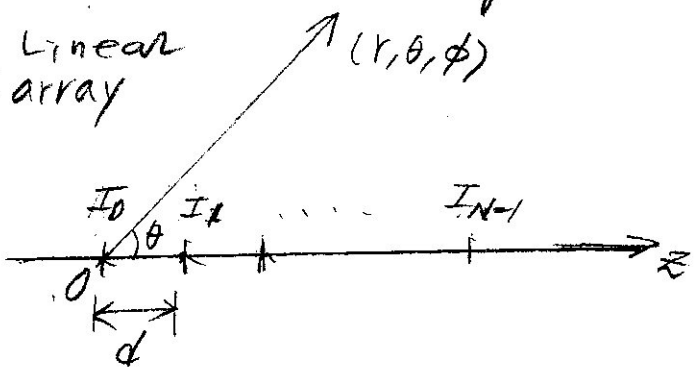
Adaptive Beam-forming Array Antenna

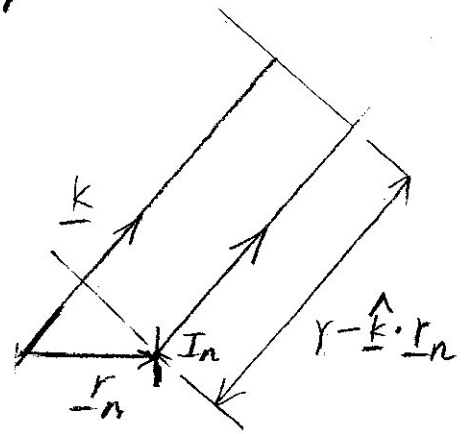
1. Linear array



2. Array factor, linear array

$$\underline{E}_n = f(\theta,\phi)\frac{I_n}{4\pi r}e^{-jk(r-\hat{\underline{k}}\cdot\underline{r}_n)}$$

$$= \underbrace{\frac{e^{-jkr}}{4\pi r}}_{\text{Free space Green's function}}\ \underbrace{f(\theta,\phi)}_{\text{element factor}}\ \underbrace{I_n e^{j\underline{k}\cdot\underline{r}_n}}_{\text{array factor contribution}}$$



$$\underline{r}_n = nd\hat{\underline{z}}$$

$$\underline{k}\cdot\underline{r}_n = k(nd)\cos\theta$$

$$E = E_1 + \cdots + E_n = \frac{e^{-jkr}}{4\pi r}f(\theta,\phi)\underbrace{\sum_{n=0}^{N-1} I_n e^{jn(kd\cos\theta)}}_{\text{array factor}}$$

$$AF = \sum_{n=0}^{N-1} I_n e^{jn\psi}, \quad \psi = kd\cos\theta$$

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3. Element factor, small dipole

