

# Wideband Microstrip Coupled-Line Ring Hybrids for High Power-Division Ratios

Hee-Ran Ahn, *Senior Member, IEEE*, and Sangwook Nam, *Senior Member, IEEE*

**Abstract**—Wideband coupled-line ring hybrids are presented for high power-division ratios. For this, an equivalent circuit to make the characteristic impedance of a transmission-line section lower is firstly investigated, based on a  $\Pi$ -type lumped-element equivalent circuits. Design formulas of four types of equivalent circuits ( $L_{\Pi}$ -,  $L_T$ -,  $L_{S2}$ -, and  $L_{S1}$ -types) are then derived, and  $N$  is introduced for more design flexibility and wideband performance. The  $L_{S2}$ -type with  $N = 1$  is composed of one transmission-line section and two identical series inductances at both ends, which cannot be used for the ring hybrids with the high power-division ratios. To avoid the problem, modified  $L_{S2}$ -type ( $ML_{S2}$ -type) with  $N$  is additionally suggested, and that with  $N = 1$  is intensively discussed. Using two of those,  $L_{S1}$ -type with  $N = 2$  and  $ML_{S2}$ -type with  $N = 1$ , the coupled-line ring hybrids with 13- and 11-dB power-division ratios are fabricated. The measured results show good agreement with those predicted.

**Index Terms**—Arbitrary power divisions, coupled-line ring hybrids with high power-division ratios, coupled transmission-line sections with very high characteristic impedances, equivalent circuits of high-impedance transmission-line sections, high-impedance transmission-line sections, rat-race couplers with arbitrary power divisions.

## I. INTRODUCTION

THE RING hybrids [1]–[11] consist of three  $90^\circ$  and one  $270^\circ$  transmission-line sections, the  $270^\circ$  section of which may be replaced with a set of  $90^\circ$  coupled transmission-line sections for wideband performance [2], [3], [11]. The ring hybrids have been used for various applications, such as phase shifters [12], antenna arrays, mixers, and balancing amplifiers, and for those applications, arbitrary power divisions are needed. In this case, if the power-division ratios are very high, i.e., more than 9 dB, the transmission-line sections with the characteristic impedances higher than  $150 \Omega$  are indispensable [2], [5]–[9] under the assumption of  $50\text{-}\Omega$  termination impedances. The characteristic impedances higher than  $150 \Omega$  are, however, not easy to implement with microstrip formats, and therefore, the restric-

TABLE I  
COMPARISONS IN TERMS OF POWER-DIVISION RATIOS

Refs	Bandwidths ( $P_{dB} \pm 0.5\text{dB}$ )	$P_{dB}$	Fabrication
[2]	$\geq 180\%$	4 dB	uniplanar
[5]	$\approx 23.6\%$	3.98 dB	microstrip
[6]	$\approx 21.2\%$	11.3 dB	microstrip
[7]	$\approx 2\text{--}3\%$	6 dB	microstrip
[8]	$\approx 9.0\%$	9 dB	microstrip
[9]	less than 10%	6 dB	microstrip
[10]	$\approx 22.2\%$	6 dB	microstrip
This work	$\approx 108\%$	13 dB	microstrip

tion on the fabrication [6] with such high power-division ratios may be inevitable. There have been previous publications treating arbitrary power divisions [2], [5]–[10] and are compared in Table I, in terms of power-division ratios ( $P_{dB}$ ). For comparisons, if the bandwidths are not specified, they were simulated based on design data given in the papers, and the bandwidths were extracted on the basis of the simulation results.

The bandwidth [2] is greater than 180% due to a  $180^\circ$  phase shifter of coplanar strip twist, but  $P_{dB}$  is 4 dB. The typical ring hybrid in [5] has the bandwidth of 23.6%, while that of the modified structure in [6] is 21.2% where  $P_{dB} = 11.3$  dB is measured in [6], but perfect matching does not appear. In addition, several others in [7]–[10] are available, but the bandwidths are smaller since the two [7], [9] are reduced, and the one in [8] is designed to achieve the desired power divisions only at the design center frequency. The design in [10] may start from one ring hybrid consisting of two  $270^\circ$  and two arbitrary lengths of transmission-line sections, and two  $270^\circ$  transmission-line sections are simplified as two variable capacitors to control the power divisions. Therefore, the bandwidth is not small even with smaller size. The ring hybrids in this paper are fabricated for  $P_{dB} = 11$  and 13 dB, and  $P_{dB} = 13$  dB is compared. In this case, the bandwidth is 108%.

The ring hybrid to be treated in this paper consists of one set of coupled transmission-line sections and are therefore called the coupled-line ring hybrid [3]. The coupled-line sections [2], [3] have inherently a  $180^\circ$  phase shifter, and thus the  $180^\circ$  phase difference between  $90^\circ$  coupled-line sections and single transmission-line sections can be achieved in much wider range of frequencies than between  $90^\circ$  and  $270^\circ$  single transmission-line sections, which is already demonstrated in [3, Fig. 2]. That is, the coupled-line ring hybrid consisting of three  $90^\circ$  sections and one  $90^\circ$  coupled-line sections has much wider bandwidths than a conventional ring hybrid.

The coupled-line sections are, for the first time, applied for the ring hybrids by March [11], but the application is valid

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only and only if the coupling coefficient is  $-3$  dB. To employ the coupled-line sections with no restriction on the coupling coefficients, new design formulas are derived [3], and high-impedance transmission-line sections generated by them are treated [13]. The reason for the bandwidth of 108% in Table I even with the  $180^\circ$  phase shifter is because of the coupling coefficient of  $-8.5$  dB.

