Lab 4 – Op Amp Filters

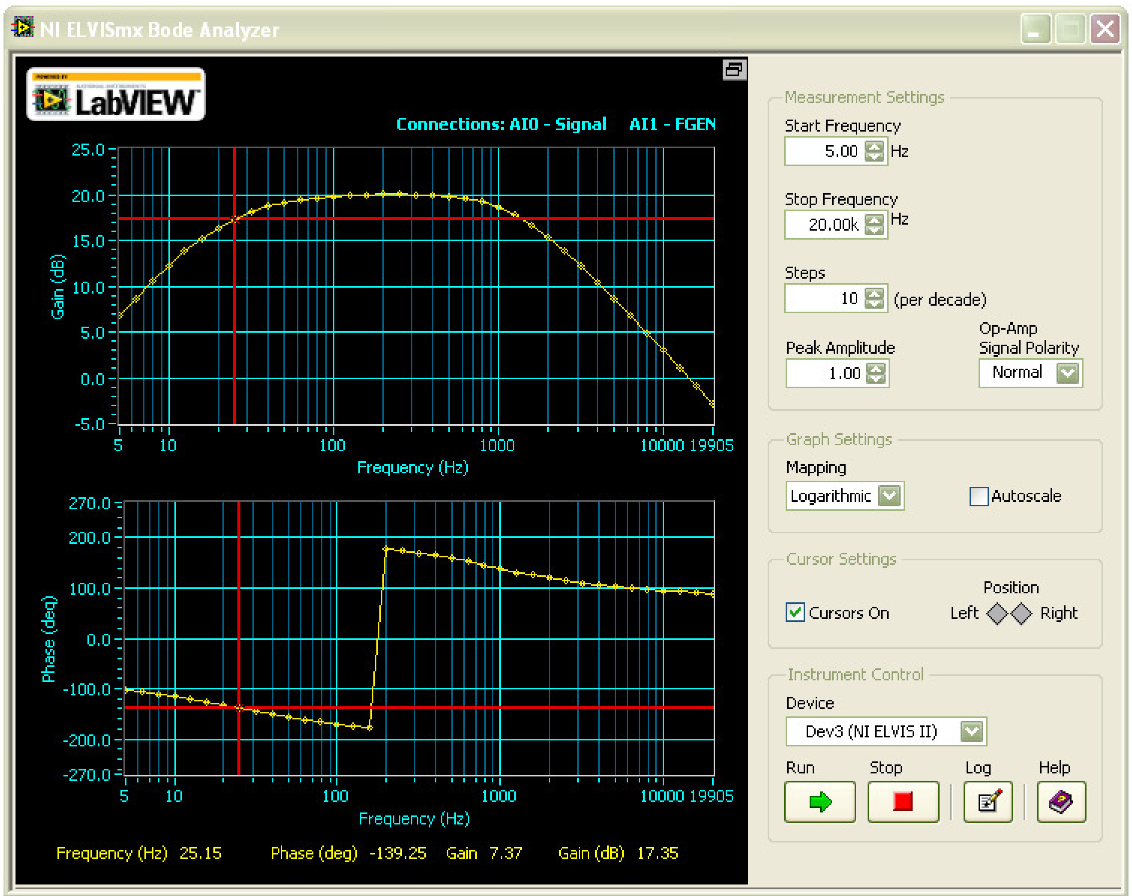


Figure 4.0. Frequency Characteristics of a BandPass Filter

Adding a few capacitors and resistors to the basic operational amplifier (op amp) circuit can yield many interesting analog circuits such as active filters, integrators, and differentiators. Filters are used to pass specific frequency bands, integrators are used in proportional control, and differentiators are used in noise suppression and waveform generation circuits.

Goal: This lab uses the NI ELVIS II suite of instruments to measure the characteristics of lowpass, highpass, and bandpass filters. Simulate these filters using Multisim with the measured component values. In the lab challenge at the end of this chapter, Multisim is used to design a second order active filter.

Required Soft Front Panels (SFPs)
Digital multimeter (DMM[Ω,C])
Function generator (FGEN)
Oscilloscope (Scope)
Impedance analyzer (Imped)
Bode analyzer (Bode)

Required Components
10 kΩ resistor R_1, (brown, black, orange)
100 kΩ resistor R_f, (brown, black, yellow)
1 μF capacitor C_1
0.01 μF capacitor C_f
741 op amp

Exercise 4.1: Measuring the Circuit Component Values

Complete the following steps to measure the values of the individual components:

1. Launch NI ELVIS II.
2. Select the DMM icon from the Instrument Measurement strip.
3. Select DMM[Ω] to measure the resistors.
4. Select DMM[C] to measure the capacitors.
5. Fill in the following information.

   R_1 ____________ Ω (10 kΩ nominal)
   R_f ____________ Ω (100 kΩ nominal)
   C_1 ____________ μF (1 μf nominal)
   C_f ____________ μF (0.01 μf nominal)

6. Close the DMM.

End of Exercise 4.1

Exercise 4.2: Frequency Response of the Basic Op Amp Circuit

Complete the following steps to build and perform measurements on an op amp.

1. On the workstation protoboard, build a simple 741 inverting op amp circuit with a gain of 10 as shown in Figure 4.1.

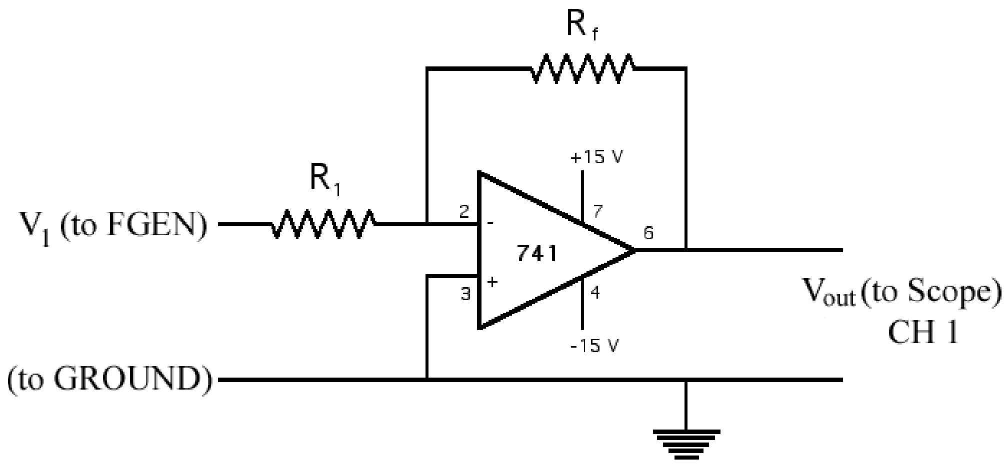


Figure 4.1. Schematic Diagram of a 741 Inverting Op Amp Circuit with a Gain of 10

The circuit looks like Figure 4.1 on the NI ELVIS II protoboard.

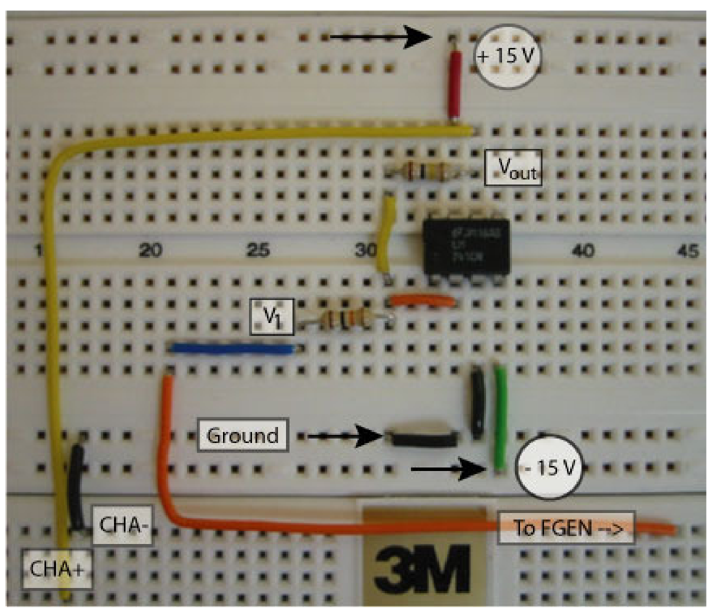


Figure 4.2. 741 Inverting Op Amp Circuit with a Gain of 10 on an NI ELVIS protoboard

Note: The op amp uses both the +15 and -15 VDC power supplies. These are found on the protoboard pin sockets labeled as "DC Power Supplies +15V, -15V & GROUND."

2. Connect the function generator [FGEN] pin socket to the input resistor R_1, and the input resistor to the op amp input.

3. Connect the [Ground] pin socket to pin 3 of the op amp.

4. Connect the op amp output voltage, V_{out}, to the oscilloscope BNC input connector [BNC 2 +] and the function generator output to BNC input [BNC 1 +]. Connect the ground to the pin sockets [BNC 1 -] and [BNC 2 -]. Using BNC cables, connect the BNC 1 and BNC 2 sockets to the CH0 and CH1 (respectively) inputs on the left side of the ELVIS.

Note: If you do not have BNC Cables, connect the circuit input and output into AI0 and AI1 respectively.

5. From the NI ELVIS II Instrument Launcher strip, select the function generator (**FGEN**) icon and the oscillosco**pe** (**Scope**) icon.

Note: By default, on the oscilloscope, the Channel 0 Settings Source is set to Scope Ch 0 and the Channel 1 Settings Source is set to Scope Ch 1. These are your op amp input and output signals, respectively. If you used the analog input channels instead, select the appropriate channels in the drop down menu under "Source".

6. To view the signals, click on the enable boxes.

7. On the function generator panel, set the following parameters:

Waveform:	**Sine wave**
Peak Amplitude:	**0.2 pp**
Frequency:	**1000 Hz**
DC Offset:	**0.0 V**

8. Check your circuit and then apply power to the NI ELVIS II protoboard.

9. Click on [Run] for both the FGEN and Scope SFPs.

10. Set the trigger to Edge, CH 0, Level 0.0 and the Time/Div to 1 ms.

11. Measure the amplitude of the op amp input (CH 0) and output (CH 1) on the oscilloscope window.

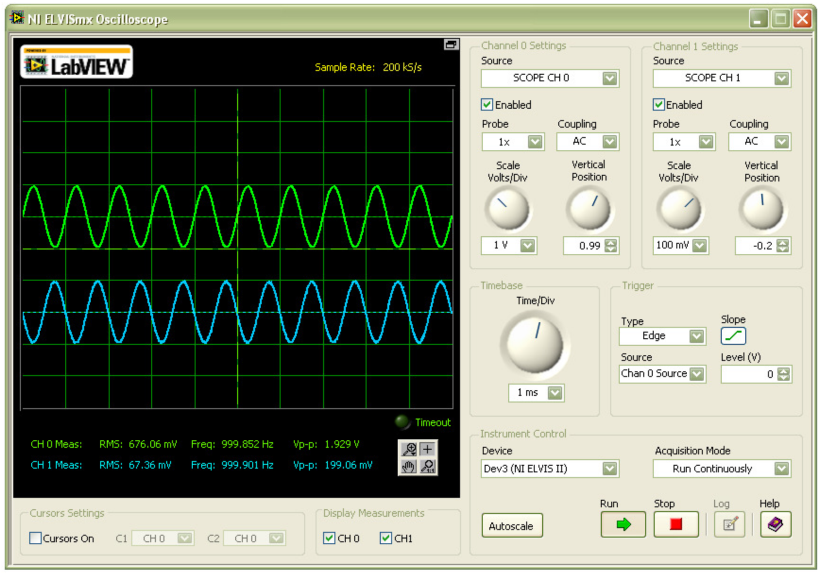


Figure 4.3. Inverting Op Amp input and output signals

Note the output signal is inverted as expected with respect to the input signal.

12. Calculate the voltage gain (the amplitude ratio, CH1/CH0).

13. Try a range of frequencies from 100 Hz to 10 kHz.

How do your measurements agree with the theoretical gain of (R_f/R_1)?

Is the ratio still the same at 100 kHz?

14. Close the FGEN and Scope windows.

End of Exercise 4.2

Exercise 4.3: Measuring the Op Amp Frequency Characteristic

The best way to study the AC characteristic response curve of an op amp is to measure its Bode plot. The Bode plot is basically a plot of gain (dB) and phase (degrees) as a function of log frequency. The transfer function for an inverting op amp circuit is given by:

$$V_{out} = - (R_f/R_1)\ V_1$$

where V_{out} is the op amp output and V_1 is the circuit input. The gain is the quantity (R_f/R_1). The minus sign inverts the output signal with respect to the input signal. On a Bode plot, one expects a straight line with a magnitude of 20 x log (gain). For a gain of 10, the Bode amplitude should be 20 dB.

Complete the following step to measure the Bode plot of the Op Amp circuit:

1. From the NI ELVIS II Instrument Launcher strip, select Bode Analyzer **(Bode)** icon.

2. Connect the signals, input (V_1) and output (V_{out}), to the analog input pins as follows:

V_1+	AI 0+	(from the FGEN output)
V_1 -	AI 0-	(from GROUND)
V_{out}+	AI 1+	(from the op amp output)
V_{out} -	AI 1 -	(from GROUND)

3. On the Bode analyzer, set the scan parameters as follows:

Start:	**5 (Hz)**
Stop:	**20000 (Hz)**
Steps:	**10 (per decade)**

4. Apply power to the protoboard.

5. Click [Run] and observe the Bode plot for the inverting op amp circuit.

6. Take a close look at the phase response.

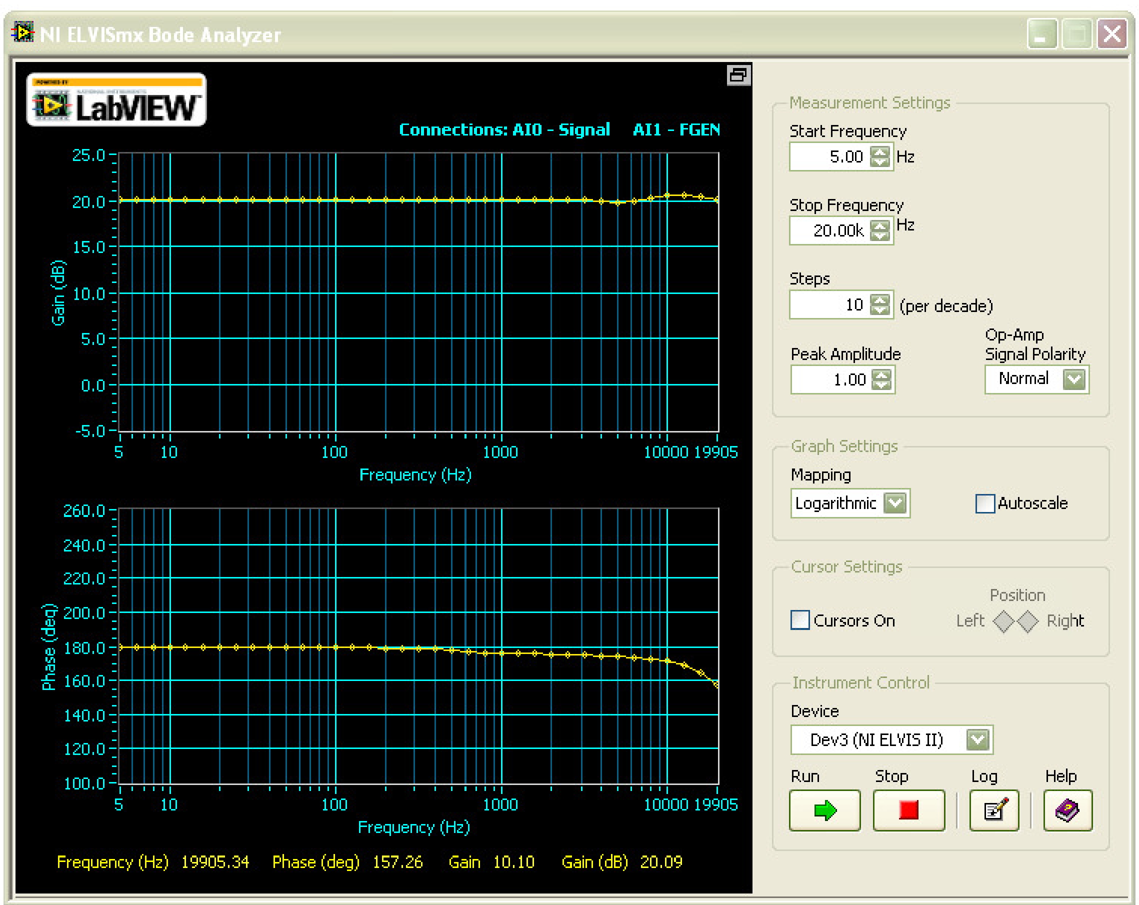


Figure 4.4. Bode Plot Measurements of an Inverting Op Amp with a gain of 10

The gain (20 dB) is flat and independent of frequency until approximately 10,000 Hz, where it starts to roll off as shown in Figure 4.4. This Bode plot is typical for a 741 op amp inverting circuit. At high frequencies, the amplifier response depends on its internal circuitry as well as any external components.