$$f(\theta,\phi) = \sin\theta$$

$$\underline{EF = f(\theta,\phi) = \sin\theta}$$

4. Total pattern factor

$$PF = EF \times AF$$

$$PF = \sin\theta \sum_{n=0}^{N-1} I_n e^{jn\psi}, \quad \psi = kd\cos\phi$$

5. Broadside array, maximum element spacing

$$I_0 = I_1 = \cdots = I_{N-1} = 1$$

$$AF = 1 + e^{j\psi} + \cdots + e^{j(N-1)\psi}$$

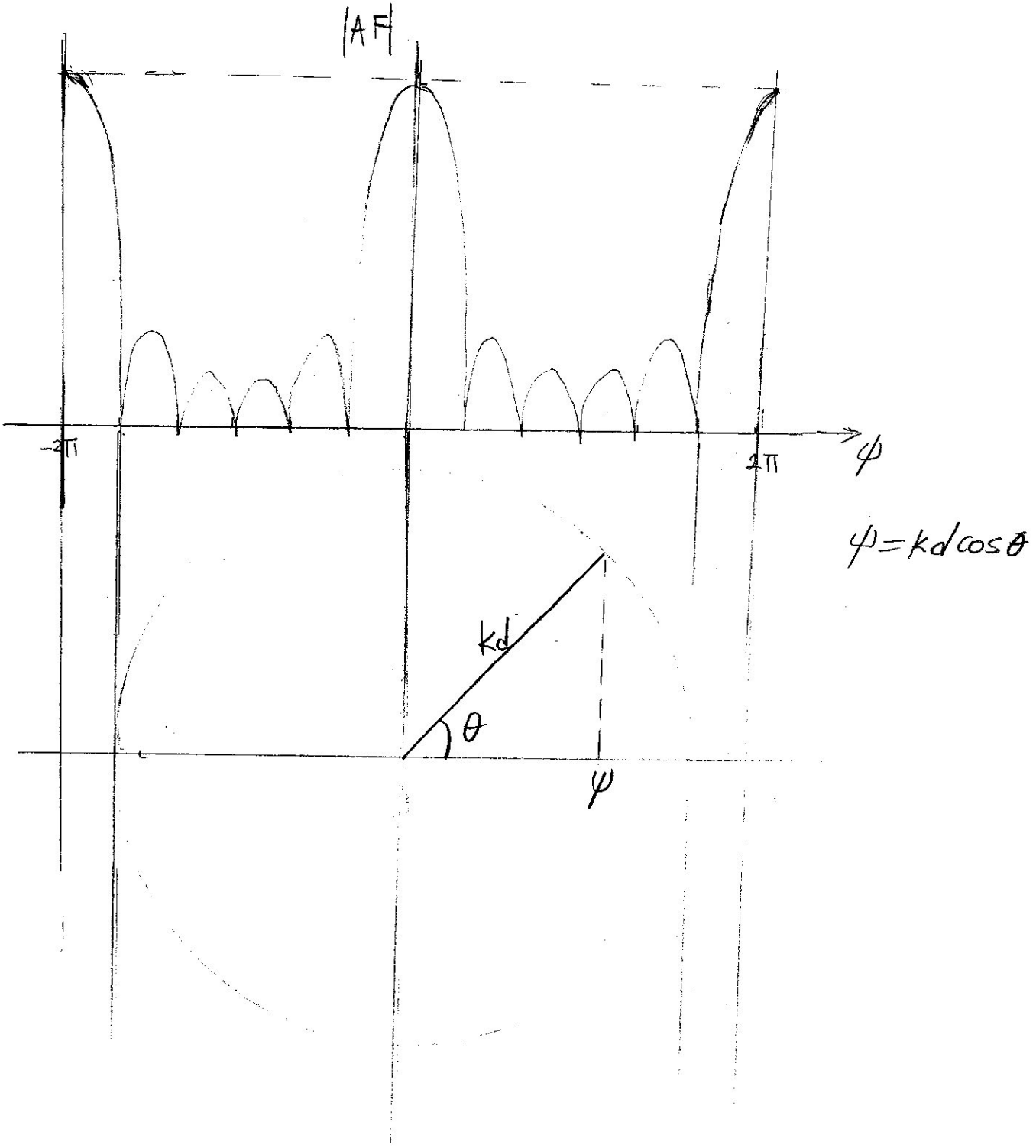
$$e^{j\psi}AF = \quad e^{j\psi} + \quad + e^{j(N-1)\psi} + e^{jN\psi}$$

$$(1-e^{j\psi})\,EF = 1 - e^{jN\psi}$$

$$AF = \frac{1-e^{jN\psi}}{1-e^{j\psi}} = \frac{e^{jN\psi/2}\left(e^{-jN\psi/2} - e^{jN\psi/2}\right)}{e^{j\psi/2}\left(e^{-j\psi/2} - e^{j\psi/2}\right)}$$