The  $180^\circ$  phase shifters may be obtained from a coplanar waveguide (CPW) inverter [14, Fig. 7(a)], CPW-slot line transitions [4, Fig. 3.23(d) and (e)] and [15, Fig. 9], and coplanar strip twist [2, Fig. 7]. They are, however, not microstrip lines and require air-bridges, for which careful implementation is demanded.

For the high power-division ratios, one single and one set of coupled transmission-line sections with high values of the characteristic impedances are necessary. To make a single transmission-line section with high characteristic impedances feasible, four types of equivalent circuits are discussed, and an integer  $N$  is introduced. The unit element of the first type is named  $L_{\Pi}$ -type, which is composed of one transmission-line section and two identical shunt inductances, being connected at both ends of the transmission-line section. The second one is designated as  $L_T$ -type, which consists of two identical transmission-line sections and one shunt inductance, being connected at the center of the two transmission-line sections. The third one is called  $L_{S2}$ -type, being made of one transmission-line section and two inductances, which are connected in series to both ends of the transmission-line section. The fourth is denominated  $L_{S1}$ -type, which is comprised of two identical transmission-line sections and one inductance, connected at the center of the two transmission-line sections in series. The equivalent circuits proposed in this paper are those with  $N$ 's unit elements connected in cascade for wideband performance and design flexibility. For the coupled-line sections, the design formulas in [3], [13], [16], and [17] are adopted.

In the conventional designs,  $L_T$ - [18] and  $L_{S2}$ -types [19] are suggested, but both are confined to the narrow bandwidth with  $N = 1$ . Furthermore, the  $L_{S2}$ -type [19] cannot be applied for the ring hybrid with the high power-division ratios, due to two inductors located at both ends, even though the characteristic impedances of the transmission-line section in the  $L_{S2}$ -type may be lower than that of an original transmission-line section. Therefore, the application in [19] is limited to the characteristic impedances of 35.35 to 50  $\Omega$ , the operating frequencies more than 20 GHz and monolithic microwave integrated circuit (MMIC) (CPW) technology.

The two inductances of the  $L_{S2}$ -type are located at both sides, each value of which is inversely proportional to the operating frequency, but proportional to the characteristic impedances of the original transmission-line section. If operating frequencies are less than 5 GHz and the characteristic impedances of the original transmission line sections are higher than 180  $\Omega$ , the inductance values are so high that they should be realized with lumped-element inductors. In this case, since a 50- $\Omega$  transmission-line section can be wider than two times the lengths of a lumped inductor, it is almost impossible to correctly fabricate the  $L_{S2}$ -type with microstrip technology, due to the difficulty in making a  $Y$ -junction, i.e., producing a

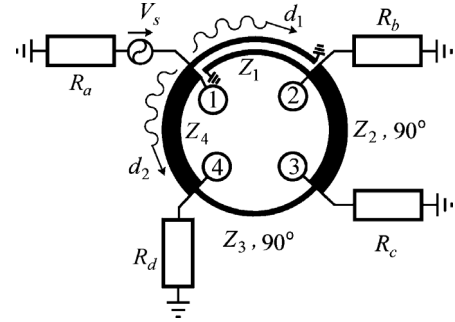


Fig. 1. Coupled-line ring hybrid with arbitrary termination impedances of  $R_a$ ,  $R_b$ ,  $R_c$ , and  $R_d$ .

port or connecting another transmission-line section or diodes. To make the single transmission-line section feasible with the similar  $L_{S2}$ -type, modified  $L_{S2}$ -type ( $ML_{S2}$ -type) is suggested. The  $ML_{S2}$ -type has one  $L_{S2}$ -type and two additional identical transmission-line sections connected at both ends of the  $L_{S2}$ -type. Since the two transmission-line sections of the  $ML_{S2}$ -type can be chosen arbitrarily, design flexibility may be improved. To fulfill engineering perspective that the simplest is generally the most preferable, the  $ML_{S2}$ -type with  $N = 1$  is intensively discussed and compared with  $L_{S2}$ -types with  $N = 1$  and 2. The compared results show that the frequency performance of the  $ML_{S2}$ -type with  $N = 1$  is better than the  $L_{S2}$ -type with  $N = 2$ , even with less number of inductances.

High-impedance transmission-line sections [20]–[22] were treated, in addition to the conventional  $L_T$ -type [18], but they require more process steps to design and etch ground planes, being unsuitable for pure microstrip technology. Moreover, all the applications by the conventional methods [16]–[18], [20]–[22] are not for the ring hybrids.

To verify the suggested theory, two microstrip coupled-line ring hybrids with the power-division ratios of 13 and 11 dB are fabricated, and the measured results show good agreement with those predicted.

## II. COUPLED-LINE RING HYBRIDS

The coupled-line ring hybrid for arbitrary power-division ratios is depicted in Fig. 1 where it is terminated in arbitrary real impedances of  $R_a$ ,  $R_b$ ,  $R_c$ , and  $R_d$  at ports ①, ②, ③, and ④. The ring hybrid consists of three  $90^\circ$  transmission-line sections and one set of  $90^\circ$  coupled transmission-line sections terminated in two short circuits in a diagonal direction. The effective characteristic impedance of the coupled transmission-line sections is  $Z_1$ , while those of the three single sections are  $Z_2$ ,  $Z_3$ , and  $Z_4$ .