End of Exercise 4.3

Exercise 4.4: Highpass Filter

A low frequency cutoff point, f_L, for a simple RC series circuit is given by the equation:

$$2\pi f_L = 1/RC$$

where f_L is measured in hertz. The low-frequency cutoff point is the frequency where the gain (dB) has fallen by -3 dB. This (-3 dB) point occurs when the impedance of the capacitor equals that of the resistor.

1. Add a 1 μF capacitor, C_l, in series with the 1 kΩ input resistor, R_1, in the op amp circuit as shown in Figure 4.5.

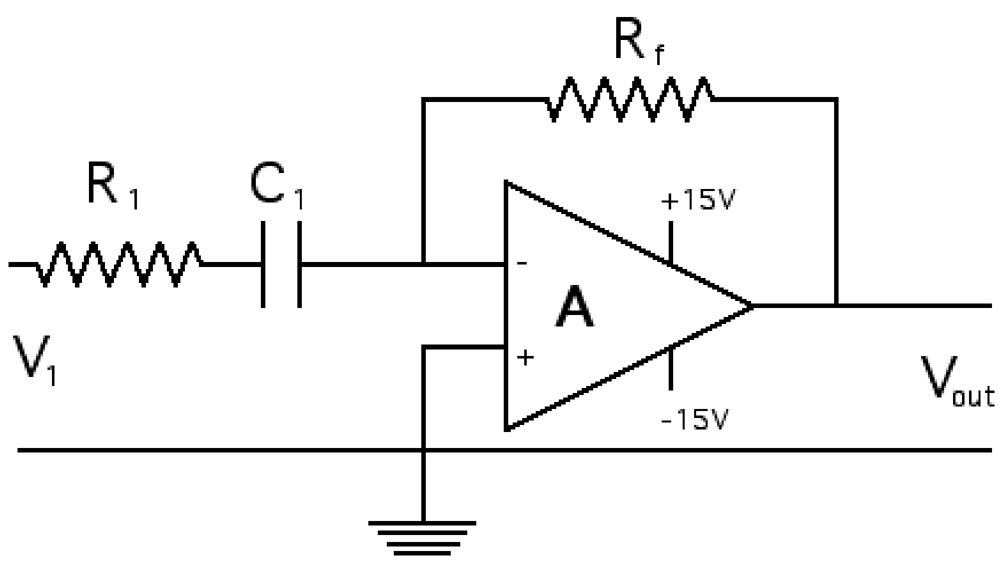


Figure 4.5. Highpass Op Amp Filter Circuit Design

The highpass op amp filter equation has a low-frequency cutoff point, f_L, where the gain has fallen to -3 dB. In other words, when $\mathbf{X_c = R}$:

$$2\pi f_L = 1/ R_1C_1$$

Figure 4.6 shows this circuit on an NI ELVIS protoboard.

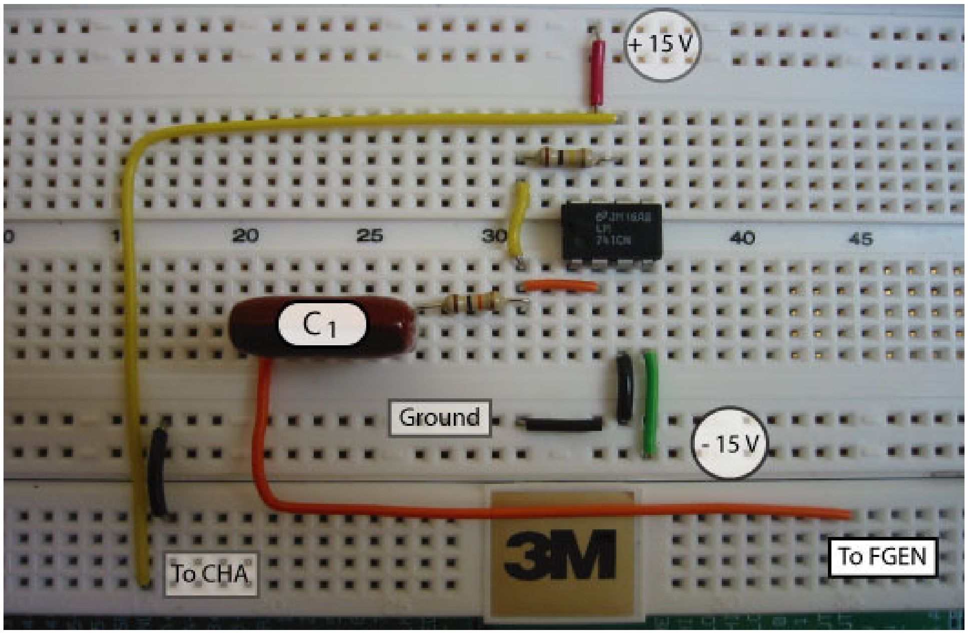


Figure 4.6. Highpass Op Amp Filter on NI ELVIS protoboard

2. Run a second Bode plot using the same scan parameters as in Exercise 4.3.

3. Observe that the low-frequency response is attenuated while the high-frequency response is similar to the basic op amp Bode plot.

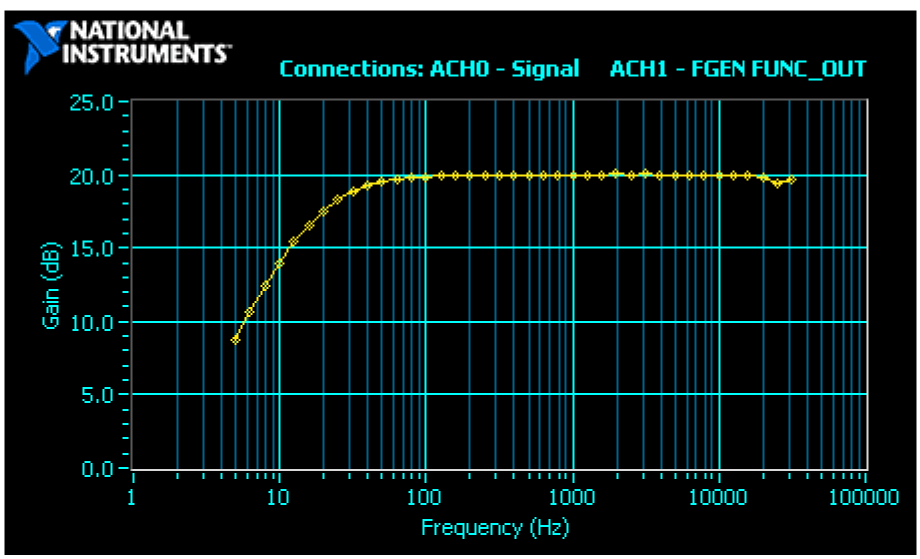


Figure 4.7. Bode Measurement of Highpass Op Amp circuit

4. Use the cursor function to find the low-frequency cutoff point, that is, the frequency at which the amplitude has fallen by -3 dB or the phase change is 45 degrees.

5. Compare your results with the following theoretical predication:

$$2\pi f_L = 1/ R_1C_1$$

End of Exercise 4.4

Exercise 4.5: Lowpass Filter

The high-frequency roll-off in the op amp circuit is due to the internal capacitance of the 741 chip being in parallel with the feedback resistor, R_f. If you add an external capacitor, C_f, in parallel with the feedback resistor, R_f, you can reduce the upper frequency cutoff point. It turns out that you can predict this new cutoff point from the following equation:

$$2\pi f_U = 1/R_f C_f$$

Complete the following steps to perform an additional frequency measurement on the op amp circuit:

1. Short the input capacitor (do not remove it because you will use it in Exercise 4.6).

2. Add the feedback capacitor, C_f, (0.01 µf) in parallel with the 100 kΩ feedback resistor.

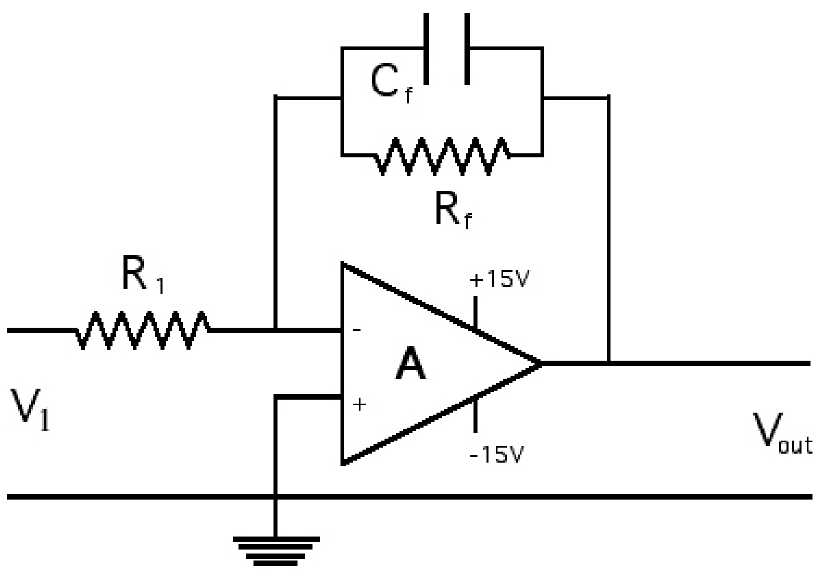


Figure 4.8. Lowpass Op Amp Filter Circuit Design

3. Run a third Bode plot using the same scan parameters.

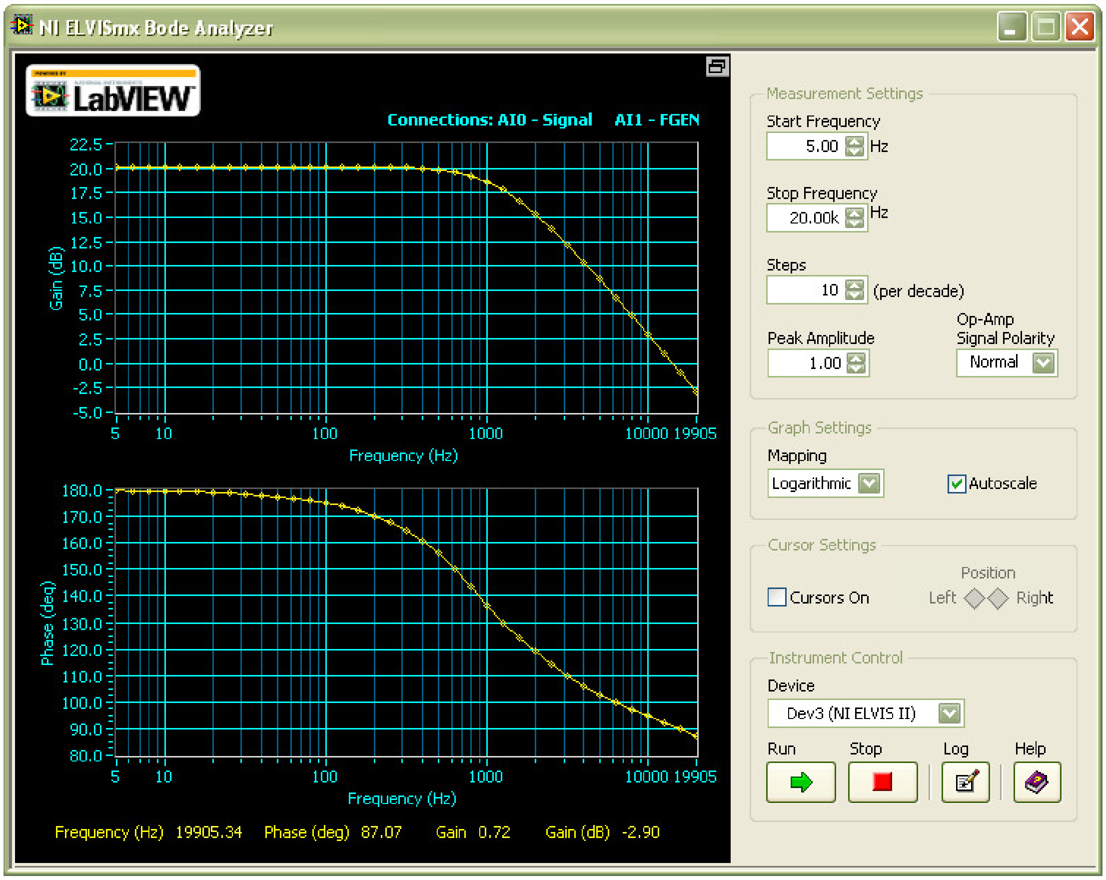


Figure 4.9. Bode Measurement of Lowpass Op Amp circuit

Figure 4.9 shows that the high-frequency response is attenuated much more than the basic op amp response.

4. Use the cursor function to find the high-frequency cutoff point, that is, the frequency at which the amplitude has fallen by -3 dB or the phase change is 45 degrees.

5. Compare your results with the following theoretical prediction:

$$\mathbf{2\pi f_U = 1/\ R_f\ C_f}$$

Note: the 90-degree phase change from the very low-frequency range to the upper-frequency range. This is as expected for a single-pole RC filter stage.

End of Exercise 4.5

Exercise 4.6: Bandpass Filter

If you allow both an input capacitor and a feedback capacitor in the op amp circuit, the response curve has both a low-cutoff frequency, f_L, and a high-cutoff frequency, f_U. The frequency range ($f_U - f_L$) is called the bandwidth. For example, a good stereo amplifier has a bandwidth of at least 20,000 Hz.

Figure 4.10 shows a bandpass filter on an NI ELVIS II protoboard.