$$|AF| = \frac{\sin(N\psi/2)}{N\sin(\psi/2)}$$

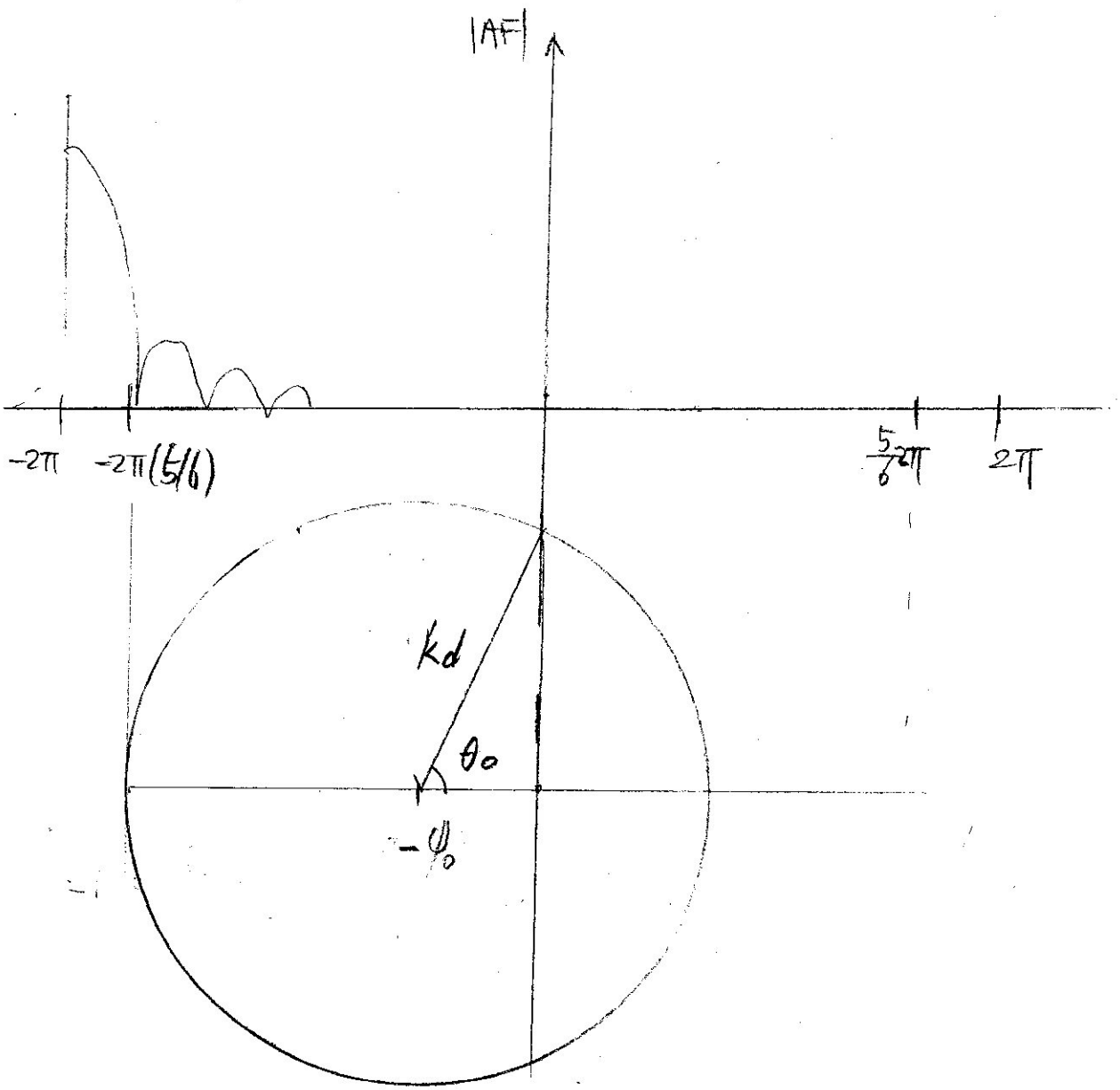
$N = 6$



$$kd_{max} = \frac{2\pi}{\lambda} d_{max} = 2\pi - \frac{2\pi}{N}, \quad d_{max} = \frac{N-1}{N}\lambda$$