If the scattering parameter ratio of  $|S_{41}|/|S_{21}|$  is  $d_2/d_1$ , as indicated in Fig. 1, the design equations for  $Z_1$ ,  $Z_2$ ,  $Z_3$ , and  $Z_4$  [2] are

$$Z_1 = \sqrt{1 + 10 \frac{P_{dB}}{10}} \sqrt{R_a R_b} \quad (1a)$$

$$Z_2 = \sqrt{\frac{1 + 10 \frac{P_{dB}}{10}}{10 \frac{P_{dB}}{10}}} \sqrt{R_b R_c} \quad (1b)$$

$$Z_3 = \sqrt{1 + 10 \frac{P_{dB}}{10}} \sqrt{R_c R_d} \quad (1c)$$

TABLE II  
CHARACTERISTIC IMPEDANCES WITH DIFFERENT  $P_{dB}$  UNDER THE  
ASSUMPTION OF EQUAL TERMINATION IMPEDANCES OF  $50 \Omega$

$P_{dB}$	$Z_1 = Z_3 (\Omega)$	$Z_2 = Z_4 (\Omega)$
9 dB	149.53 $\Omega$	53.05 $\Omega$
10 dB	165.83 $\Omega$	52.44 $\Omega$
11 dB	184.32 $\Omega$	51.95 $\Omega$
12 dB	205.24 $\Omega$	51.56 $\Omega$
13 dB	228.87 $\Omega$	51.24 $\Omega$

$$Z_4 = \sqrt{\frac{1 + 10^{\frac{P_{dB}}{10}}}{10^{\frac{P_{dB}}{10}}}} \sqrt{R_a R_d} \quad (1d)$$

where  $P_{dB}$  is the power-division ratio in dB and expressed as  $20 \log d_2/d_1$ .

$R_a = R_b = R_c = R_d = 50 \Omega$  being assumed, the characteristic impedances of  $Z_1, Z_2, Z_3,$  and  $Z_4$  are calculated by varying  $P_{dB}$  and listed in Table II. The characteristic impedances of  $Z_1$  and  $Z_3$  are proportional to the power-division ratios. To implement the ring hybrids with  $P_{dB} \geq 9$  dB,  $90^\circ$  transmission-line sections, which cannot be easily realized in microstrip technology, are indispensable as computed in Table II. The way to realize such transmission-line sections with the characteristic impedances higher than  $150 \Omega$  will be discussed in terms of single and coupled transmission-line sections.

### III. SINGLE TRANSMISSION-LINE SECTIONS

In the case that the characteristic impedance of a single transmission-line section is too high to realize with the microstrip format, ways to make such high-impedance transmission-line sections feasible will be introduced in terms of a  $\Pi$ -type lumped-element equivalent circuit. Design formulas for the transmission-line section will then be derived based on the lumped-element approach.

#### A. Lumped-Element Approach

A  $\Pi$ -type lumped-element equivalent circuit of a transmission-line section with the characteristic impedance of  $Z_0$  and the electrical length of  $\Theta_u$  is depicted in Fig. 2 where the  $\Pi$ -type consists of two identical shunt capacitances of  $C_0$  and one series inductance of  $L_0$ . Since the image impedance of the  $\Pi$ -type is equal to the characteristic impedance of the transmission-line section,  $Z_0$  and  $\Theta_u$  may be expressed with the lumped elements [4] such as

$$Y_0 = \sqrt{\omega C_0 \left( \frac{2}{\omega L_0} - \omega C_0 \right)} \quad (2a)$$

$$\cos \Theta_u = 1 - \omega^2 C_0 L_0 \quad (2b)$$

where  $Y_0 = Z_0^{-1}$ .

The characteristic impedances of  $Z_0$  were calculated by fixing  $\omega L_0$  at  $128.6 \Omega$  and varying  $\omega C_0$ , and the calculation results are plotted in Fig. 2 where the characteristic impedances are proportional to  $(\omega C_0)^{-1}$ , and  $(\omega C_0)^{-1} = 549.5 \Omega$  is needed for  $Z_0 = 200 \Omega$ , as indicated in Fig. 2. If the characteristic impedance of an original transmission-line section is too high to fabricate, the lower value of  $Z_0$  may be obtained by decreasing  $(\omega C_0)^{-1}$  in the  $\Pi$ -type, keeping the characteristics

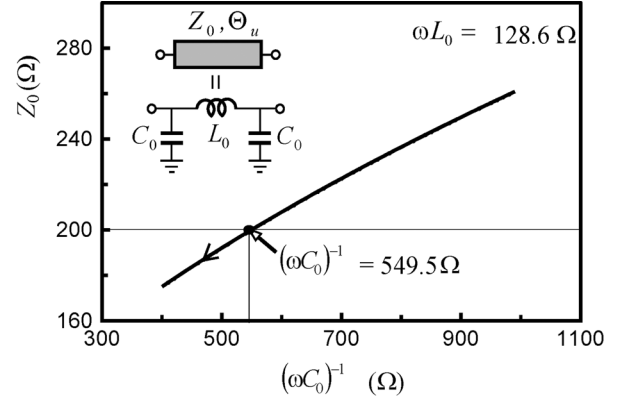


Fig. 2.  $\Pi$ -type equivalent circuit and its characteristic impedance variation.

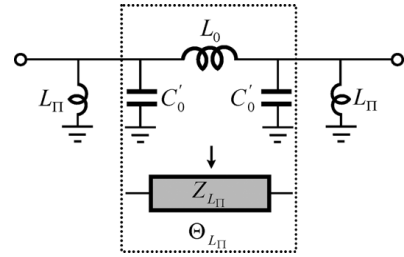


Fig. 3.  $\Pi$ -type lumped-element equivalent circuit with inductances added.

the same as those of the original transmission-line section. A way to increase  $\omega C_0$  for the  $\Pi$ -type is adding inductances to the original circuit.

