Lecture 2

Review of Maxwell's Equations, EM Energy and Power

Optional Reading: Steer – Appendix D, or Pozar – Section 1.2,1.6, or any text on Engineering Electromagnetics (e.g., Hayt/Buck)

Time-domain Maxwell's Equations: Faraday's Law

- Faraday's law in integral form

$$V = -E = -\frac{d\Phi}{dt} \Rightarrow \boxed{\oint_C \mathbf{E}\cdot d\mathbf{l} = -\frac{d}{dt}\iint_{S_C} \mathbf{B}\cdot d\mathbf{s}}$$

electromotive force (EMF)

Are the two expressions below equivalent? If not, can you state what the relationship is between the two?

$$\frac{d}{dt}\iint_{S_C} \mathbf{B}\cdot d\mathbf{s} \overset{?}{=} \iint_{S_C} \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{s}$$

Hayt/Buck, section 9.1

Hint: Consider both the *transformer* EMF and the *motional* EMF.

- Faraday's law in differential (point-wise) form

$$\iint_{S_C} (\nabla\times\mathbf{E})\cdot d\mathbf{s} = -\iint_{S_C} \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{s} \quad \Rightarrow \quad \boxed{\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}}$$

Time-domain Maxwell’s Equations: Ampère’s Law

- Ampère’s law in integral form

$$\oint_C \mathbf{H}\cdot d\mathbf{l} = \iint_{S_C}\left(\mathbf{J}+\frac{\partial \mathbf{D}}{\partial t}\right)\cdot d\mathbf{s} = \overset{\textit{source}}{I^s} + \overset{\textit{conduction}}{I_\sigma} + \underset{\textit{displacement}}{I_D}$$

- Ampère’s law in differential (point-wise) form

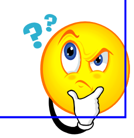
$$\nabla\times\mathbf{H} = \underbrace{\mathbf{J}^s + \overbrace{\sigma\mathbf{E}}^{\mathbf{J}_\sigma}}_{\mathbf{J}} + \underbrace{\frac{\partial \mathbf{D}}{\partial t}}_{\mathbf{J}_D}$$

Maxwell’s correction: displacement current

Show that Ampère’s law of magnetostatics does not hold for time-varying EM fields (it is inconsistent with the conservation of charge).

$$\nabla\times\mathbf{H} = \mathbf{J} \overset{?}{\Leftrightarrow} \nabla\cdot\mathbf{J} = -\frac{\partial \rho}{\partial t}$$

Show that Ampère’s law with Maxwell’s correction observes the conservation of charge for time-varying fields.



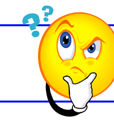
All Four Time-domain Maxwell's Equations

the curl MEs	integral form	differential form
Faraday's Law	$\oint_C \mathbf{E}\cdot d\mathbf{l} = -\iint_{S_C} \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{s}$	$\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
Ampère's Law	$\oint_C \mathbf{H}\cdot d\mathbf{l} = \iint_{S_C} \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\right)\cdot d\mathbf{s}$	$\nabla\times\mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \underbrace{\sigma\mathbf{E} + \mathbf{J}^i}_{\mathbf{J}}$
Gauss' Law of Electricity	$\oiint_S \mathbf{D}\cdot d\mathbf{s} = \iiint_{V_S} \rho_v dv = Q_{\text{free}}$	$\nabla\cdot\mathbf{D} = \rho_v$
Gauss' Law of Magnetism	$\oiint_S \mathbf{B}\cdot d\mathbf{s} = 0$	$\nabla\cdot\mathbf{B} = 0$

Gauss laws follow from the conservation of charge and the curl MEs

Time-domain Maxwell’s Equations and Charge Conservation

Prove that Gauss’ law of electricity follows from Ampère’s law and the conservation of electrical charge.



Prove that Gauss’ law of magnetism follows from Faraday’s law.