$$\underline{d_{max} = \frac{5}{6}\lambda = 0.833\lambda}$$

6. Scanned array, $\theta_0 = 45°$, maximum distance



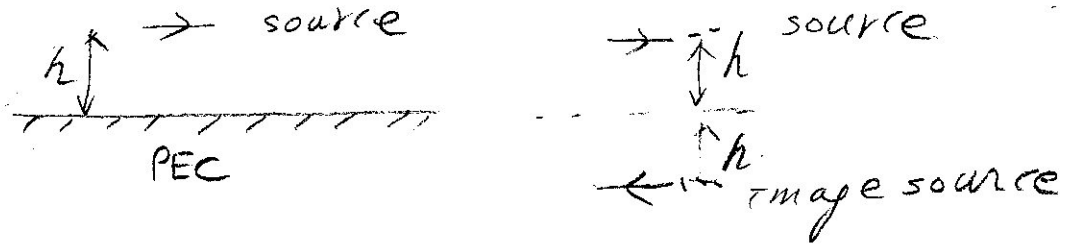
$$-kd_{max} - \psi_0 = -2\pi\left(1 - \frac{1}{N}\right)$$

$$\frac{2\pi}{\lambda}d + \frac{2\pi}{\lambda}d\cos\theta_0 = 2\pi\frac{N-1}{N}$$

$$d = \frac{1}{1+\cos\theta_0}\frac{N-1}{N}\lambda$$

$N=6$, $\theta_0 = 45°$, $d = d_{max} = \frac{1}{1+1/\sqrt{2}}\frac{5}{6}\lambda = \underline{0.488\lambda}$

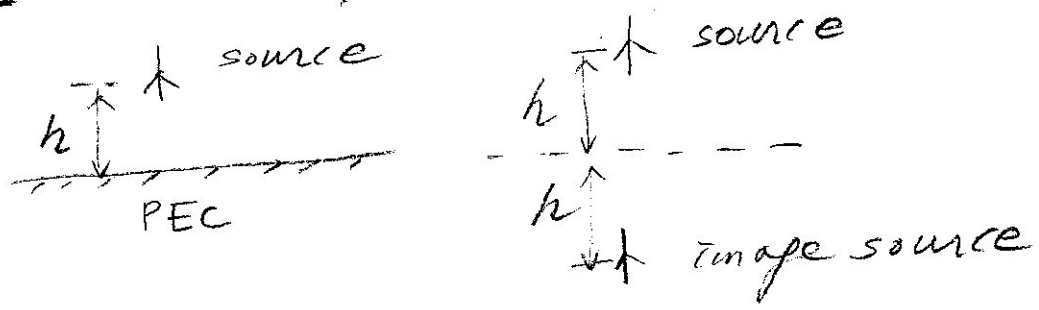
7. horizontal dipole above a ground



$2h = \frac{\lambda}{2}$: source and image source in phase

$$h_{opt} = \frac{\lambda}{4}$$

8. vertical dipole above a ground

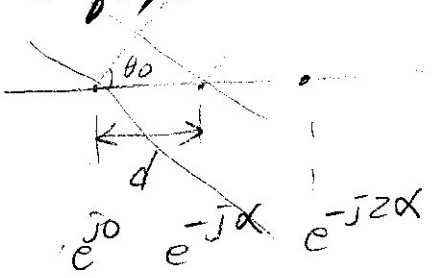


$2h = \lambda$: in-phase condition

$$h_{opt} = \frac{\lambda}{2}$$

9. inter-element phase shift, scanned array

$kd\cos\theta_0 = \alpha$



$\theta_0 = 45^\circ$

$\alpha = \frac{2\pi}{\lambda}(0.488\lambda)\cos 45^\circ = 2.168$ rad

$= 124.2^\circ$

10. Scanned array, pattern factor

$\theta_0 = 45°$, $d = 0.488\lambda$

$kd = 0.976\pi$

$kd\cos\theta_0 = 0.690\pi$