A lumped-element circuit is described in Fig. 3, consisting of one series inductance of  $L_0$  and two parallel resonators with each having an inductance of  $L_{\Pi}$  and a capacitance of  $C'_0$ . An inductance of  $L_0$  and two identical capacitances of  $C'_0$  form another  $\Pi$ -type circuit, which can be equivalent to a transmission-line section with the characteristic impedance of  $Z_{L\Pi}$  and the electrical length of  $\Theta_{L\Pi}$  assumed. If the lumped-element circuit in Fig. 3 is the same as that of the  $\Pi$ -type in Fig. 2, the capacitance of  $C'_0$  is

$$\omega C'_0 = \omega C_0 + \frac{1}{\omega L_{\Pi}}. \quad (3)$$

The characteristic impedances of  $Z_{L\Pi}$  and the electrical length of  $\Theta_{L\Pi}$  were calculated by varying the inductances of  $L_{\Pi}$  to see how much to deviate from the original transmission-line section. The calculation results are plotted in Fig. 4 where  $\omega L_0$  is fixed at  $Z_0 \sin \Theta_u$ , where  $Z_0 = Y_0^{-1} = 200 \Omega$  and  $\Theta_u = 40^\circ$ , which means that the characteristic impedance and the electrical length of the original transmission-line section are  $Z_0 = 200 \Omega$  and  $\Theta_u = 40^\circ$ .

In Fig. 4, the characteristic impedance of  $Z_{L\Pi}$  has a minimum value of  $128.5 \Omega$  when  $L_{\Pi}$  is around  $28$  nH and close to  $200 \Omega$  when  $L_{\Pi}$  is close to infinite. The electrical length of  $\Theta_{L\Pi}$  is inversely proportional to the inductance of  $L_{\Pi}$  and approaches to  $40^\circ$ , when  $L_{\Pi}$  has a very large value. In other words, with no inductance of  $L_{\Pi}$  added, the characteristic impedance of  $Z_{L\Pi}$  and the electrical length of  $\Theta_{L\Pi}$  becomes the same as those of the original one. Fig. 4 results from the calculation, and it

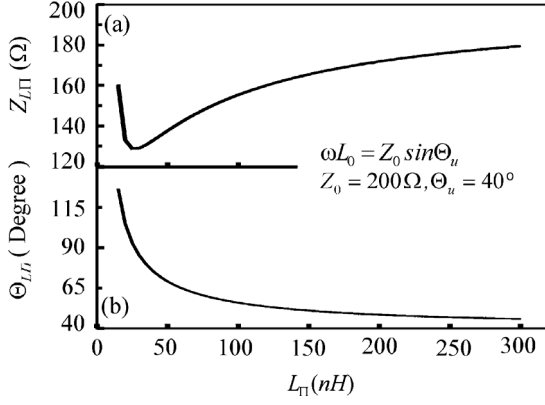


Fig. 4. Calculation results of characteristic impedance of  $Z_{L_{\Pi}}$  and electrical length of  $\Theta_{L_{\Pi}}$ .

is found that a lower characteristic impedances of  $Z_{L_{\Pi}}$  can be obtained by adding less value of  $L_{\Pi}$  and that the electrical length of  $\Theta_{L_{\Pi}}$  increases inversely with  $L_{\Pi}$ .

With this concept, three more equivalent circuits, by which the characteristic impedances of transmission-line sections can be lowered, may be derived using  $T$ -type and halves of  $\Pi$ - and  $T$ -types ( $L$ -sections) of the lumped-element equivalent circuits.

### B. Design Formulas

A transmission-line section with the characteristic impedance of  $Z_0$  and the electrical length of  $\Theta/N$  is depicted in Fig. 5(a), and its four types of equivalent circuits are detailed in Fig. 5(b)–(d). The characteristic impedances of the transmission-line sections in the equivalent circuits in Fig. 5(b)–(d) are intended to be lower than  $Z_0$  of the original transmission-line section, and  $N$  in Fig. 5(a) is a positive integer. To distinguish from each other, the four equivalent circuits are called  $L_{\Pi}$ -,  $L_T$ -,  $L_{S2}$ -, and  $L_{S1}$ -types. The  $L_{\Pi}$ -type in Fig. 5(b) consists of one transmission-line section with the characteristic impedance of  $Z_{L_{\Pi}}$  and the electrical length of  $\Theta_{L_{\Pi}}$ , and two identical shunt inductances of  $L_{\Pi}$ , as demonstrated in Figs. 3 and 5(b). The  $L_T$ -type in Fig. 5(c) comprises two identical transmission-line sections with the characteristic impedance of  $Z_{L_T}$  and the electrical length of  $\Theta_{L_T}/2$ , and one shunt inductance of  $L_T$ . The  $L_{S2}$ -type in Fig. 5(d) is composed of one transmission-line section with the characteristic impedance of  $Z_{L_{S2}}$  and the electrical length of  $\Theta_{L_{S2}}$ , and two identical series inductances of  $L_{S2}$ . The  $L_{S1}$ -type in Fig. 5(e) includes two identical transmission-line sections with the characteristic impedance of  $Z_{L_{S1}}$  and the electrical length of  $\Theta_{L_{S1}}/2$ , and a series inductance of  $L_{S1}$ . If the characteristic impedances of  $Z_{L_{\Pi}}$ ,  $Z_{L_T}$ ,  $Z_{L_{S2}}$ , and  $Z_{L_{S1}}$  in Fig. 5(b)–(d) are lower than that of the original transmission-line section, the four circuits in Fig. 5 are good candidates for the coupled-line ring hybrid with high power-division ratios. The design formulas will be derived using even- and odd-mode excitation analyses.