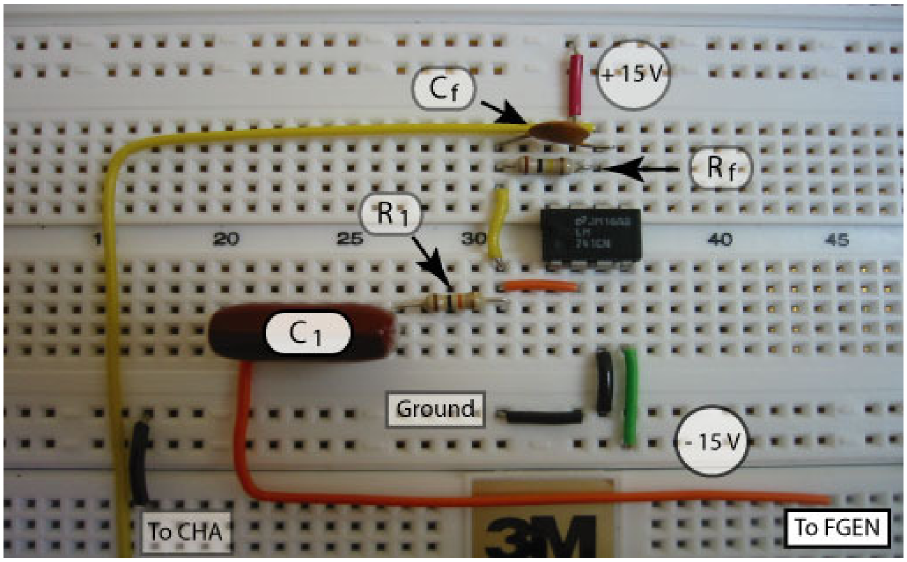


Figure 4.10. Bandpass Op Amp circuit on NI ELVIS protoboard

1. Remove the short on C_1.

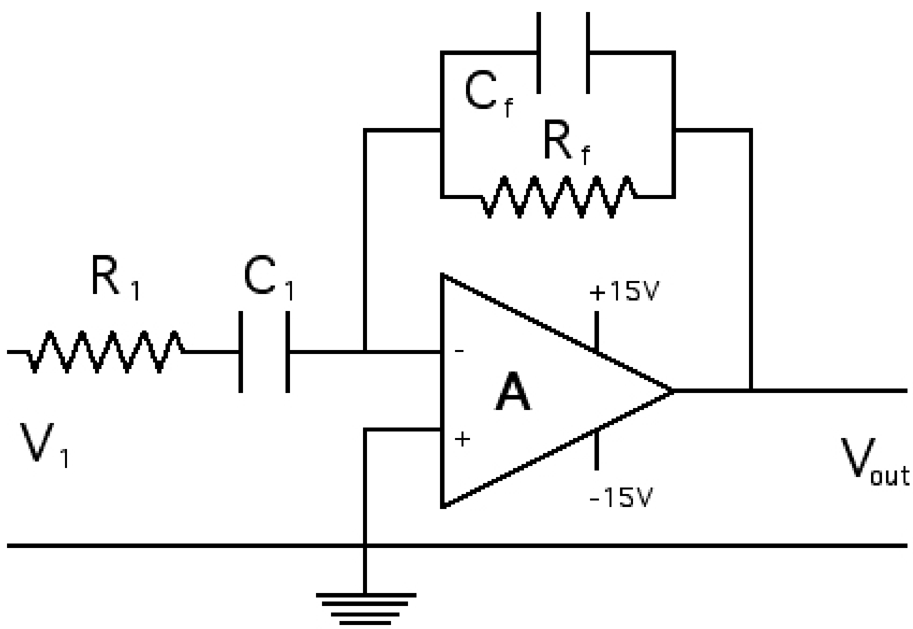


Figure 4.11. Bandpass Op Amp Filter Circuit Design

2. Run a fourth Bode plot using the same scan parameters.

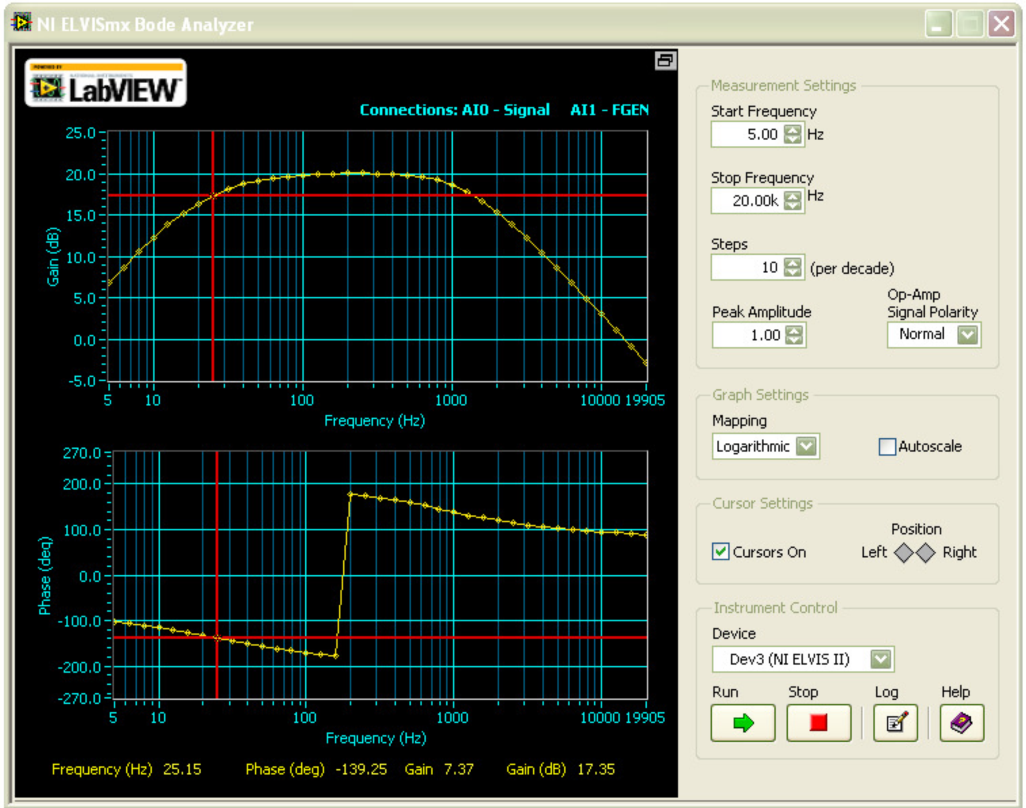


Figure 4.12. Bode Measurement of Bandpass Op Amp circuit

Using the cursors, draw a line between the -3 dB points. All frequencies with an amplitude above this line are contained within the frequency pass band.

How does this bandwidth measurement agree with the theoretical prediction of ($f_U - f_L$)?

End of Exercise 4.6

For Further Study

The generalized op amp transfer curve is given by the following phasor equation

$$\mathbf{V}_{out} = (\mathbf{Z}_f/\mathbf{Z}_1)\mathbf{V}_{\mathbf{in}}$$

where the impedance values for the four circuits are:

Op Amp	$\mathbf{Z}_f$	$\mathbf{Z}_1$	**Gain**
Basic	R_f	R_1	R_f/R_1
Highpass	R_f	$R_1 + X_{C1}$	$R_f/(R_1 + X_{C1})$
Lowpass	$R_f + X_{Cf}$	R_1	$(R_f + X_{Cf})/R_1$

Bandpass	$R_f + X_{Cf}$	$R_1 + X_{C1}$	$(R_f + X_{Cf})/(R_1 + X_{C1})$

Table 4.0. Impedance Values for the FourOp Amp Circuits

At any frequency, you can use the impedance analyzer (Imped) to measure the impedances $\mathbf{Z}_f$ and $\mathbf{Z}_1$. A LabVIEW program can calculate the ratio of two complex numbers. The magnitude of the ratio $|\mathbf{Z}_f/\mathbf{Z}_1|$ is the gain.

Note: You could also use the impedance analyzer to find the frequencies where R_1 equals X_{C1} and R_f equals X_{Cf} to verify that the lower- and upper-frequency cutoff points from the Bode plot are equal to these frequencies.

Multisim Challenge: Design a Second-Order Lowpass Filter

In Exercise 4.5, you built a lowpass filter with a single capacitor in the feedback loop. At high frequencies beyond the cutoff point, the gain falls off linearly with a slope of 6 dB/octave. Some applications require a sharper roll-off. You can accomplish this using a filter design with two or more capacitors in the filter design.

1. Design a second-order lowpass filter with a -3 dB cutoff point, f_c, at 1000 Hz.

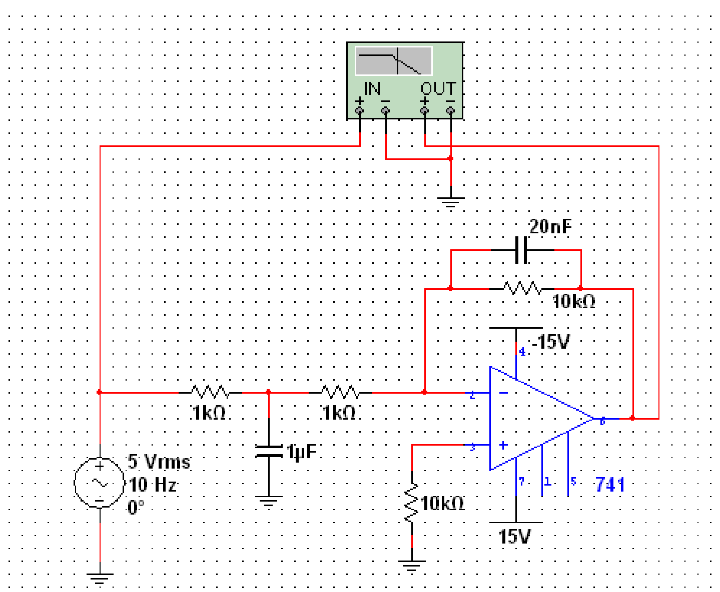


Figure 4.13. Multisim solution of a Second-order Op Amp Filter

This filter has two cutoff points

$$f_{c1} = (R_1 \| R_2)/(2\pi C_1) \quad \text{and} \quad f_{c2} = (2\pi R_3 C_2)^{-1}$$

In the special case when $f_{c1} = f_{c2} = f_c$, the gain expression for this filter becomes

$$|G| = \frac{-R_3/(R_1+R_2)}{1+(f/f_c)^2}$$

2. Pick resistors and capacitors to satisfy the special case requirement that

$$f_{c1} = f_{c2} = 1000 \text{ Hz}$$

3. Launch the Multisim program Two Pole Active Filter.

4. Double-click on the Bode analyzer icon to open a results window.

5. Run this program and view the Bode plot.

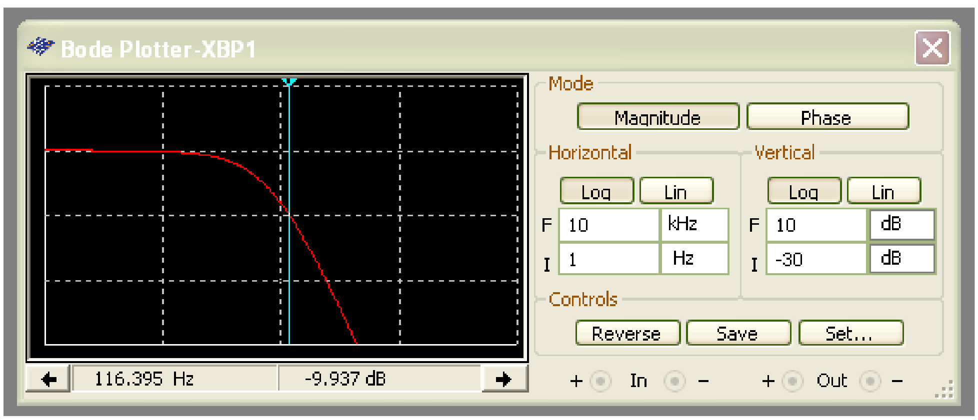


Figure 4.14. Frequency Response of a Second-order Op Amp Filter

6. From the graph of the gain, estimate the slope of the roll-off curve (should be 40 dB/decade).

7. Modify this program with your component values.

8. Compare the slope of the roll-off curve with the previous result in Exercise 4.5 for a single-pole lowpass filter.

9. If you have the time and components, try building a real two-pole circuit on the NI ELVIS II protoboard.

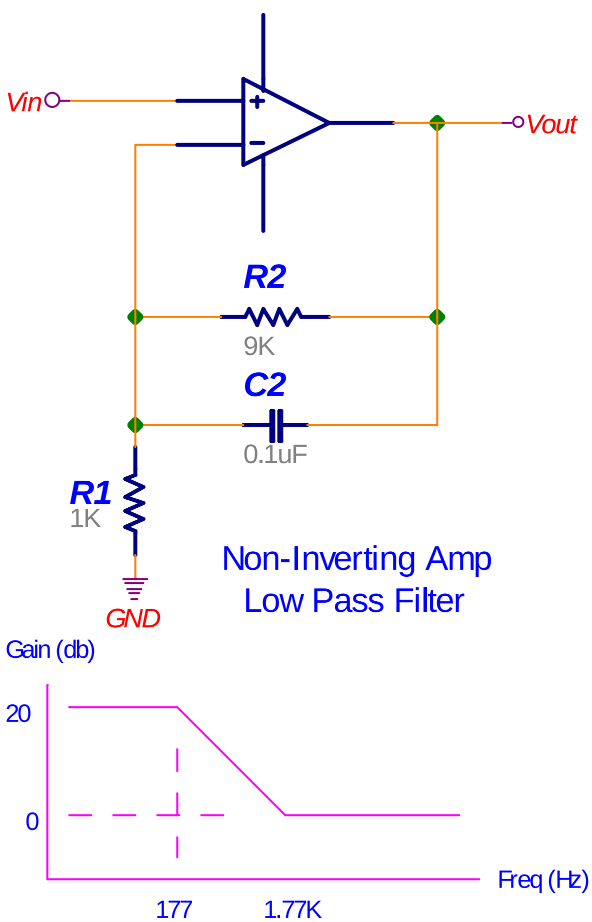
How well does the Bode plot of the theoretical design match your real circuit? Refer to the Lab 3 Multisim Challenge to recall how to overlay in Excel a theoretical design curve with a measured real curve.

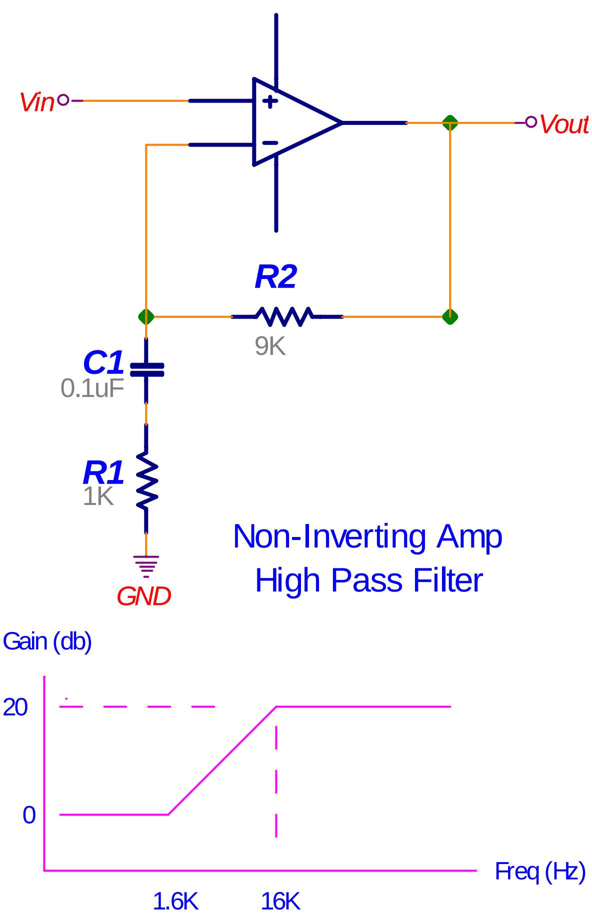
OP-AMP Filter Examples:

The two examples below show how adding a capacitor can change a non-inverting amplifiers frequency response. If the capacitor is removed you're left with a standard non-inverting amplifier with a gain of 10 (1 + R2/R1). Recall that the capacitors impedance depends on frequency (**$X_c = 1/(2\pi fC)$**) and the corner frequency of an RC filter is **$f_c = 1/(2\pi RC)$**.

In first circuit the capacitor is placed in parallel with the feedback resistor (R2). At low frequencies ($f << f_c$) the capacitors impedance (Xc) is much greater than R2 and therefore the parallel combination of R2 & Xc is about R2 (i.e. R2|| Xc = R2 when $f << f_c$). As frequency increases towards the corner frequency the impedance of the capacitor decreases and becomes comparable to that of the resistor. This lowers the impedance of the parallel combination of R2 & Xc and therefore the gain begins decreasing. When $f >> f_c$, R2|| Xc = Xc causing the gain to drop. In this case the gain bottoms out at one since the gain equation is 1 + R2/R1.

The second circuit has the capacitor in series with R1. When $f << f_c$ the capacitors reactance is large and R1 + Xc = Xc. Therefore the gain is 1 + R2/ Xc which = 1 when Xc >> R2. When $f >> f_c$ the capacitors reactance is small and R1 + Xc = R1. Therefore the gain is 1 + R2/ R1 which = 10 when Xc << R1.





Faculty of Engineering
Department of Electronics and Communications

GEE336
Electronic Circuits II

Lecture #7
Active Filters

Instructor:
Dr. Ahmad El-Banna

SUMMER 2015

Agenda

- Basic Filter Responses
- Filter Response Characteristics
- Active LPF, HPF, BPF & BSF
- Active Filters Based on Two-Integrators Loop
- Active Filters Based Upon Inductor replacement

BASIC FILTER RESPONSES

Intro.

