} I_2 = U_1 + Z_t^{12} \frac{V_2}{Z_L} \qquad (35)$$

$$V_2 = U_2 + V_{21} = U_2 - Z_t^{21} I_1 = U_2 + Z_t^{21} \frac{V_1}{Z_L} \qquad (36)$$

For an N-element receiving antenna array, the uncoupled array manifold U_1, U_2, …, and U_N (the isolation terminal voltages) can be calculated from the received array manifold V_1, V_2, …, and V_N (contaminated with the coupled voltages) using the following formula:

$$
\begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_N \end{bmatrix} = \begin{bmatrix} 1 & -\dfrac{Z_t^{12}}{Z_L} & \cdots & -\dfrac{Z_t^{1N}}{Z_L} \\ -\dfrac{Z_t^{21}}{Z_L} & 1 & \cdots & -\dfrac{Z_t^{2N}}{Z_L} \\ \vdots & \vdots & \ddots & \vdots \\ -\dfrac{Z_t^{N1}}{Z_L} & -\dfrac{Z_t^{N2}}{Z_L} & \cdots & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} \qquad (37)
$$

Example 2

A seven-element monopole array is used to detect the directions of two incoming signals (plane waves) at $\phi_1 = 62.4°$ and $\phi_2 = 111.9°$, respectively. The received antenna terminal voltages, V_1, V_2, V_3, V_4, V_5, V_6, and V_7 are analyzed by the direction-finding algorithm, MUSIC. The result is shown on next page. Three MUSIC power spectra are obtained with the uncoupled voltages determined by three different methods: (i) using the uncoupled voltages calculated by the receiving mutual impedances, (ii) using the uncoupled voltages calculated by the (conventional) mutual impedances, and (iii) using the coupled voltages directly. The average SNR of the received voltages is 39.1 dB.
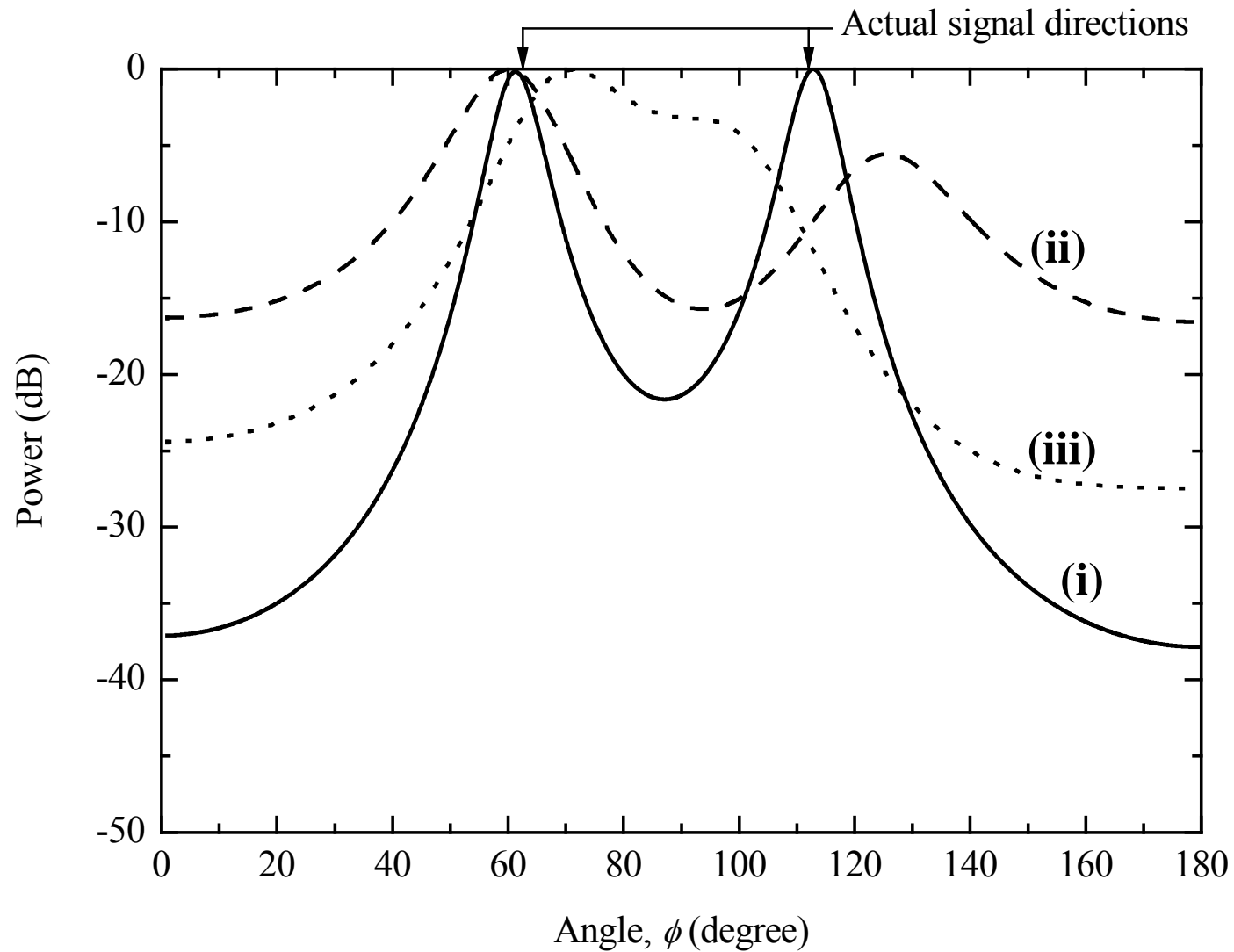
The MUSIC power spectra for the estimation of two signal sources at $\phi_1 = 62.4°$ and $\phi_2 = 111.9°$

From this figure, it shows that the case of using the receiving mutual impedances (case (i)) for calculating the uncoupled voltages produces the most accurate direction finding results. As direction finding arrays are typical receiving arrays, this example tells that the antenna mutual coupling effect in a receiving antenna array has to be analyzed by using the receiving mutual impedances and not the (conventional) mutual impedances.

(For more details on this example, please see ref. [8].)

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