The even- and odd-mode impedances  $Z_{OC}$  and  $Z_{SC}$  of the transmission-line section in Fig. 5(a) are

$$Z_{OC} = -jZ_0 \cot \frac{\Theta}{2N} \quad (4a)$$

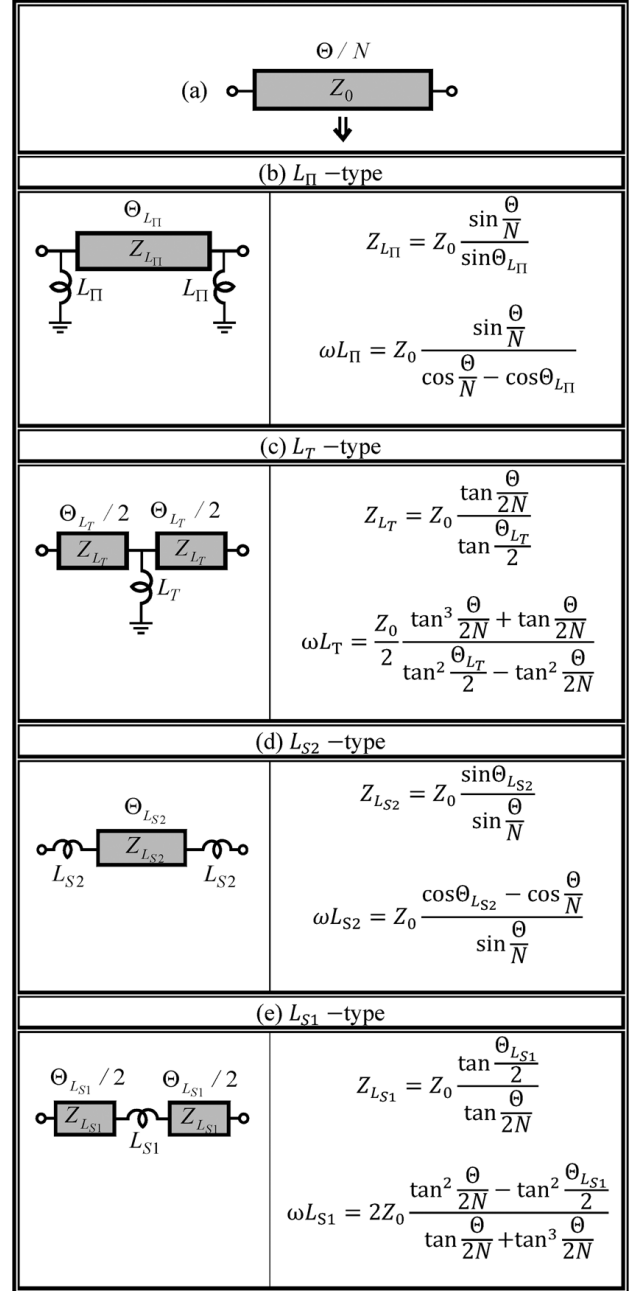


Fig. 5. Equivalent circuits. (a) Transmission-line section with  $\Theta/N$ . (b)  $L_{\Pi}$ -type. (c)  $L_T$ -type. (d)  $L_{S2}$ -type. (e)  $L_{S1}$ -type.

$$Z_{SC} = jZ_0 \tan \frac{\Theta}{2N}. \quad (4b)$$

For the  $L_{\Pi}$ -type in Fig. 5(b), the even- and odd-mode admittances,  $Y_{e-L_{\Pi}}$  and  $Y_{o-L_{\Pi}}$  are

$$Y_{e-L_{\Pi}} = \frac{1}{j\omega L_{\Pi}} + jY_{L_{\Pi}} \tan \frac{\Theta_{L_{\Pi}}}{2} \quad (5a)$$

$$Y_{o-L_{\Pi}} = \frac{1}{j\omega L_{\Pi}} - jY_{L_{\Pi}} \cot \frac{\Theta_{L_{\Pi}}}{2} \quad (5b)$$

where  $Y_{L_{\Pi}} = Z_{L_{\Pi}}^{-1}$ .

For the  $L_T$ -type in Fig. 5(c), the even- and odd-mode impedances,  $Z_{e\_L_T}$  and  $Z_{o\_L_T}$  are

$$Z_{e\_L_T} = jZ_{L_T} \frac{2\omega L_T + Z_{L_T} \tan \frac{\Theta_{L_T}}{2}}{Z_{L_T} - 2\omega L_T \tan \frac{\Theta_{L_T}}{2}} \quad (6a)$$

$$Z_{o\_L_T} = jZ_{L_T} \tan \frac{\Theta_{L_T}}{2}. \quad (6b)$$

For the  $L_{S2}$ -type in Fig. 5(d), the even- and odd-mode impedances  $Z_{e\_L_{S2}}$  and  $Z_{o\_L_{S2}}$  are

$$Z_{e\_L_{S2}} = j\omega L_{S2} - jZ_{L_{S2}} \cot \frac{\Theta_{L_{S2}}}{2} \quad (7a)$$

$$Z_{o\_L_{S2}} = j\omega L_{S2} + jZ_{L_{S2}} \tan \frac{\Theta_{L_{S2}}}{2}. \quad (7b)$$

For the  $L_{S1}$ -type in Fig. 5(e), the even- and odd-mode impedances  $Z_{e\_L_{S1}}$  and  $Z_{o\_L_{S1}}$  are

$$Z_{e\_L_{S1}} = -jZ_{L_{S1}} \cot \frac{\Theta_{L_{S1}}}{2} \quad (8a)$$

$$Z_{o\_L_{S1}} = jZ_{L_{S1}} \frac{\frac{\omega L_{S1}}{2} + Z_{L_{S1}} \tan \frac{\Theta_{L_{S1}}}{2}}{Z_{L_{S1}} - \frac{\omega L_{S1}}{2} \tan \frac{\Theta_{L_{S1}}}{2}}. \quad (8b)$$