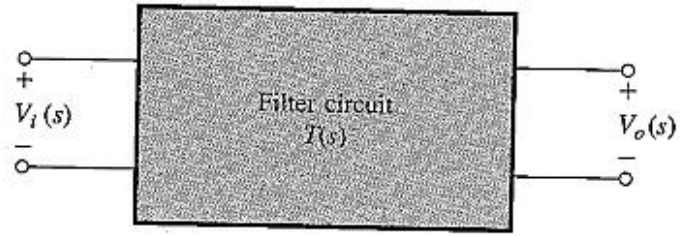
- **Filters** are circuits that are capable of **passing** signals with certain selected **frequencies** while **rejecting** signals with **other** frequencies.
- This property is called **selectivity**.
- **Active** filters use **transistors** or **op-amps** combined with passive RC, RL, or RLC circuits.

- The **passband** of a filter is the range of frequencies that are allowed to pass through the filter with **minimum attenuation**.
- The **critical frequency**, (also called the **cutoff frequency**) defines the **end of the passband** and is normally specified at the point where the response drops (**70.7%**) from the passband response.

- Following the passband is a region called the **transition region** that leads into a region called the **stopband**.
- There is **no precise point** between the transition region and the stopband.

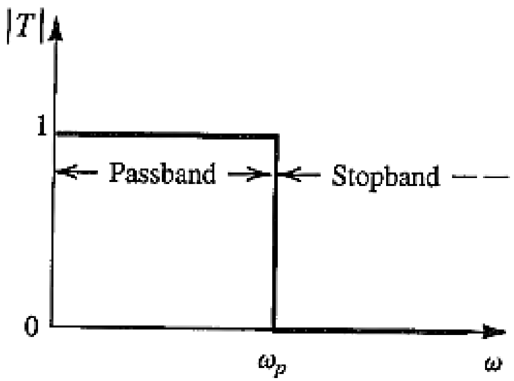
Basic Filter Responses

- **Ideal Response**

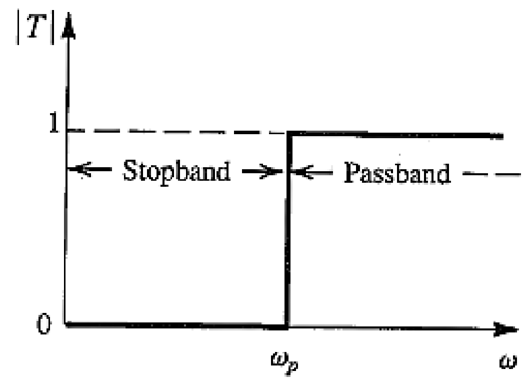


Filter transfer function

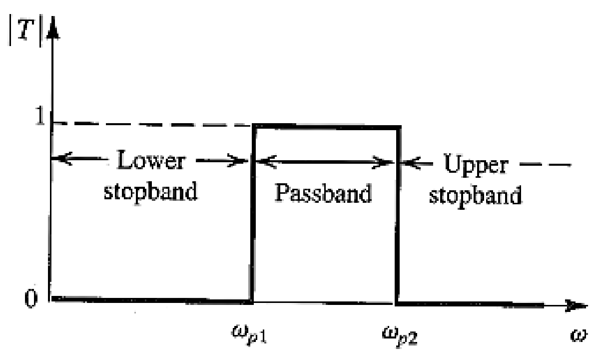
$$T(s) \equiv \frac{V_o(s)}{V_i(s)}$$



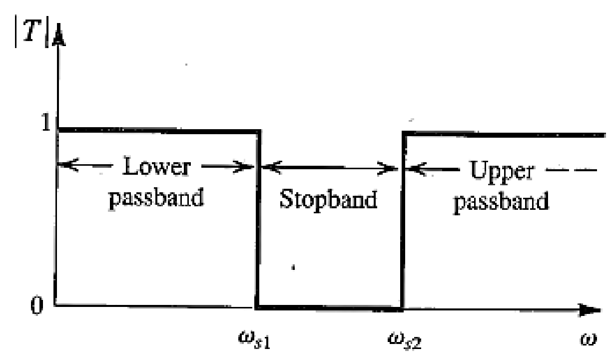
(a) Low-pass (LP)



(b) High-pass (HP)



(c) Bandpass (BP)



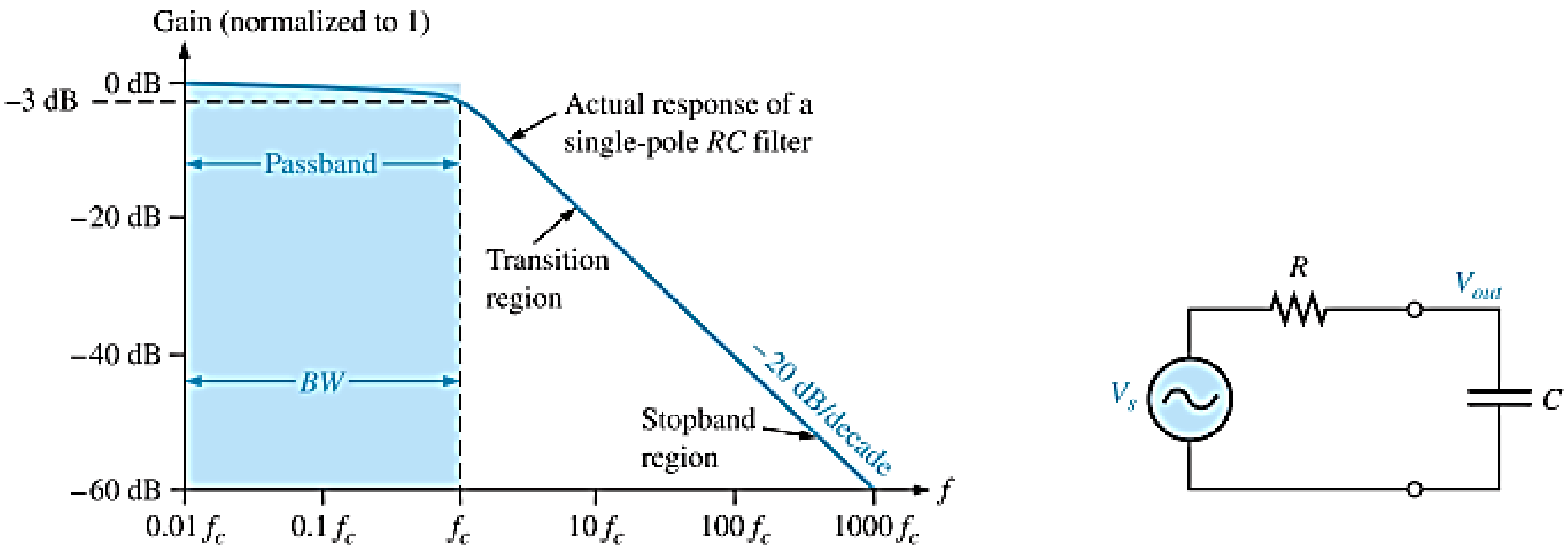
(d) Bandstop (BS)

Basic Filter Responses

- Actual filter responses depend on the **number of poles**, a term used with filters to describe the **number of RC circuits** contained in the filter.
- The -20 dB/decade **roll-off** rate for the gain of a basic RC filter means that at a frequency of $10\,f_c$, the output will be -20dB (10%) of the input.
- This roll-off rate is **not a good filter characteristic** because too much of the unwanted frequencies (beyond the passband) are allowed through the filter.

Basic Filter Responses

- **Low-Pass Filter Response**



(a) Comparison of an ideal low-pass filter response (blue area) with actual response. Although not shown on log scale, response extends down to $f_c = 0$.

(b) Basic low-pass circuit

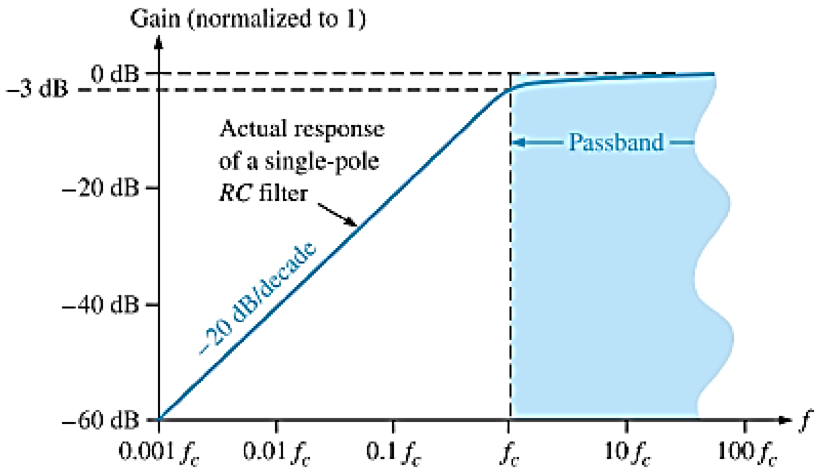
$$BW = f_c$$

$$f_c = \frac{1}{2\pi RC}$$

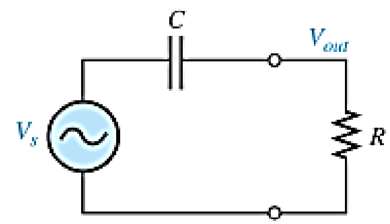
$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{1/sc}{R + 1/sc} = \frac{1}{1 + scR}$$

Basic Filter Responses..

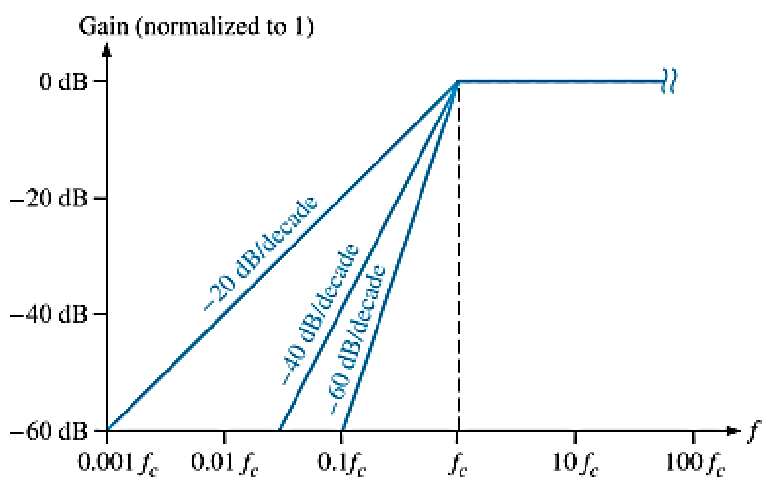
- **High-Pass Filter Response**



(a) Comparison of an ideal high-pass filter response (blue area) with actual response



(b) Basic high-pass circuit



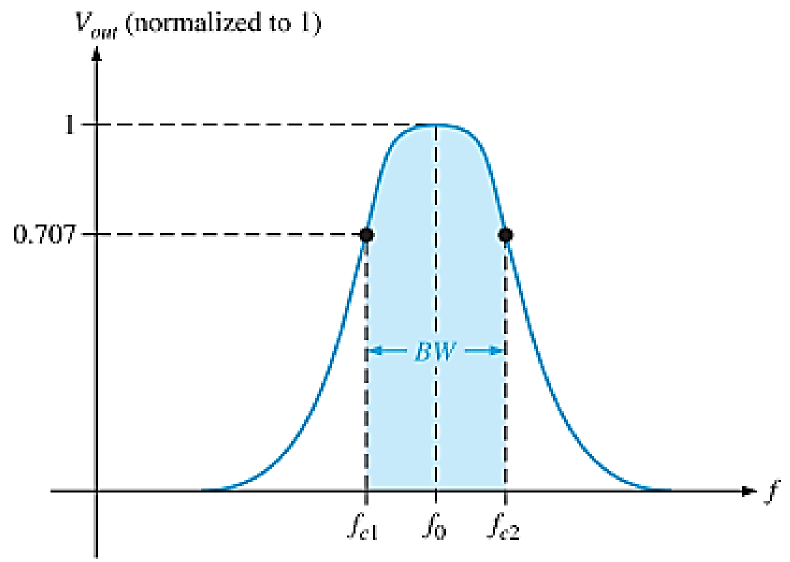
$$f_c = \frac{1}{2\pi RC}$$

Basic Filter Responses...

- **Band-Pass Filter Response**

$$BW = f_{c2} - f_{c1}$$

$$f_0 = \sqrt{f_{c1} f_{c2}}$$



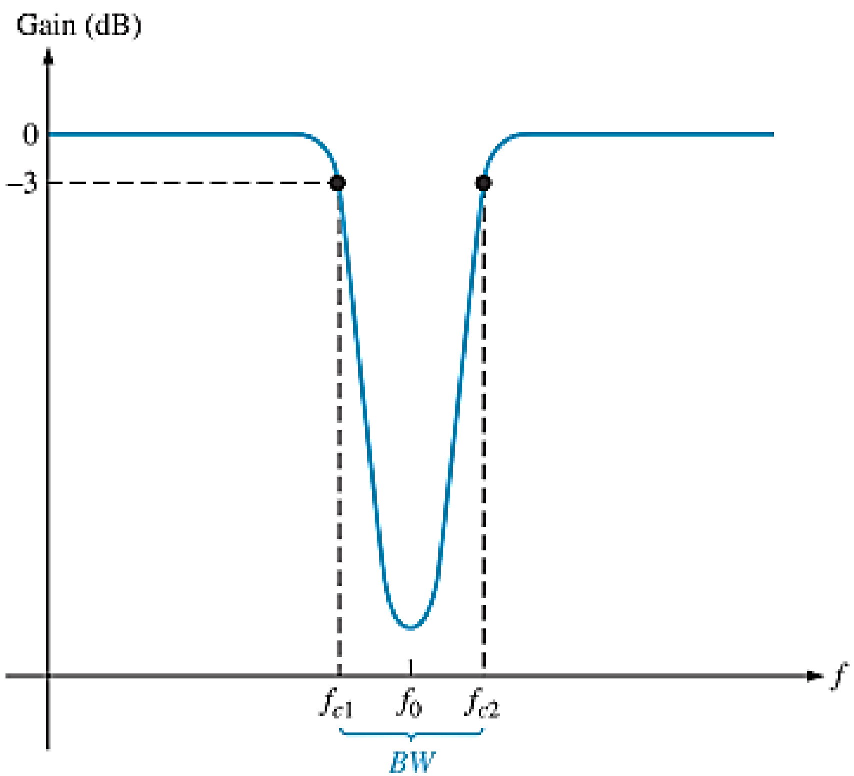
- The **quality factor** (Q) of a band-pass filter is the ratio of the center frequency to the bandwidth.
- The higher the value of Q, the narrower the bandwidth and the better the selectivity for a given value of f_0.
- Band-pass filters are sometimes classified as **narrow-band** (Q>10) or **wide-band** (Q<10).

$$Q = \frac{f_0}{BW}$$

Basic Filter Responses....

- **Band-Stop Filter Response**

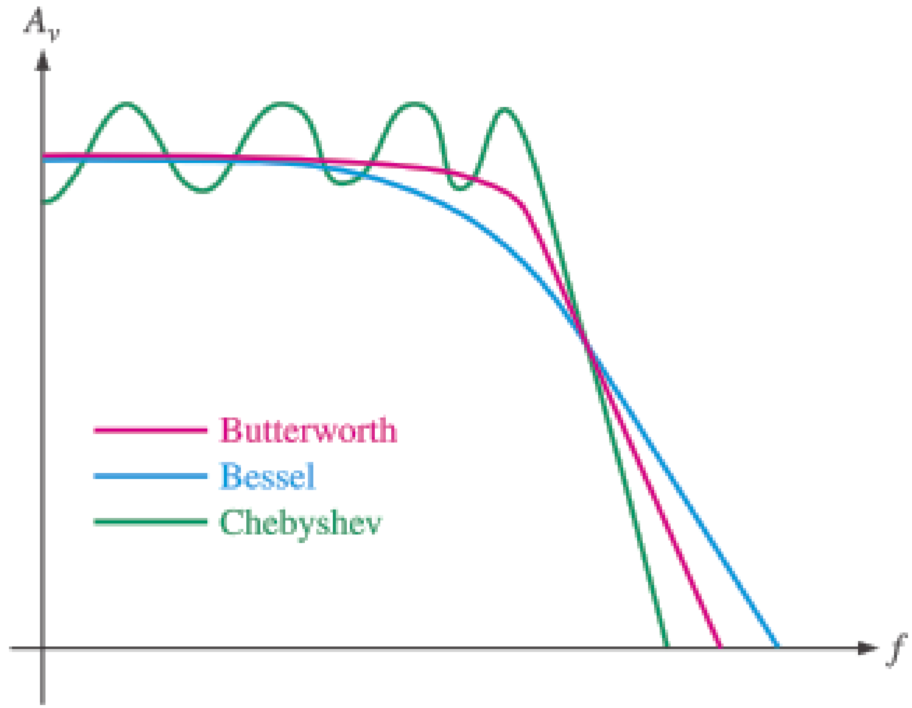
  also known as notch, band-reject, or band-elimination filter.