Constitutive Relations

- Maxwell's equations are 4 but only 2 of them are independent (the two curl equations)
- there are 4 unknown vectors in the 2 curl Maxwell equations – we need two more vector equations for a complete solution
- the constitutive EM equations are essential in describing the EM field interaction with matter
  - in vacuum (SI): $\mathbf{D} = \varepsilon_0 \mathbf{E},\ \mathbf{J} = 0,\ \mathbf{B} = \mu_0 \mathbf{H}$

$$\mu_0 = 4\pi \times 10^{-7}\ \text{H/m}$$

$$\varepsilon_0 \approx 8.854187817 \times 10^{-12}\ \text{F/m}$$

$$\varepsilon_0 = \frac{1}{c^2 \mu_0},\ c = 2.99792458\ldots \times 10^8\ \text{m/s}$$

  - in matter:

$$\mathbf{D} = F_p(\mathbf{E}, \mathbf{H}),\ \mathbf{J} = F_c(\mathbf{E}, \mathbf{H}),\ \mathbf{B} = F_m(\mathbf{E}, \mathbf{H})$$

Constitutive Relations (2)

- in microwave engineering we often assume that materials are isotropic, linear and dispersion-free:

$$\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P} = \varepsilon_0\varepsilon_r\mathbf{E},\ \ \varepsilon_r = 1 + \chi_e$$
$$\mathbf{J}_\sigma = \sigma\mathbf{E}$$
$$\mathbf{B} = \mu_0\mathbf{H} + \mathbf{M} = \mu_0\mu_r\mathbf{H},\ \mu_r = 1 + \chi_m$$

- the above assumption does not hold for many materials, for example
  - water and living tissue
  - plasma
  - magnetized plasma
  - ferrites
  - piezoelectric crystals

Constitutive Relations (3)

Describe how the constitutive relations of the following types of materials are described mathematically

- heterogeneous
- nonlinear
- anisotropic
- bi-anisotropic
- dispersive

Each of the materials listed as examples in the previous slide is either heterogeneous, or nonlinear, or anisotropic, or dispersive. Which property applies to which material?

Time-harmonic EM Analysis: Field Phasors

$$\boxed{\begin{aligned}\mathbf{E}(x,y,z,t) &= \mathrm{Re}\left\{\tilde{\mathbf{E}}(x,y,z)e^{j\omega t}\right\}\\ \mathbf{H}(x,y,z,t) &= \mathrm{Re}\left\{\tilde{\mathbf{H}}(x,y,z)e^{j\omega t}\right\}\end{aligned}}$$

- time-domain field vector

$$\begin{aligned}\mathbf{E}(x,y,z,t) = \hat{\mathbf{x}}E_x(x,y,z)\cos[\omega t + \varphi_x(x,y,z)] &+ \\ \hat{\mathbf{y}}E_y(x,y,z)\cos[\omega t + \varphi_y(x,y,z)] &+ \\ \hat{\mathbf{z}}E_z(x,y,z)\cos[\omega t + \varphi_z(x,y,z)]&\end{aligned}$$

- vector-field phasor (frequency-domain field vector)

$$\begin{aligned}\tilde{\mathbf{E}} = \hat{\mathbf{x}}E_x(x,y,z)e^{j\varphi_x(x,y,z)} &+ \\ \hat{\mathbf{y}}E_y(x,y,z)e^{j\varphi_y(x,y,z)} &+ \\ \hat{\mathbf{z}}E_z(x,y,z)e^{j\varphi_z(x,y,z)}&\end{aligned} \quad \Rightarrow \quad \begin{aligned}\tilde{\mathbf{E}}_{(x,y,z)} = \hat{\mathbf{x}}\tilde{E}_x(x,y,z) &+ \\ \mathbf{y}\tilde{E}_y(x,y,z) &+ \\ \hat{\mathbf{z}}\tilde{E}_z(x,y,z)&\end{aligned}$$

Maxwell Equations in Phasor Form

- phasor of the time-derivative of a function

$$f(x,y,z,t) \doteq F(x,y,z) \;\Rightarrow\; \boxed{\frac{\partial f_{(x,y,z,t)}}{\partial t} \doteq j\omega F(x,y,z)}$$

- spatial derivatives

$$\boxed{\frac{\partial f}{\partial \xi} \doteq \frac{\partial F}{\partial \xi} \;,\quad \xi = x, y, z}$$

time domain	frequency domain
$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \tilde{\mathbf{E}} = -j\omega \tilde{\mathbf{B}}$
$\nabla \times \mathbf{H} = \dfrac{\partial \mathbf{D}}{\partial t} + \sigma \mathbf{E} + \mathbf{J}^{s}$	$\nabla \times \tilde{\mathbf{H}} = \underbrace{j\omega \tilde{\mathbf{D}} + \sigma \tilde{\mathbf{E}}}_{\text{usually given as } j\omega\tilde{\mathbf{D}} \text{ only}} + \tilde{\mathbf{J}}^{s}$
$\nabla \cdot \mathbf{D} = \rho_v$	$\nabla \cdot \tilde{\mathbf{D}} = \tilde{\rho}_v$
$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \tilde{\mathbf{B}} = 0$

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The Continuity Relation in Phasor Form

Write the continuity equation in phasor form.