The design equations for the circuits in Fig. 5(b)–(e) are obtained by equating the even- and odd-mode impedances of the transmission-line section in (4) to each of those in (5)–(8) and summarized in Fig. 5 where  $\Theta = 90^\circ$  with  $N = 1$  is excluded for the  $L_{\Pi}$ -type because  $Z_{L_{\Pi}}$  is greater than  $Z_0$  with any value of  $\Theta_{L_{\Pi}}$  for  $\Theta = 90^\circ$ . To have the characteristic impedances of  $Z_{L_{\Pi}}$ ,  $Z_{L_T}$ ,  $Z_{L_{S2}}$ , and  $Z_{L_{S1}}$  lower than  $Z_0$ ,  $\Theta_{L_{\Pi}}$  and  $\Theta_{L_T}$  should be longer, whereas  $\Theta_{L_{S2}}$  and  $\Theta_{L_{S1}}$  should be shorter than  $\Theta/N$ . Finally, the equivalent circuits for the electrical length of  $\Theta$  of the transmission-line section are depicted in Fig. 6.

### C. Characteristic Impedances and Frequency Responses

*a)  $L_{\Pi}$ - and  $L_T$ -Types:* Based on the design equations in Fig. 5(b) and (c), the characteristic impedances of  $Z_{L_{\Pi}}$  and  $Z_{L_T}$  were calculated by varying  $N$ . The calculation results are listed in Table III and plotted in Fig. 7. In this case, the characteristic impedance and electrical length of the original transmission-line section are  $Z_0 = 180 \Omega$  and  $\Theta = 90^\circ$ .

The resulting characteristic impedances of  $Z_{L_{\Pi}}$  and  $Z_{L_T}$  in Fig. 6 are, in any case, smaller than or equal to  $180 \Omega$  and inversely proportional to the total electrical length of  $N\Theta_{L_{\Pi}}$  or  $N\Theta_{L_T}$ . If it is assumed that a characteristic impedance of  $120 \Omega$  may be realized without any difficulty, when  $N = 5$  in Fig. 7(a),  $N\Theta_{L_{\Pi}} = 138^\circ$  is needed for the  $L_{\Pi}$ -type, while  $N\Theta_{L_T} = 133.6^\circ$  for the  $L_T$ -type. When  $N = 2$  in Fig. 7(d), the required electrical length for the characteristic impedance of  $120 \Omega$  is  $N\Theta_{L_T} = 127.4^\circ$  for the  $L_T$ -type, but no solution exists for the  $L_{\Pi}$ -type. With increasing  $N$ , the difference between  $Z_{L_{\Pi}}$  and  $Z_{L_T}$  becomes smaller, and the two values are about the same when  $N = 5$  in Fig. 7(a).

The  $L_{\Pi}$ - and  $L_T$ -types with  $N = 2$  and 1 were designed at a design center frequency of 1 GHz, and frequency responses of  $|S_{11}|$  are compared in Fig. 8 where solid lines are those with  $N = 2$ , and the dotted line is that with  $N = 1$  [18]. In

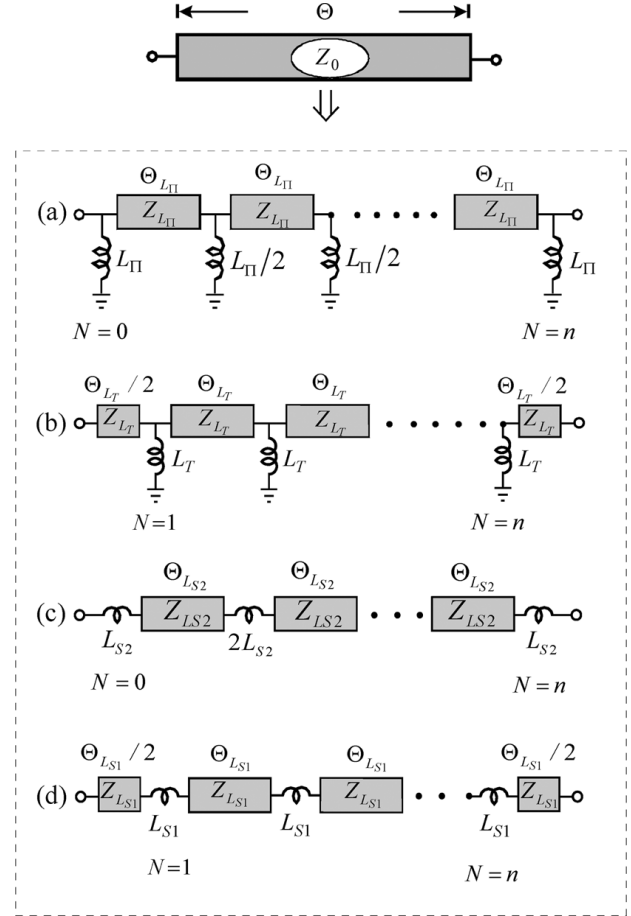


Fig. 6. Final equivalent circuits of a transmission-line section with  $Z_0$  and  $\Theta$ . (a)  $L_{\Pi}$ -type. (b)  $L_T$ -type. (c)  $L_{S2}$ -type. (d)  $L_{S1}$ -type.