FILTER RESPONSE CHARACTERISTICS

FILTER RESPONSE CHARACTERISTICS

- Each type of filter response (low-pass, high-pass, band-pass, or band-stop) can be tailored by ***circuit component values*** to have either a
  - **Butterworth,**
  - **Chebyshev, or**
  - **Bessel** characteristic.
- Each of these characteristics is identified by the **shape of the response curve**, and each has an advantage in certain applications.



<u>The Butterworth Characteristic</u>

- The Butterworth characteristic provides a **very flat amplitude response** in the passband and a roll-off rate of -20 dB/decade/pole.
- The **phase response is not linear**, and the phase shift (thus, time delay) of signals passing through the filter varies nonlinearly with frequency.
- Therefore, a **pulse** applied to a Butterworth filter will **cause overshoots** on the output because each frequency component of the pulse's rising and falling edges experiences a different time delay.

FILTER RESPONSE CHARACTERISTICS..

The Chebyshev Characteristic

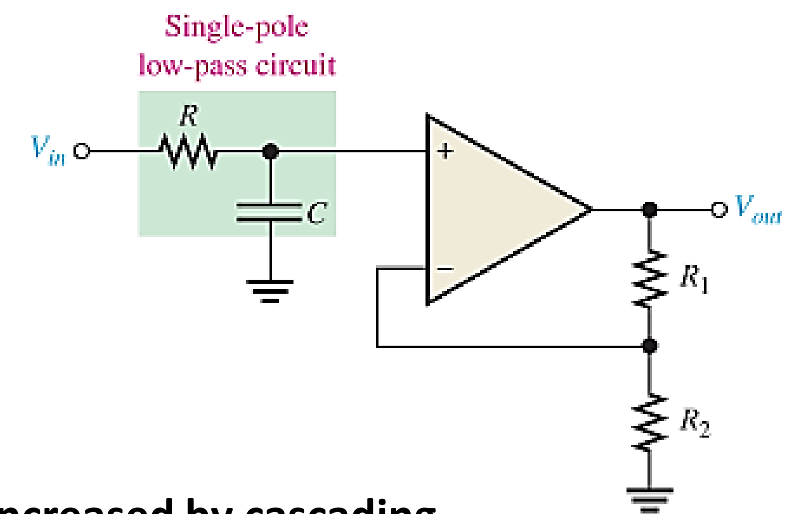
- Filters with the Chebyshev response characteristic are useful when a **rapid roll-off** is required because it provides a roll-off rate greater than -20 dB/decade/pole.
- This is a **greater rate** than that of the Butterworth, so filters can be implemented with the Chebyshev response with **fewer poles** and **less complex** circuitry for a given roll-off rate.
- This type of filter response is characterized by overshoot or **ripples in the passband** (depending on the number of poles) and an even **less linear phase response** than the Butterworth.

The Bessel Characteristic

- The Bessel response exhibits a **linear phase characteristic**, meaning that the phase shift increases linearly with frequency.
- The result is almost **no overshoot on the output** with a pulse input.
- It has the **slowest roll-off** rate.

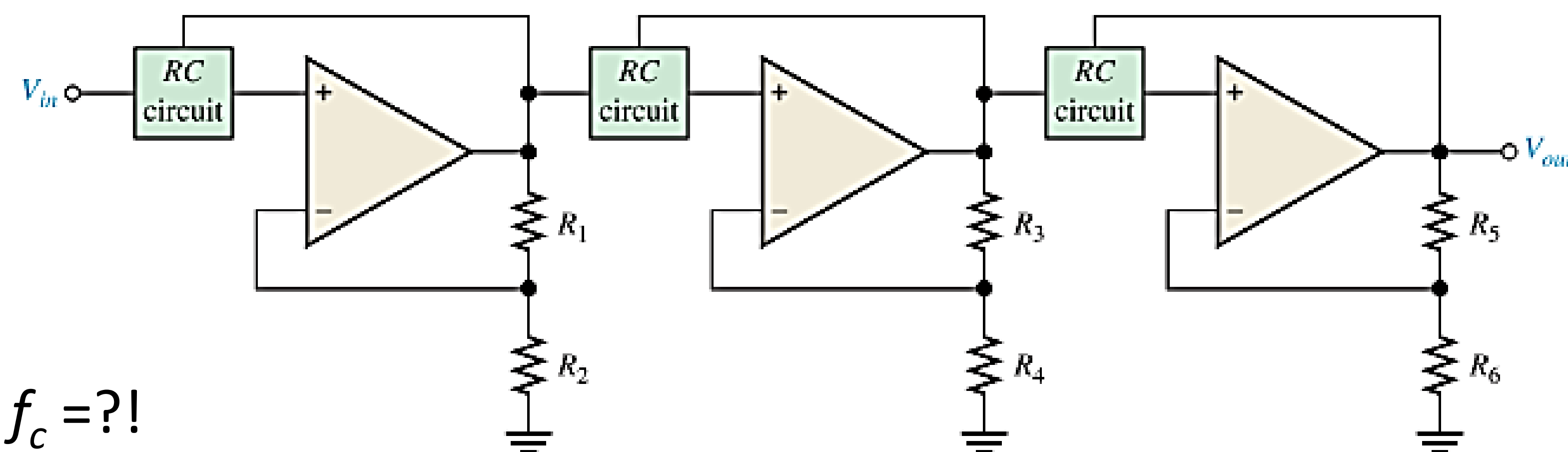
Critical Frequency and Roll-Off Rate

$$f_c = \frac{1}{2\pi RC}$$



- The number of filter **poles** can be **increased by cascading**.

<u>Example:</u> Third-order (three-pole) filter



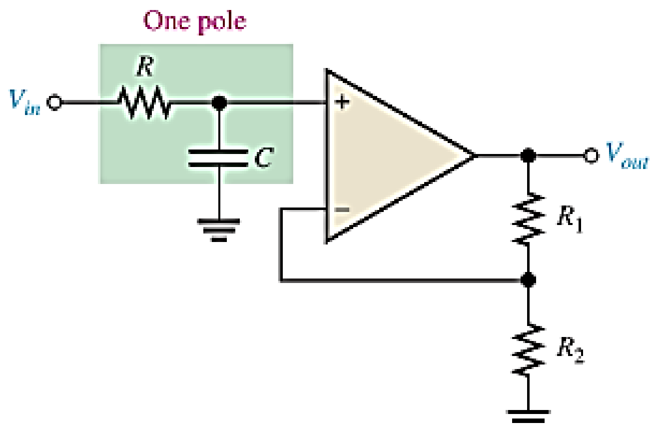
f_c =?!

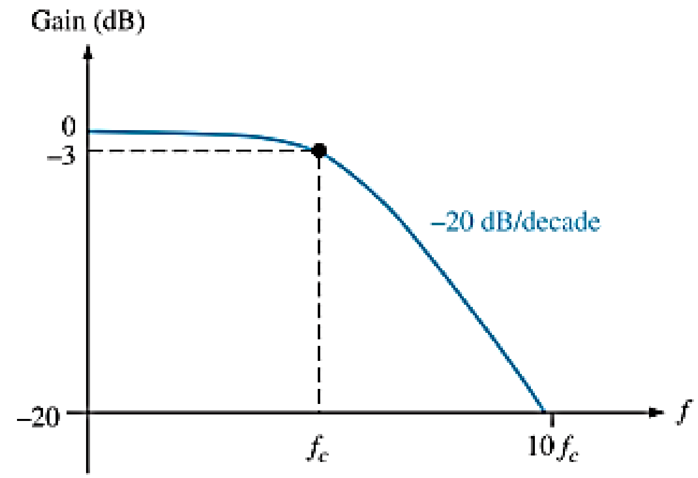
ACTIVE LOW-PASS FILTERS

Advantages of Op-Amp Active Filters

- Filters that use **op-amps** as the **active element** provide several **advantages** over passive filters (R, L, and C elements only).
  - The op-amp provides **gain**, so the **signal is not attenuated** as it passes through the filter.
  - The high input impedance of the op-amp **prevents excessive loading of the driving source**.
  - The low output impedance of the op-amp **prevents the filter from being affected by the load** that it is driving.
  - Active filters are also **easy to adjust over a wide frequency range** without altering the desired response.

Single-Pole LPF



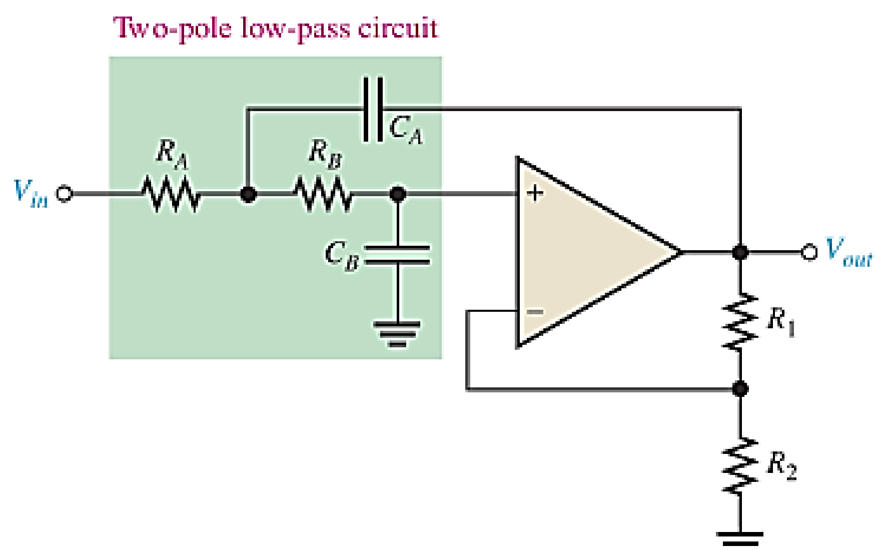


$$A_{cl(NI)} = \frac{R_1}{R_2} + 1$$

$$f_c = \frac{1}{2\pi RC}$$

2-Pole LPF
The Sallen-Key LPF (2^{nd} Order)

- It is used to provide **very high Q factor and passband gain without the use of inductors**.
- It is also known as a **VCVS** (voltage-controlled voltage source) filter.



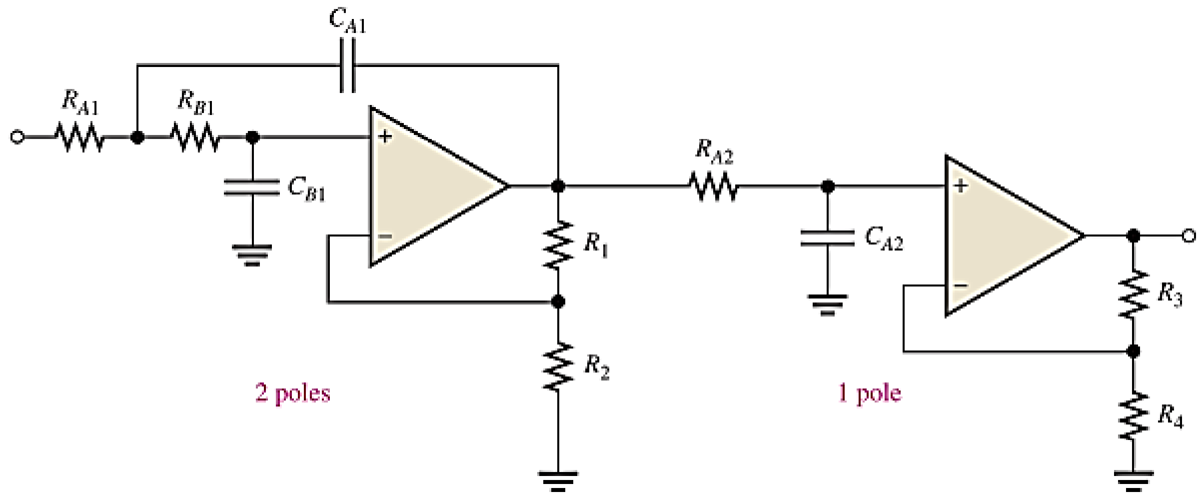
$$f_c = \frac{1}{2\pi\sqrt{R_A R_B C_A C_B}}$$

$$f_c = \frac{1}{2\pi RC} \quad @ \; R_A = R_B = R \text{ and } C_A = C_B = C.$$

<u>Assignment:</u>
Derive the f_c equation.

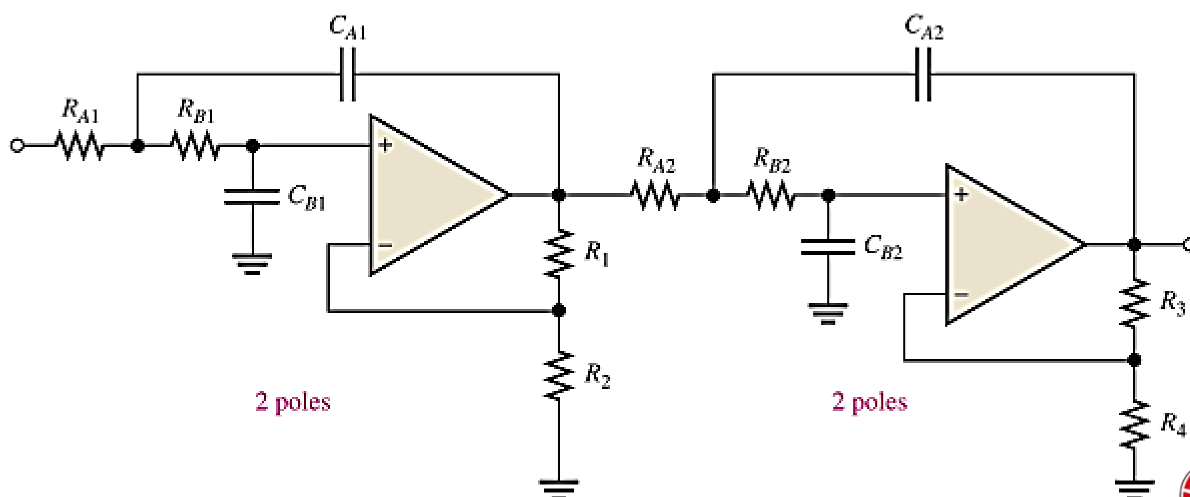
Cascaded LPF

- A **three-pole** filter is required to get a **third-order** low-pass response.



(a) Third-order configuration

- A **four-pole** filter is **preferred** because it uses the same number of op-amps to achieve a faster roll-off.