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$

Prove that the equation

$$\nabla \times \tilde{\mathbf{H}} = j\omega \underbrace{\tilde{\varepsilon}\tilde{\mathbf{E}}}_{\tilde{\mathbf{D}}} + \sigma \tilde{\mathbf{E}} + \tilde{\mathbf{J}}^{s}$$

where the permittivity is a complex number such that $\tilde{\varepsilon} = \varepsilon' - j\varepsilon''$, ($\varepsilon'$ and ε'' being positive real) can also be written as

$$\nabla \times \tilde{\mathbf{H}} = j\omega \overline{\varepsilon} \tilde{\mathbf{E}} + \tilde{\mathbf{J}}^{s}$$

where

$$\overline{\varepsilon} = \tilde{\varepsilon} - j\frac{\sigma}{\omega} = \varepsilon' - j\left(\varepsilon'' + \frac{\sigma}{\omega}\right)$$

Constitutive Relations in the Frequency Domain

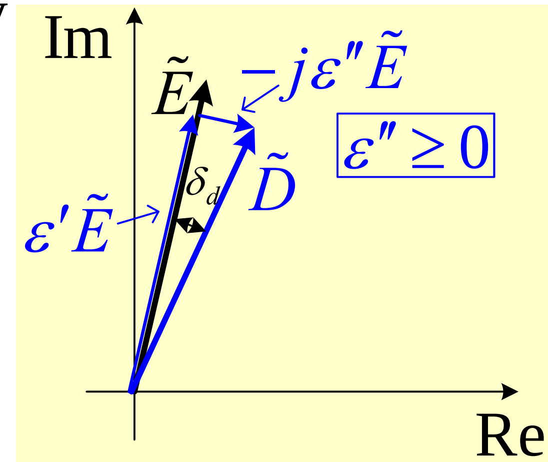
- dielectric polarization and complex permittivity

$$\tilde{\mathbf{D}} = \tilde{\varepsilon}\tilde{\mathbf{E}},\ \tilde{\varepsilon} = \varepsilon' - j\varepsilon'' \text{ or } \tilde{\varepsilon} = \varepsilon_0(\varepsilon_r' - j\varepsilon_r'')$$

$$\tilde{\varepsilon} = \varepsilon'(1 - j\tan\delta_d),\ \ \tan\delta_d = \varepsilon''/\varepsilon'$$

$$\mathbf{E}(x,y,z,t) = \hat{\mathbf{e}}E(x,y,z)\cos(\omega t)$$

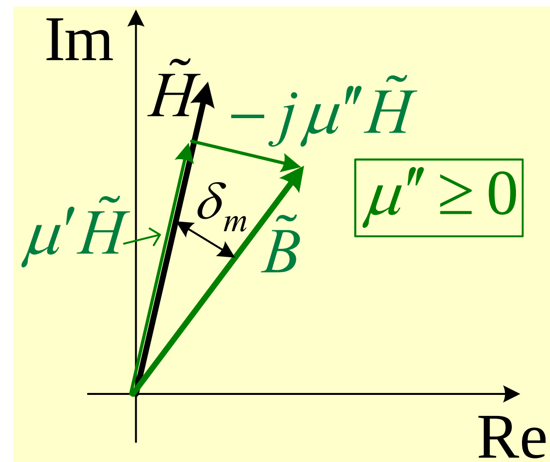
$$\mathbf{D}(x,y,z,t) = \hat{\mathbf{d}}\,|\tilde{\varepsilon}|\,E(x,y,z)\cos(\omega t - \delta_d)$$



- magnetization and complex permeability

$$\tilde{\mathbf{B}} = \tilde{\mu}\tilde{\mathbf{H}},\ \tilde{\mu} = \mu' - j\mu'' \text{ or } \tilde{\mu} = \mu_0(\mu_r' - j\mu_r'')$$

$$\tilde{\mu} = \mu'(1 - j\tan\delta_m),\ \ \tan\delta_m = \mu''/\mu'$$



Why are the imaginary parts of ε and μ negative?

Hint: Consider a capacitor. The flux density **D** relates to the charge Q on the plates while **E** relates to the voltage V applied to the plates.