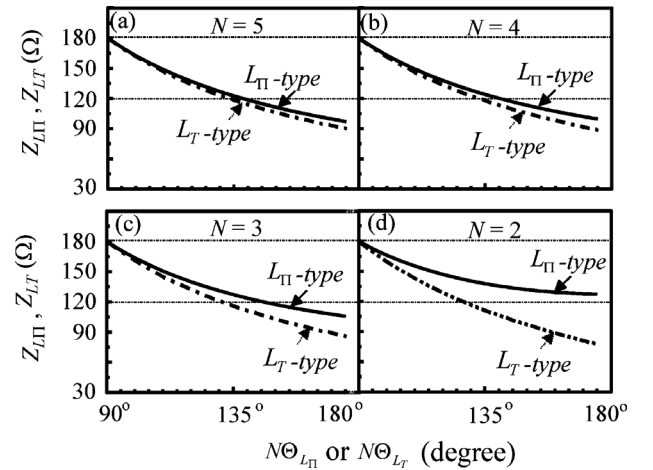


Fig. 7. Characteristic impedances of  $Z_{L_{\Pi}}$  and  $Z_{L_T}$  for a transmission-line section with  $Z_0 = 180 \Omega$  and  $\Theta = 90^\circ$ . (a)  $N = 5$ . (b)  $N = 4$ . (c)  $N = 3$ . (d)  $N = 2$ .

this case, the characteristic impedance and the electrical length of the original transmission-line section are  $Z_0 = 180 \Omega$  and  $\Theta = 90^\circ$ , and  $N\Theta_{L_{\Pi}} = N\Theta_{L_T} = 135^\circ$  was chosen for the simulations. If a bandwidth is defined as a frequency range where return loss is greater than 15 dB, the bandwidths of the  $L_{\Pi}$ - and  $L_T$ -types with  $N = 2$  are 81% (0.9–1.71 GHz) and

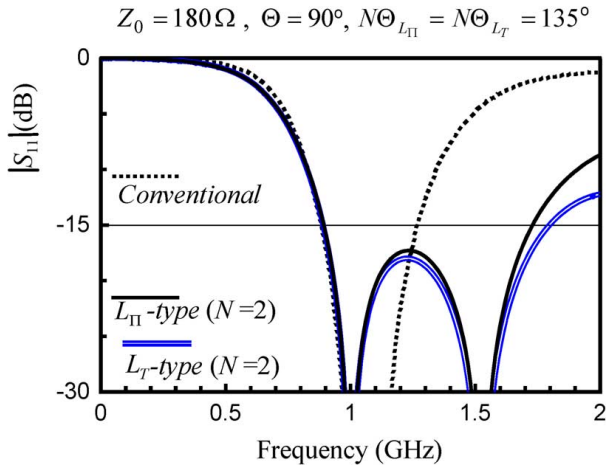


Fig. 8. Comparisons between the conventional ( $L_T$ -type with  $N = 1$ ),  $L_{II}$ - and  $L_T$ -types with  $N = 2$ .

TABLE III  
DESIGN DATA OF  $L_{II}$ - AND  $L_T$ -TYPES OF EQUIVALENT CIRCUITS  
WITH  $Z_0 = 180 \Omega$  AND  $\Theta = 90^\circ$

$N$	$N\Theta_{L_{II}} = N\Theta_{L_T} = 135^\circ$		$N\Theta_{L_{II}} = N\Theta_{L_T} = 120^\circ$	
	$Z_{L_{II}} (\Omega)$	$Z_{L_T} (\Omega)$	$Z_{L_{II}} (\Omega)$	$Z_{L_T} (\Omega)$
$N = 2$	137.77	111.59	146.97	129.14
$N = 3$	127.28	116.44	140.02	132.51
$N = 4$	123.99	118.03	137.77	133.62
$N = 5$	122.52	118.75	136.76	134.13

90% (0.9–1.8 GHz), respectively, while that with  $N = 1$  is only 36% (0.9–1.26 GHz). The reason for the wider bandwidths with  $N = 2$  is because of another resonance frequency made by increasing the number of  $N$ .

Based on the data in Table III, frequency responses of  $L_{II}$ - and  $L_T$ -types are plotted in Fig. 9 where  $N\Theta_{L_{II}} = N\Theta_{L_T} = 135^\circ$  is fixed and the number of  $N$  is varied. All the cases in Fig. 9 are perfectly matched at the design center frequency of 1 GHz, and several other matching frequencies exist. Independent of  $N$ , the second matching frequency is around 1.53 GHz, and the second frequency with 15-dB return loss is around 1.79 GHz. With  $N$  growing, the frequency responses of both equivalent circuits are about the same, but the optimum number of  $N$  is 2 for this case.

*b) Measurements of  $L_T$ -Type With  $N = 2$ :* To verify the  $L_T$ -type with  $N = 2$ , one with  $Z_0 = 180 \Omega$  and  $\Theta = 90^\circ$  was designed at a design center frequency of 2 GHz and fabricated on a substrate (RT/Duroid 5880,  $\epsilon_r = 2.2$ ,  $H = 0.787$  mm). If  $N = 2$  is chosen,  $Z_{L_T}$  in Fig. 6(b) becomes 111.59  $\Omega$  with  $N\Theta_{L_T} = 135^\circ$ , which can be realized without any problem. The inductance of  $L_T$  is implemented with a short stub with the characteristic impedance of 120  $\Omega$ . The fabricated  $L_T$ -type is displayed in Fig. 10. Since the circuit in Fig. 10 is terminated in 180  $\Omega$ , there is no way to directly measure it. After measuring the circuit with the termination impedances of 50  $\Omega$ , the measured data are transformed into those with 180- $\Omega$  termination impedances using Agilent's Advanced Design System (ADS). The results measured and predicted are compared in Fig. 11 where quite good agreement between the two types of results is obtained.