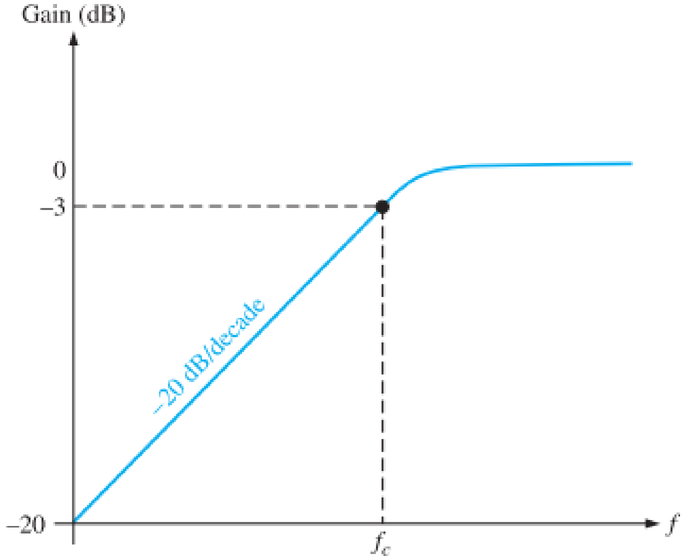


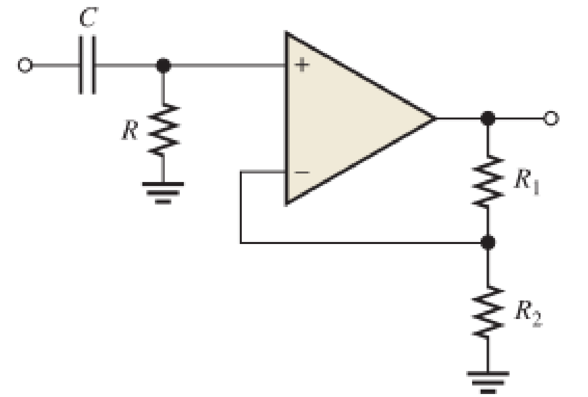
(b) Fourth-order configuration

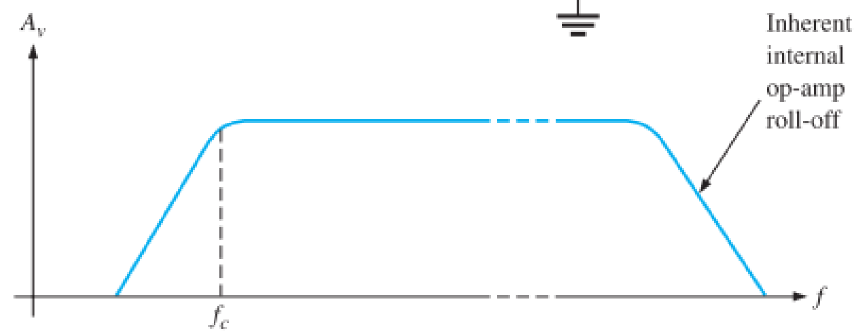
In high-pass filters, the roles of the capacitor and resistor are reversed in the RC circuits.

ACTIVE HIGH-PASS FILTERS

Single Pole HPF

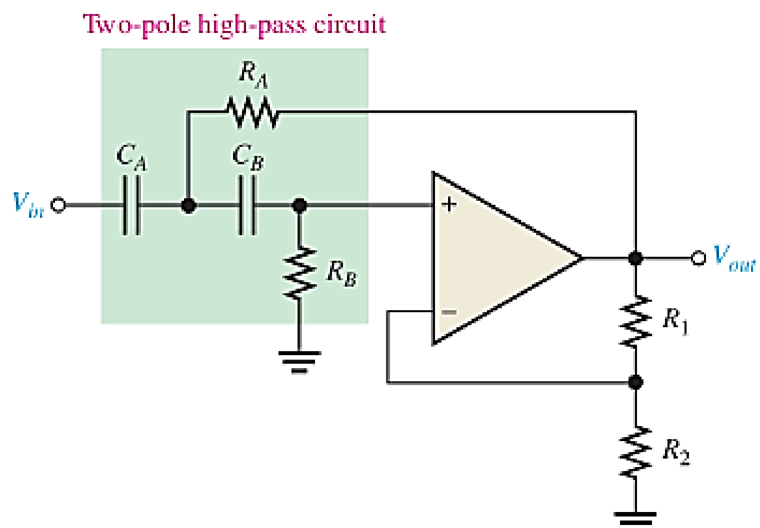




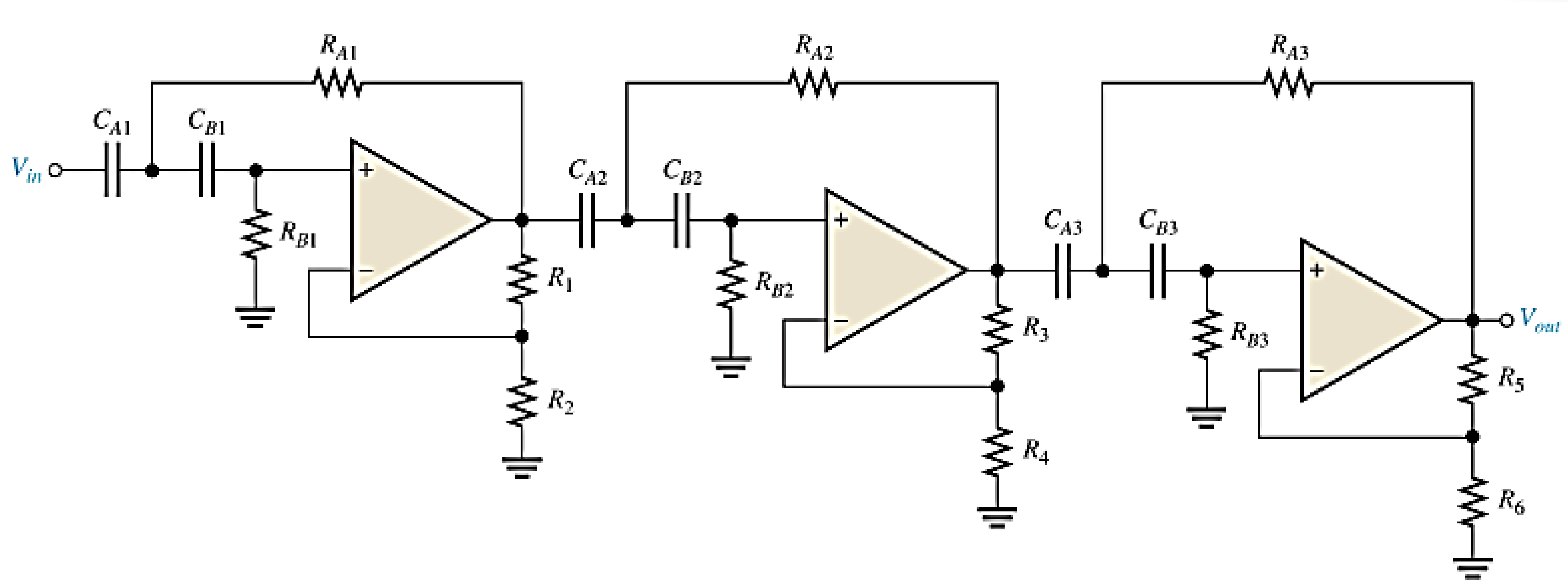


(b) Nonideal

Sallen-Key HPF



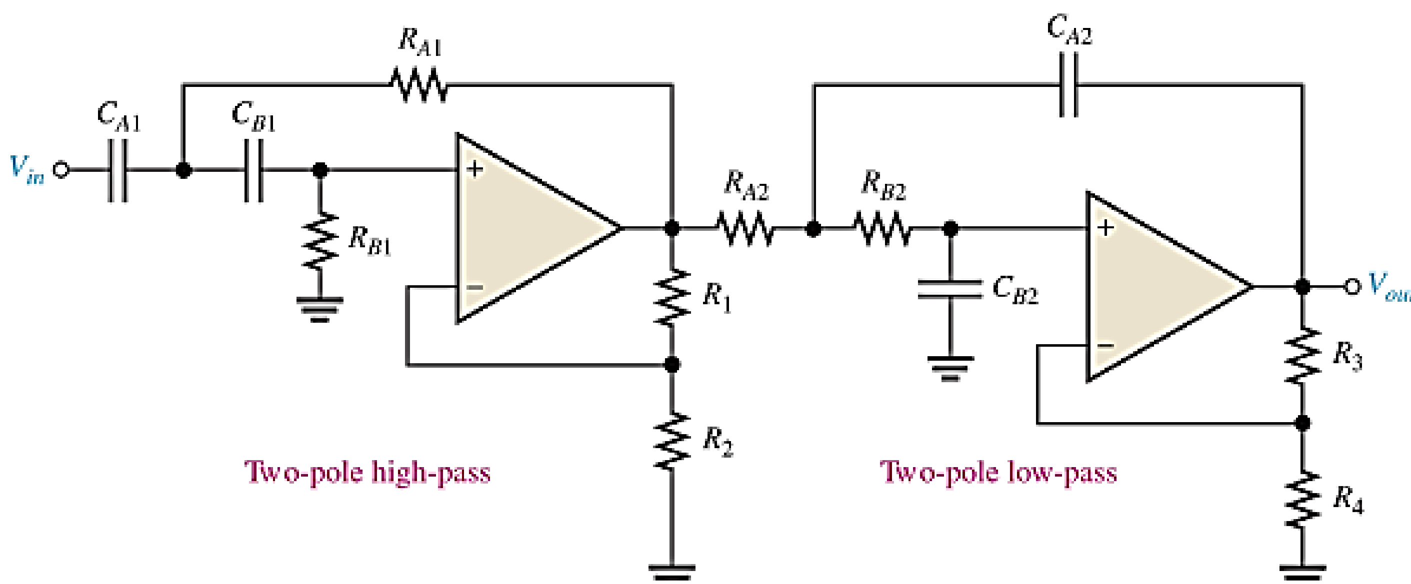
Cascaded HPF



Order = ?
roll-off = ?

ACTIVE BAND-PASS FILTERS

Cascaded Low-Pass and High-Pass Filters



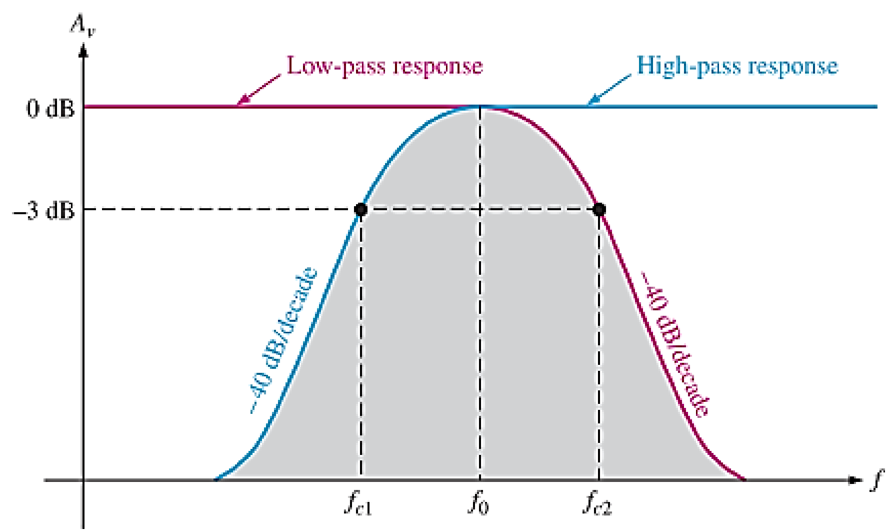
$$f_{c1} = \frac{1}{2\pi\sqrt{R_{A1}R_{B1}C_{A1}C_{B1}}}$$

$$f_{c2} = \frac{1}{2\pi\sqrt{R_{A2}R_{B2}C_{A2}C_{B2}}}$$

$$f_0 = \sqrt{f_{c1}f_{c2}}$$

If equal components,

$$f_c = 1/(2\pi RC).$$



ACTIVE FILTERS BASED ON TWO-INTEGRATORS LOOP

Biquad Filter

(Two-Integrators Loop biquadratic circuit)

- "Biquad" is an abbreviation of "**biquadratic**", which refers to the fact that its **transfer function** is the ratio of two quadratic functions.

- To derive the biquad circuit, consider the 2nd order high pass transfer function

$$\frac{V_{hp}}{V_i} = \frac{Ks^2}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

Cross multiply and reform,

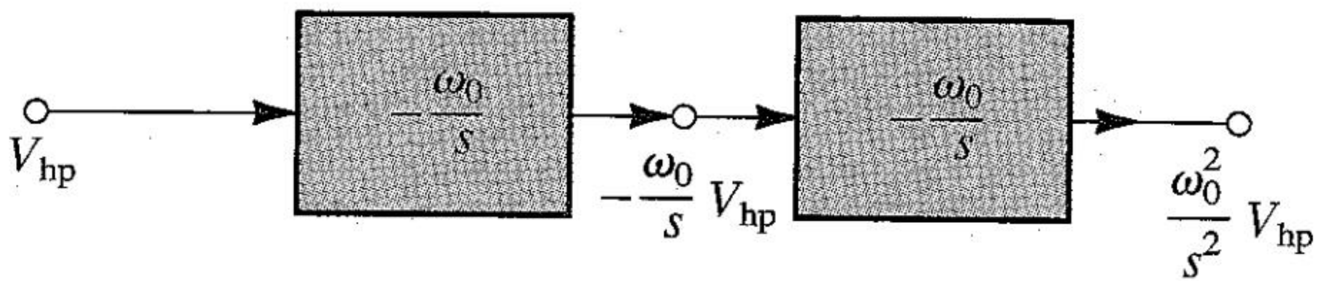
$$V_{hp} + \frac{1}{Q}\left(\frac{\omega_0}{s}V_{hp}\right) + \left(\frac{\omega_0^2}{s^2}V_{hp}\right) = KV_i$$

$$V_{hp} = KV_i - \frac{1}{Q}\frac{\omega_0}{s}V_{hp} - \frac{\omega_0^2}{s^2}V_{hp}$$

Biquad Filter ..

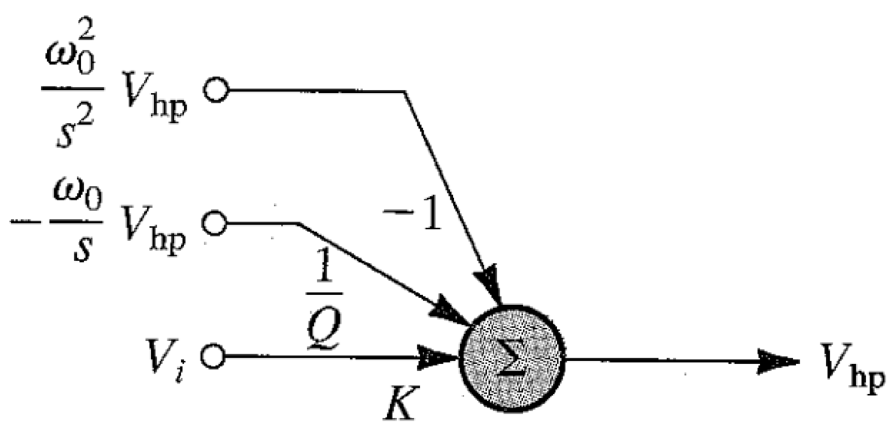
- Derivation of a block diagram realization of the two-integrator loop biquad

$$V_{hp} + \frac{1}{Q}\left(\frac{\omega_0}{s}V_{hp}\right) + \left(\frac{\omega_0^2}{s^2}V_{hp}\right) = KV_i$$



(a)

$$V_{hp} = KV_i - \frac{1}{Q}\frac{\omega_0}{s}V_{hp} - \frac{\omega_0^2}{s^2}V_{hp}$$



(b)

Biquad Filter …

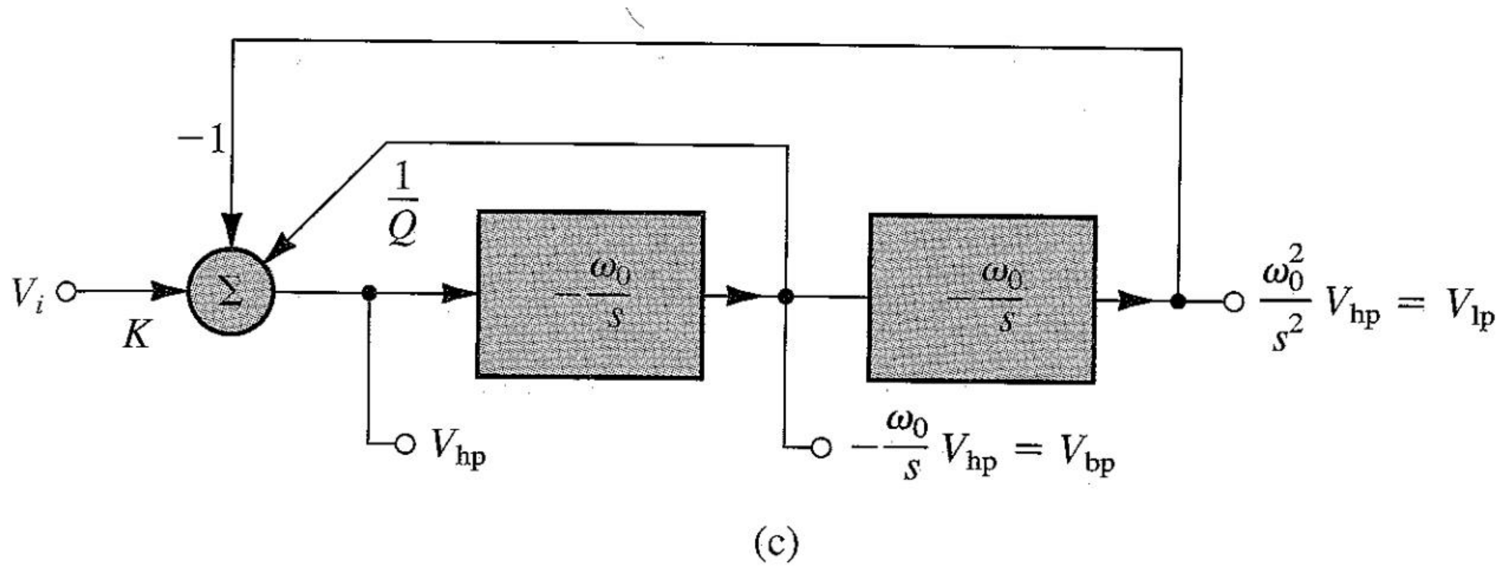


FIGURE 12.23 Derivation of a block diagram realization of the two-integrator-loop biquad.