Time-average Quadratic Value of Vector Field

- time-average “energy” (quadratic) value of harmonic field

$$\mathcal{E}_{\text{av}} = \frac{1}{T}\int_0^T \mathbf{E}(t)\cdot\mathbf{E}(t)dt = \frac{1}{T}\int_0^T \left\{[E_x(t)]^2 + [E_y(t)]^2 + [E_z(t)]^2\right\}dt$$

$$= \frac{1}{T}\int_0^T \left[E_{\text{M}x}^2 \cos^2(\omega t + \varphi_x) + E_{\text{M}y}^2 \cos^2(\omega t + \varphi_y) + E_{\text{M}z}^2 \cos^2(\omega t + \varphi_z)\right]dt =$$

$$= \frac{1}{2}(E_{\text{M}x}^2 + E_{\text{M}y}^2 + E_{\text{M}z}^2) = \frac{1}{2}|\tilde{\mathbf{E}}|^2 = \frac{1}{2}\tilde{\mathbf{E}}\cdot\tilde{\mathbf{E}}^*$$

Note: $\tilde{\mathbf{E}}\cdot\tilde{\mathbf{E}}^* = \tilde{E}_x\tilde{E}_x^* + \tilde{E}_y\tilde{E}_y^* + \tilde{E}_z\tilde{E}_z^* = E_{\text{M}x}^2 + E_{\text{M}y}^2 + E_{\text{M}z}^2$

- root-mean-square (RMS) value

$$E_{\text{rms}} = \sqrt{\mathcal{E}_{\text{av}}} = \sqrt{\frac{1}{T}\int_0^T \mathbf{E}\cdot\mathbf{E}dt} = \frac{|\tilde{\mathbf{E}}|}{\sqrt{2}} = \sqrt{\frac{\tilde{\mathbf{E}}\cdot\tilde{\mathbf{E}}^*}{2}}$$

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Time-average Quadratic Quantities: Stored Energy

- energy density as a function of time (same as in statics)

$$w_{\mathrm{e}}(t)=\frac{1}{2}\mathbf{E}(t)\cdot\mathbf{D}(t)=\frac{1}{2}\varepsilon'\mathbf{E}(t)\cdot\mathbf{E}(t),\ \mathrm{J/m^3}\quad \text{electric energy}$$

in an isotropic medium

$$w_{\mathrm{m}}(t)=\frac{1}{2}\mathbf{H}(t)\cdot\mathbf{B}(t)=\frac{1}{2}\mu'\mathbf{H}(t)\cdot\mathbf{H}(t),\ \mathrm{J/m^3}\quad \text{magnetic energy}$$

- time-average (stored) energy density (harmonic field)

$$w_{\mathrm{e,av}}=\frac{1}{T}\int_0^T w_{\mathrm{e}}(t)dt=\frac{\varepsilon'}{2}\frac{1}{T}\int_0^T \mathbf{E}(t)\cdot\mathbf{E}(t)dt=\frac{\tilde{\mathbf{E}}\cdot(\varepsilon'\tilde{\mathbf{E}}^*)}{4}=\boxed{\frac{\mathrm{Re}\{\tilde{\mathbf{E}}\cdot\tilde{\mathbf{D}}^*\}}{4}}\ \mathrm{J/m^3}$$

$$w_{\mathrm{m,av}}=\frac{1}{T}\int_0^T w_{\mathrm{m}}(t)dt=\frac{\tilde{\mathbf{H}}\cdot(\mu'\tilde{\mathbf{H}}^*)}{4}=\boxed{\frac{\mathrm{Re}\{\tilde{\mathbf{H}}\cdot\tilde{\mathbf{B}}^*\}}{4}}\ \mathrm{J/m^3}$$

Time-average Quadratic Quantities: Dissipation (Power Loss)

- dissipated power density as a function of time (Joule's law in differential form)

$$p_{\mathrm{d}}(t) = \mathbf{J}_{\sigma}(t)\cdot\mathbf{E}(t) = \sigma\mathbf{E}(t)\cdot\mathbf{E}(t),\ \mathrm{W/m^3}$$

in an isotropic medium

- time-average dissipated power density (harmonic field)

$$p_{\mathrm{d,av}} = \frac{1}{T}\int_0^T p_{\mathrm{d}}(t)dt = \sigma\frac{1}{T}\int_0^T \mathbf{E}(t)\cdot\mathbf{E}(t)dt = \frac{\tilde{\mathbf{E}}\cdot(\sigma\tilde{\mathbf{E}}^*)}{2} = \boxed{\frac{\mathrm{Re}\{\tilde{\mathbf{E}}\cdot\tilde{\mathbf{J}}_{\sigma}^*\}}{2}}\ \mathrm{W/m^3}$$