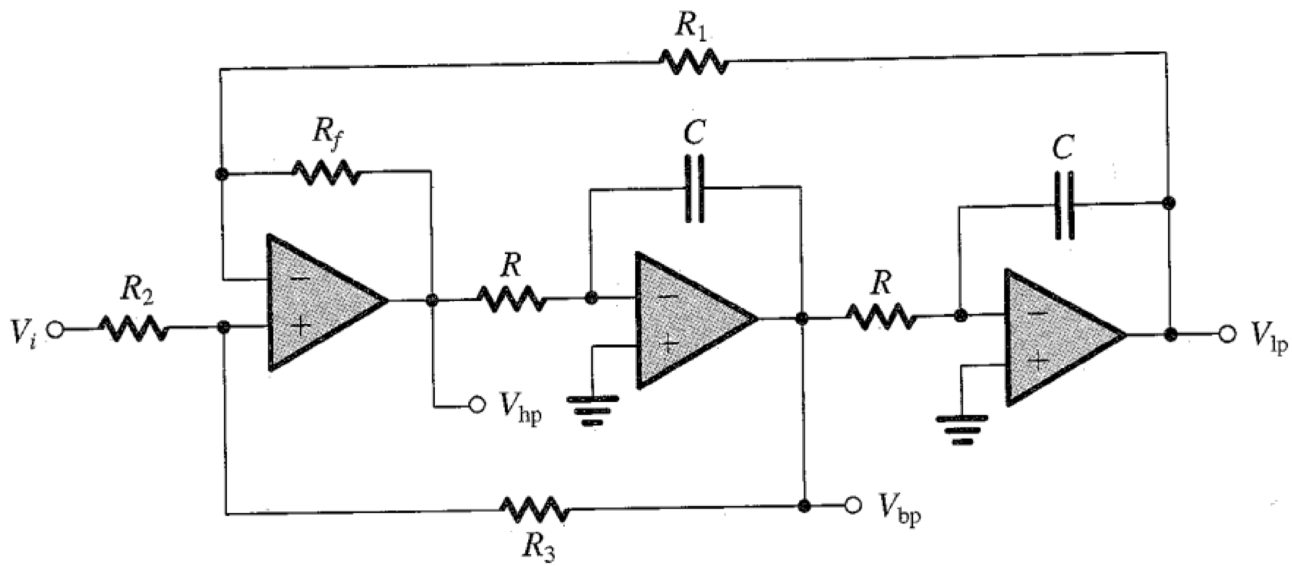
$$\frac{(-\omega_0/s)V_{hp}}{V_i} = -\frac{K\omega_0 s}{s^2 + s(\omega_0/Q) + \omega_0^2} = T_{bp}(s)$$

$$V_{hp} = KV_i - \frac{1}{Q}\frac{\omega_0}{s}V_{hp} - \frac{\omega_0^2}{s^2}V_{hp}$$

$$\frac{(\omega_0^2/s^2)V_{hp}}{V_i} = \frac{K\omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2} = T_{lp}(s)$$

Biquad Filter ….

(Universal Circuit)

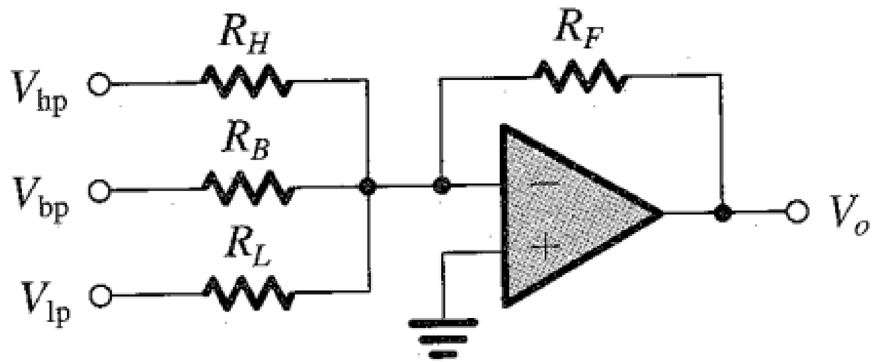


$$V_{hp} = \frac{R_3}{R_2+R_3}\left(1+\frac{R_f}{R_1}\right)V_i + \frac{R_2}{R_2+R_3}\left(1+\frac{R_f}{R_1}\right)\left(-\frac{\omega_0}{s}V_{hp}\right) - \frac{R_f}{R_1}\left(\frac{\omega_0^2}{s^2}V_{hp}\right)$$

$R_f/R_1 = 1$ $R_3/R_2 = 2Q-1$ $K = 2-(1/Q)$

Biquad Filter ….

- To obtain notch and all-pass function, the three outputs of the biquad are summed with appropriate weights



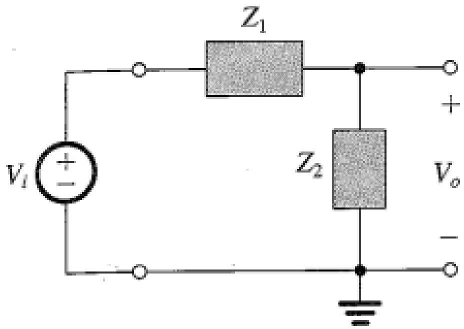
$$\frac{V_o}{V_i} = -K\frac{(R_F/R_H)s^2 - s(R_F/R_B)\omega_0 + (R_F/R_L)\omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

Notch filter as example, use

$$R_B = \infty \qquad \frac{R_H}{R_L} = \left(\frac{\omega_n}{\omega_0}\right)^2$$

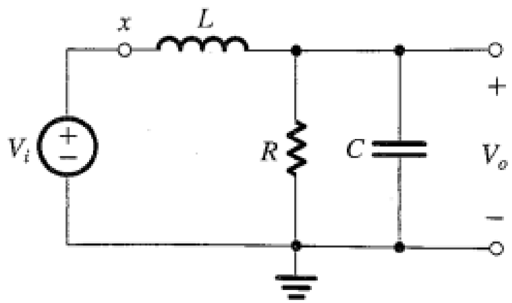
ACTIVE FILTERS BASED UPON INDUCTOR REPLACEMENT

2^{nd} order LCR Resonator



(a) General structure

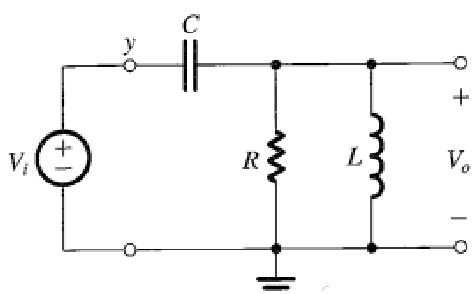
$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s)+Z_2(s)}$$



(b) LP

$$T(s) \equiv \frac{V_o}{V_i} = \frac{Z_2}{Z_1+Z_2} = \frac{Y_1}{Y_1+Y_2} = \frac{1/sL}{(1/sL)+sC+(1/R)}$$

$$= \frac{1/LC}{s^2+s(1/CR)+(1/LC)}$$

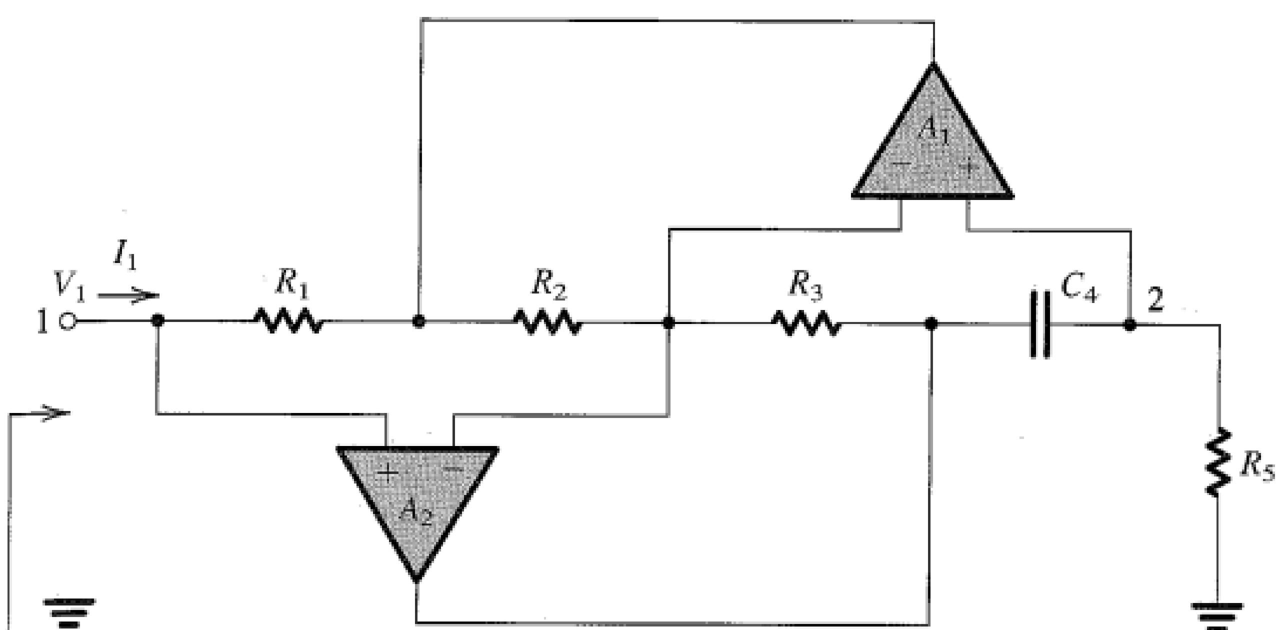


(c) HP

$$T(s) \equiv \frac{V_o}{V_i} = \frac{a_2 s^2}{s^2+s(\omega_0/Q)+\omega_0^2}$$

2^{nd} order Active Filter based on inductor replacement

The Antoniou Inductance-Simulation Circuit



$$Z_{in} \equiv \frac{V_1}{I_1} = sC_4R_1R_3R_5/R_2$$

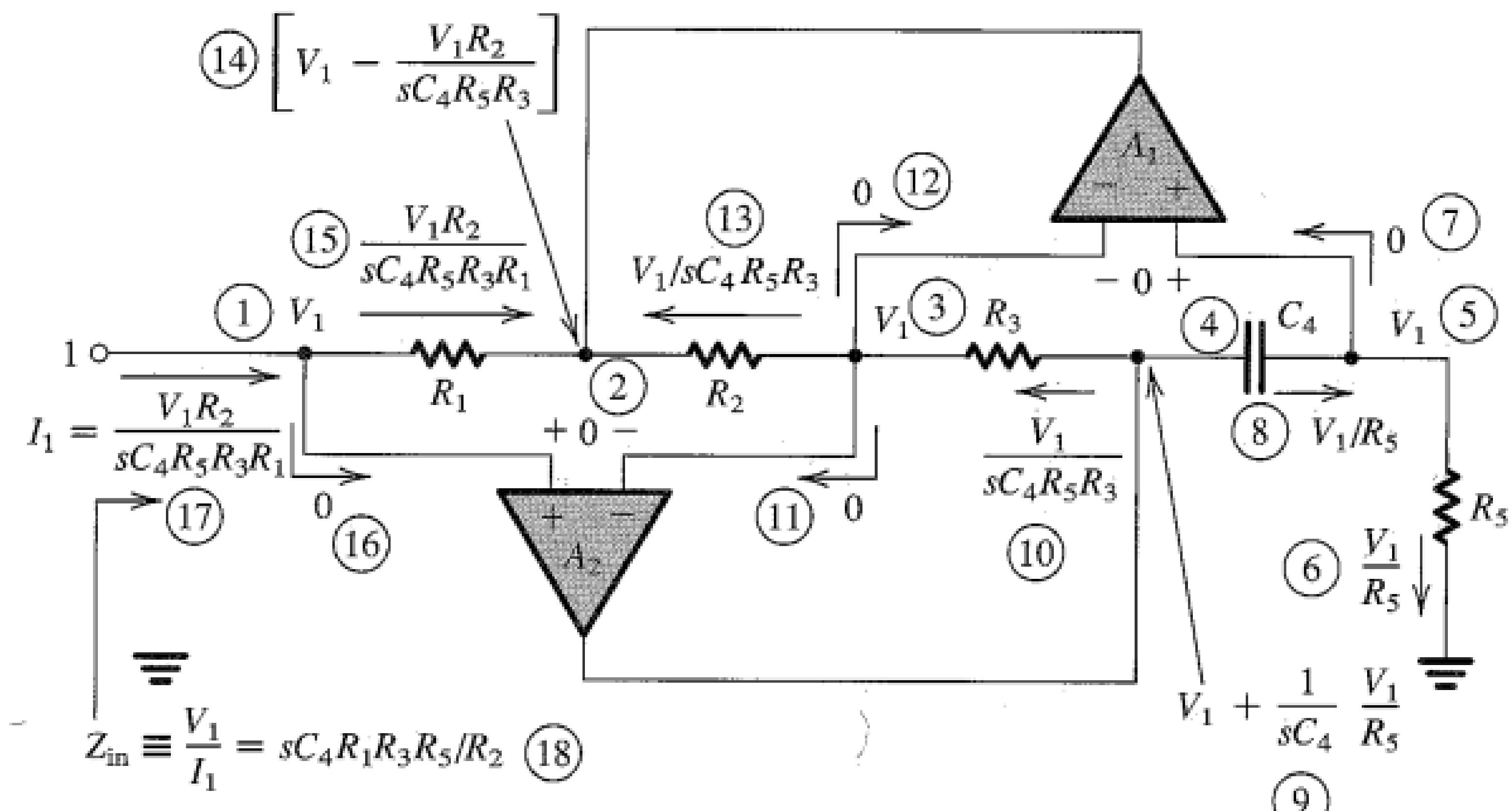
(a)

$$L = C_4R_1R_3R_5/R_2$$

selecting $R_1 = R_2 = R_3 = R_5 = R$ and $C_4 = C$,

$$L = CR^2.$$

2^{nd} order Active Filter based on inductor replacement ..

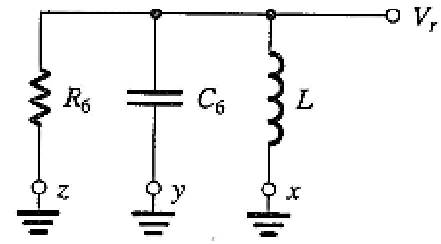


(b)

FIGURE 12.20 (a) The Antoniou inductance-simulation circuit. (b) Analysis of the circuit assuming ideal op amps. The order of the analysis steps is indicated by the circled numbers.

2nd order Active Filter based on inductor replacement …

The Op Amp–RC Resonator



(a)

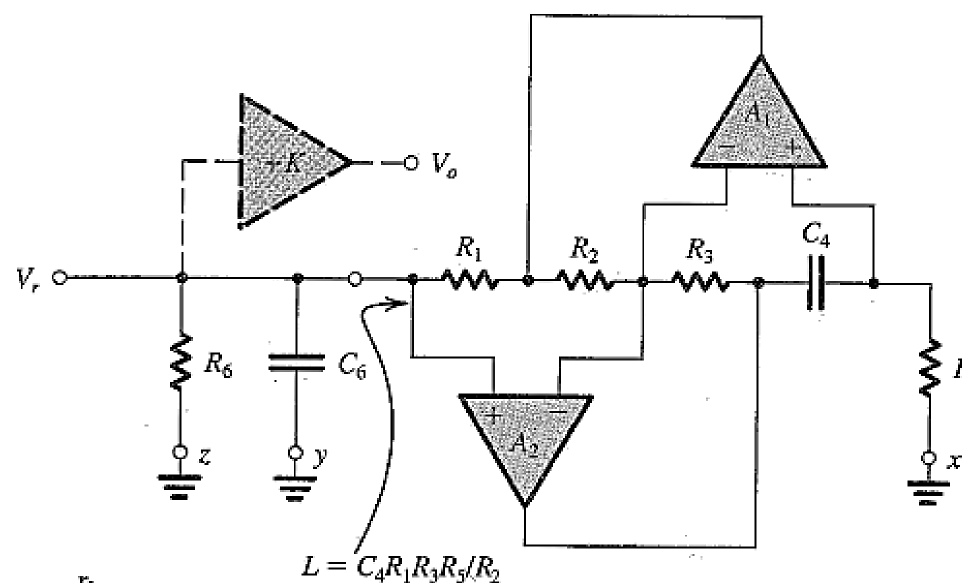
$$\omega_0 = 1/\sqrt{LC_6} = 1/\sqrt{C_4C_6R_1R_3R_5/R_2}$$

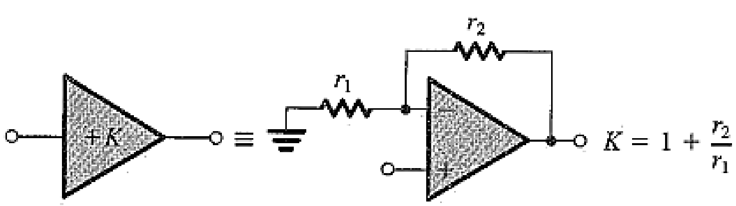
$$Q = \omega_0 C_6 R_6 = R_6\sqrt{\frac{C_6}{C_4}\frac{R_2}{R_1R_3R_5}}$$

$$C_4 = C_6 = C \text{ and } R_1 = R_2 = R_3 = R_5 = R$$

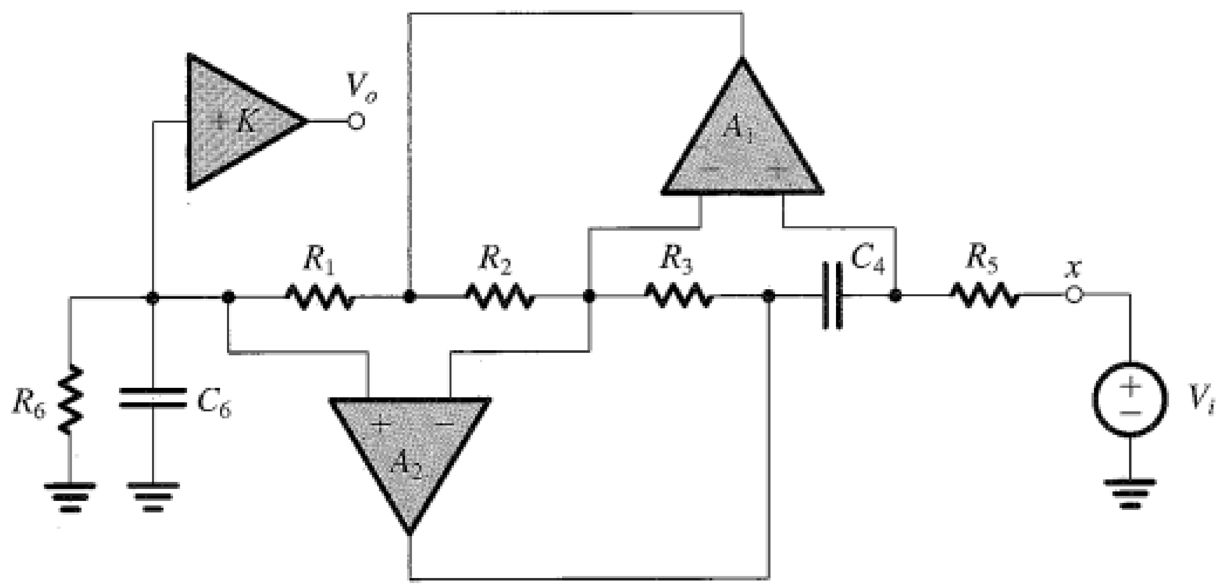
$$\omega_0 = 1/CR$$

$$Q = R_6/R$$





LPF
with inductor replacement circuit



(a) LP

$$T(s) = \frac{KR_2/C_4C_6R_1R_3R_5}{s^2 + s\frac{1}{C_6R_6} + \frac{R_2}{C_4C_6R_1R_3R_5}}$$

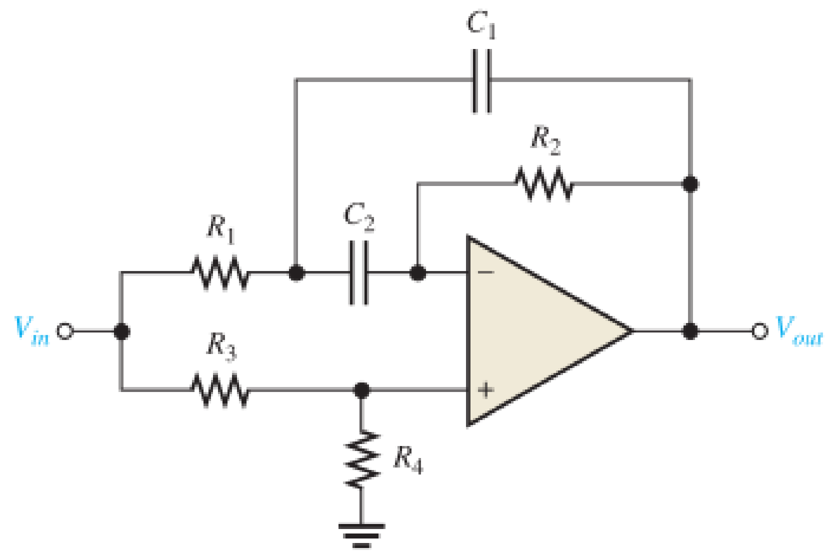
K = DC gain

$$\omega_0 = 1/\sqrt{LC_6} = 1/\sqrt{C_4C_6R_1R_3R_5/R_2}$$

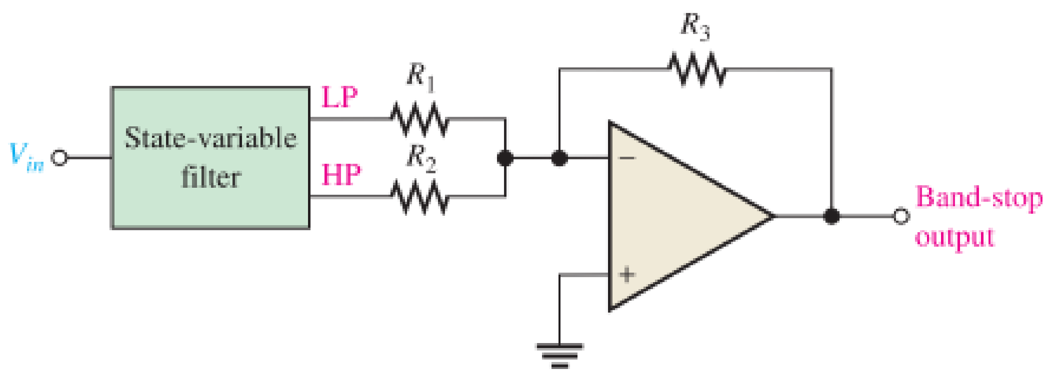
$$Q = \omega_0 C_6 R_6 = R_6\sqrt{\frac{C_6}{C_4}\frac{R_2}{R_1R_3R_5}}$$

ACTIVE BAND-STOP FILTERS

Multiple-Feedback Band-Stop Filter



State-Variable Band-Stop Filter

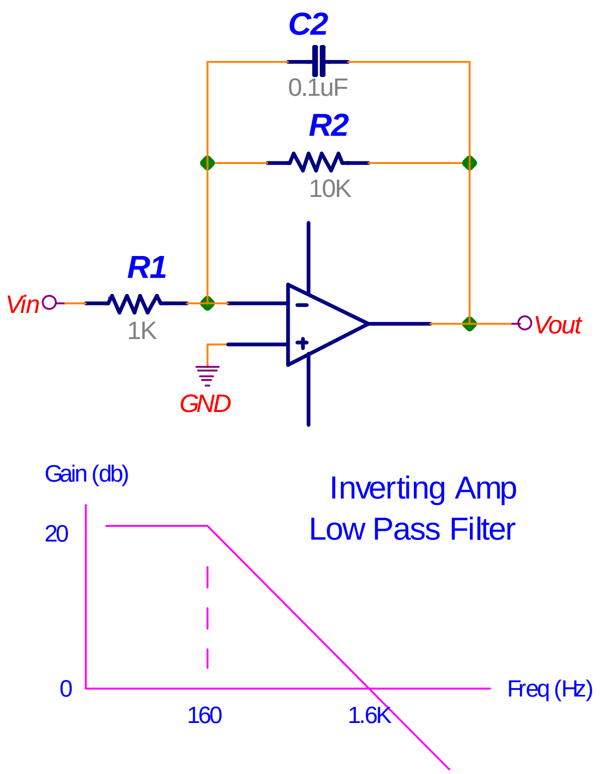


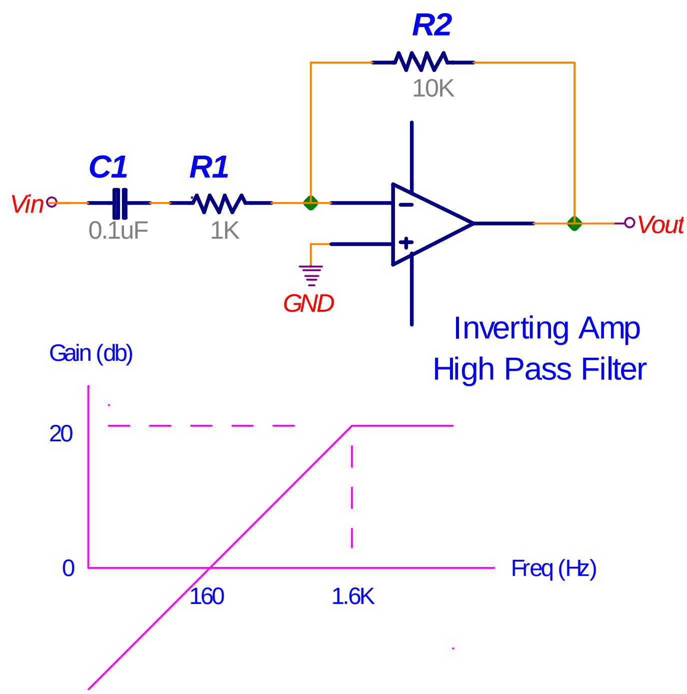
- For more details, refer to:
  - Chapter 15 at T. Floyd, **Electronic Devices**,9th edition.
  - Chapter 12 at Sedra & Smith, **Microelectronic Circuits**, 5th edition.
- The lecture is available online at:
  - http://bu.edu.eg/staff/ahmad.elbanna-courses/12884
- For inquires, send to:
  - ahmad.elbanna@feng.bu.edu.eg

High and low pass filters can be made by adding capacitors to inverting amplifiers as well.

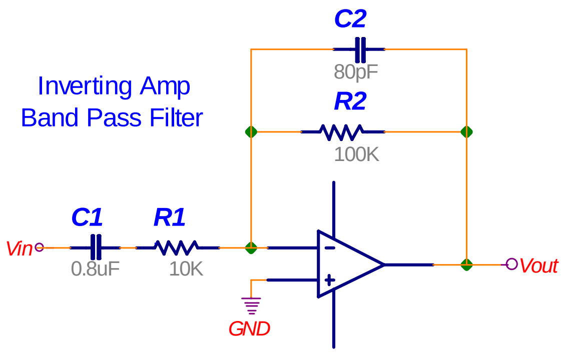
The first circuit is a low pass filter. At low frequencies the capacitors impedance is high, much higher than R2, and therefore doesn't affect the circuit ($X_C||R2 = R2$). At high frequencies the capacitors impedance is low, much lower than R2, and therefore limits the impedance of the parallel combination ($X_C||R2 = X_C$). Since the gain equation for a non-inverting amp is –R2/R1 the gain doesn't bottom out at one. The gain continues to decrease as frequency increases beyond the cutoff frequency.

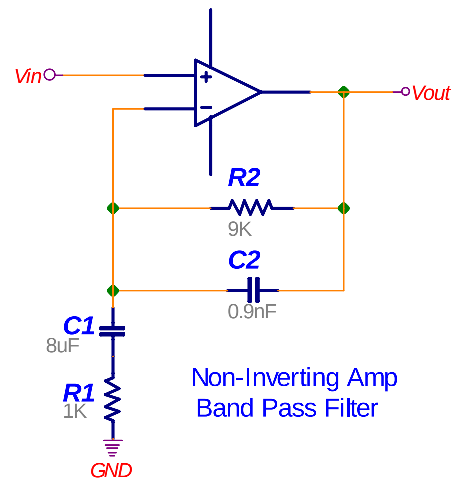
The second circuit is a high pass filter. At low frequencies (below the cutoff frequency) the capacitors impedance is high, much higher than R1, and therefore $R1 + X_C = X_C$. The gain is therefore $R2/X_C$. At high frequencies the capacitors impedance is low, much lower than R1, and therefore $R1 + X_C = R1$. The gain is therefore R2/R1.

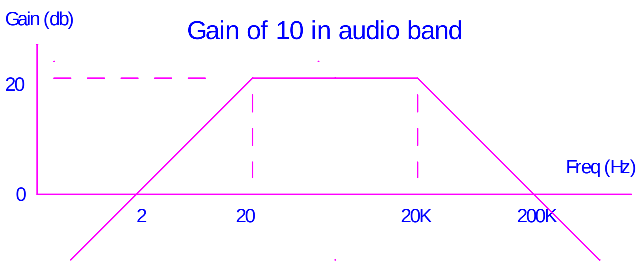


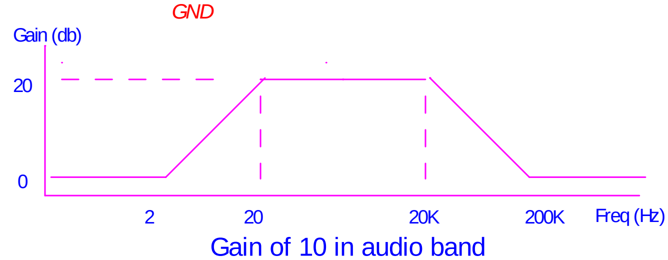


The low pass and high pass filter can be combined into a band pass filter. In the examples below the corner frequencies were chosen to be the audio band (20Hz – 20KHz). Notice the difference in the gain outside of the pass band. The gain of the inverting amplifier continues to drop as you get farther away from the pass band. The gain of the non-inverting amplifier only drops to 1 (0db).









Each gain stage can be combined with another for a larger gain and a steeper roll-off of the frequency.