Time-average Quadratic Quantities: Power Transfer

- power-flux density as a function of time

$$\mathbf{S}(t)=\mathbf{E}(t)\times\mathbf{H}(t),\ \mathrm{W/m^2} \longleftarrow \textit{Poynting's vector}$$

- time-average transferred power density (harmonic field)

$$\begin{aligned}\mathbf{E}(t)&=\hat{\mathbf{e}}E_0\sin(\omega t+\Delta\varphi)\\ \mathbf{H}(t)&=\hat{\mathbf{h}}H_0\sin(\omega t)\end{aligned} \Rightarrow \mathbf{S}_{\mathrm{av}}=\frac{1}{T}\int_0^T\mathbf{S}(t)dt=\frac{1}{T}\int_0^T\mathbf{E}(t)\times\mathbf{H}(t)dt$$

$$\begin{aligned}\mathbf{S}_{\mathrm{av}}&=(\hat{\mathbf{e}}\times\hat{\mathbf{h}})E_0H_0\frac{1}{T}\int_0^T[\sin(\omega t)\cdot\sin(\omega t+\Delta\varphi)]dt\\ &=(\hat{\mathbf{e}}\times\hat{\mathbf{h}})E_0H_0\frac{1}{2T}\int_0^T[\cos\Delta\varphi-\cos(2\omega t+\Delta\varphi)]dt\\ &=\frac{(\hat{\mathbf{e}}\times\hat{\mathbf{h}})E_0H_0}{2}\left[\cos\Delta\varphi-\frac{1}{T}\underbrace{\int_0^T\cos(2\omega t+\Delta\varphi)dt}_{0}\right]\\ &=\frac{(\hat{\mathbf{e}}\times\hat{\mathbf{h}})E_0H_0\cos\Delta\varphi}{2}=\boxed{\frac{\mathrm{Re}(\tilde{\mathbf{E}}\times\tilde{\mathbf{H}}^*)}{2}}\ \mathrm{W/m^2}\end{aligned}$$

Time-average Quadratic Quantities: Source Power

- source power density as a function of time

$$p^{\mathrm{s}}(t) = -\left[\boxed{\mathbf{J}^{\mathrm{s}}(t)\cdot\mathbf{E}(t)} + \mathbf{M}^{\mathrm{s}}(t)\cdot\mathbf{H}(t)\right]$$

fictitious

- time-average source power density (harmonic field)

$$\boxed{\begin{aligned}\mathbf{E}(t) &= \hat{\mathbf{e}}E_0\sin(\omega t + \Delta\varphi)\\ \mathbf{J}^{\mathrm{s}}(t) &= \hat{\mathbf{j}}^{\mathrm{s}}J_0\sin(\omega t)\end{aligned}}$$

$$p_{\mathrm{av}}^{\mathrm{s}} = (\hat{\mathbf{e}}\cdot\hat{\mathbf{j}}^{\mathrm{s}})E_0J_0^{\mathrm{s}}\frac{1}{T}\int_0^T\left[\sin(\omega t)\cdot\sin(\omega t + \Delta\varphi)\right]dt$$

$$= \frac{(\hat{\mathbf{e}}\cdot\hat{\mathbf{j}}^{\mathrm{s}})E_0J_0^{\mathrm{s}}\cos\Delta\varphi}{2} = \boxed{\frac{\mathrm{Re}(\tilde{\mathbf{E}}\cdot\tilde{\mathbf{J}}^{\mathrm{s}*})}{2}}\ \mathrm{W/m^3}$$

Putting Things Together: Poynting's Theorem

- power-balance equation of an EM system (derived from ME): relates quadratic quantities
  - time domain (all vector quantities are functions of time)
    - differential form

$$\nabla\cdot\left[\mathbf{E}(t)\times\mathbf{H}(t)\right]+\frac{\partial}{\partial t}\left[\frac{1}{2}\mathbf{D}(t)\cdot\mathbf{E}(t)\right]+\frac{\partial}{\partial t}\left[\frac{1}{2}\mathbf{B}(t)\cdot\mathbf{H}(t)\right]+$$
$$+\mathbf{J}_{\sigma}(t)\cdot\mathbf{E}(t)=-\left[\mathbf{E}(t)\cdot\mathbf{J}^{s}(t)+\mathbf{H}(t)\cdot\mathbf{M}^{s}(t)\right],\ \mathrm{W/m^3}$$

    - integral form

$$\underbrace{\oint\!\!\!\oint_S\left[\mathbf{E}(t)\times\mathbf{H}(t)\right]\cdot d\mathbf{s}}_{\text{term 1}}+\frac{\partial}{\partial t}\iiint_{V_S}\left[\underbrace{\frac{1}{2}\mathbf{D}(t)\cdot\mathbf{E}(t)}_{\text{term 2}}+\underbrace{\frac{1}{2}\mathbf{B}(t)\cdot\mathbf{H}(t)}_{\text{term 3}}\right]dv+$$
$$+\underbrace{\iiint_{V_S}\mathbf{J}_{\sigma}(t)\cdot\mathbf{E}(t)dv}_{\text{term 4}}=\underbrace{-\iiint_{V_S}\left[\mathbf{E}(t)\cdot\mathbf{J}^{s}(t)+\mathbf{H}(t)\cdot\mathbf{M}^{s}(t)\right]dv}_{\text{term 5}},\ \mathrm{W}$$

Putting Things Together: Poynting's Theorem – 2

Describe with one sentence the physical meaning of terms 1, 2, 3, 4 and 5 in Poynting's time-domain theorem in sl. 18.

Putting Things in Perspective: Poynting's Theorem – 3

- frequency domain
  - differential form

$$0.5\nabla\cdot(\tilde{\mathbf{E}}\times\tilde{\mathbf{H}}^*)+0.5j\omega(\underbrace{\tilde{\mathbf{H}}^*\cdot\tilde{\mathbf{B}}}_{\tilde{\mu}|\tilde{\mathbf{H}}|^2}-\underbrace{\tilde{\mathbf{E}}\cdot\tilde{\mathbf{D}}^*}_{\tilde{\varepsilon}^*|\tilde{\mathbf{E}}|})=-0.5\left(\tilde{\mathbf{E}}\cdot\tilde{\mathbf{J}}^{s*}+\tilde{\mathbf{H}}^*\cdot\tilde{\mathbf{M}}^s\right)$$

explicit active (loss) and reactive (exchange) power terms

$$\overset{\text{term 1}}{\boxed{\nabla\cdot\mathrm{Re}(\tilde{\mathbf{E}}\times\tilde{\mathbf{H}}^*)}}+\overset{\text{term 2}}{\boxed{\sigma\tilde{\mathbf{E}}\cdot\tilde{\mathbf{E}}^*}}+\overset{\text{term 3}}{\boxed{\omega\left(\mu''\tilde{\mathbf{H}}\cdot\tilde{\mathbf{H}}^*+\varepsilon''\tilde{\mathbf{E}}\cdot\tilde{\mathbf{E}}^*\right)}}=\overset{\text{term 4}}{\boxed{-\mathrm{Re}\left(\tilde{\mathbf{E}}\cdot\tilde{\mathbf{J}}^{s*}+\tilde{\mathbf{H}}^*\cdot\tilde{\mathbf{M}}^s\right)}}$$

$$\underset{\text{term 5}}{\boxed{\nabla\cdot\mathrm{Im}(\tilde{\mathbf{E}}\times\tilde{\mathbf{H}}^*)}}+\underset{\text{term 6}}{\boxed{j\omega\left(\mu'\tilde{\mathbf{H}}\cdot\tilde{\mathbf{H}}^*-\varepsilon'\tilde{\mathbf{E}}\cdot\tilde{\mathbf{E}}^*\right)}}=\underset{\text{term 7}}{\boxed{-\mathrm{Im}\left(\tilde{\mathbf{E}}\cdot\tilde{\mathbf{J}}^{s*}+\tilde{\mathbf{H}}^*\cdot\tilde{\mathbf{M}}^s\right)}}$$

Describe with one sentence the physical meaning of terms 1, 2, 3, 4, 5, 6 and 7 in Poynting's frequency-domain theorem above.

Summary

- the 4 Maxwell equations (2 *curl* and 2 *divergence*) form the basis of EM and of microwave engineering
- the 2 *div* equations (the Gauss laws) follow from the *curl* equations and the continuity of charge
- phasors are used in harmonic (single-frequency) field analysis
- the imaginary parts of the complex permittivity and permeability must be negative (or zero) indicating loss (no loss) (a positive imaginary part would indicate a "gain" material!)
- quadratic field quantities such as *energy* and *power* as well the *root-mean-square* values are conveniently calculated using the products of phasors
- Poynting's theorem describes the power balance in EM systems