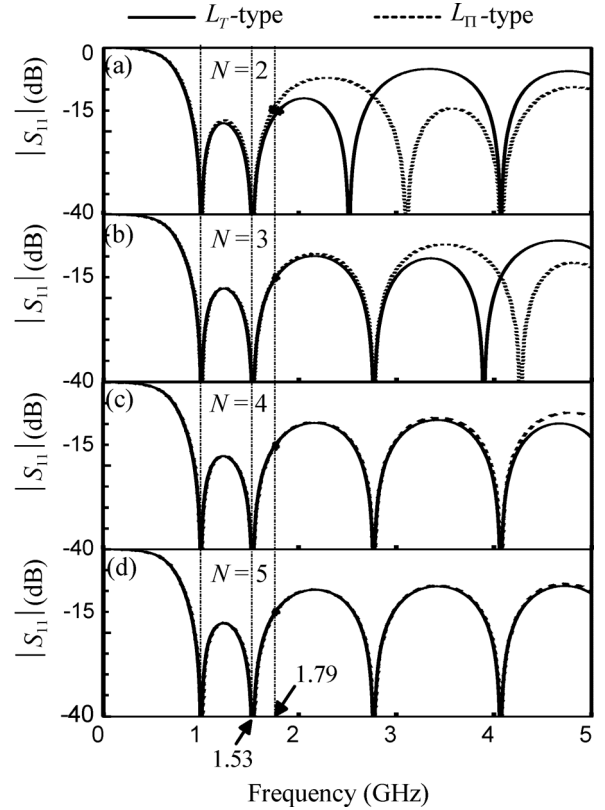


Fig. 9. Frequency responses of  $L_{II}$ - and  $L_T$ -types. (a)  $N = 2$ . (b)  $N = 3$ . (c)  $N = 4$ . (d)  $N = 5$ .

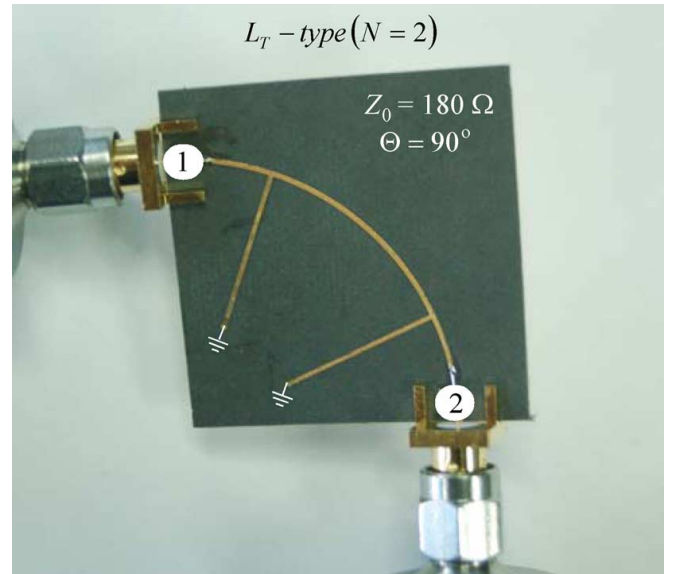


Fig. 10. Photograph of a  $L_T$ -type with  $Z_0 = 180 \Omega$  and  $\Theta = 90^\circ$ .

*c)  $L_{S2}$ - and  $L_{S1}$ -Types:* Based on the design equations in Fig. 5(d) and (e), the characteristic impedances of  $Z_{L_{S2}}$  and  $Z_{L_{S1}}$  for  $Z_0 = 180 \Omega$  and  $\Theta = 90^\circ$  were calculated by varying  $N$ . The results for  $N\Theta_{L_{S2}} = N\Theta_{L_{S1}} = 50^\circ, 60^\circ, \text{ and } 70^\circ$  are listed in Tables IV and V and plotted in Fig. 12. The characteristic impedances of  $Z_{L_{S2}}$  and  $Z_{L_{S1}}$  in Fig. 12 are proportional to the electrical lengths of  $N\Theta_{L_{S2}}$  or  $N\Theta_{L_{S1}}$  and have the maximum value of 180  $\Omega$  when  $N\Theta_{L_{S2}} = N\Theta_{L_{S1}} = 90^\circ$ . With  $N$

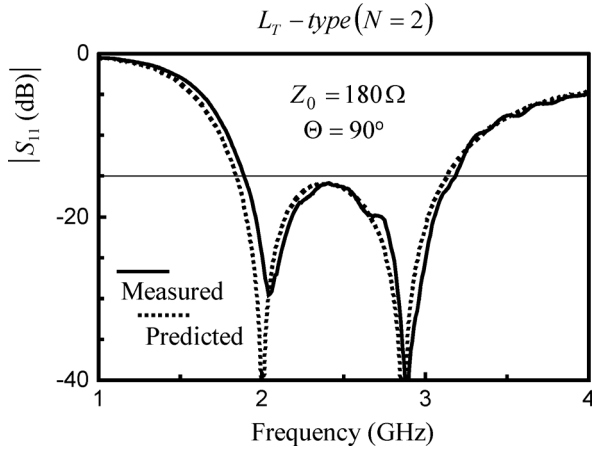


Fig. 11. Results measured and predicted are compared.

TABLE IV

DESIGN DATA OF  $L_{S2}$ - AND  $L_{S1}$ -TYPES FOR  $N\Theta_{L_{S2}} = N\Theta_{L_{S1}} = 50^\circ$  AND  $70^\circ$ , WHEN THE ORIGINAL TRANSMISSION-LINE SECTION HAVING  $Z_0 = 180 \Omega$  AND  $\Theta = 90^\circ$

N	$N\Theta_{L_{S2}} = N\Theta_{L_{S1}} = 50^\circ$		$N\Theta_{L_{S2}} = N\Theta_{L_{S1}} = 70^\circ$	
	$Z_{L_{S2}} (\Omega)$	$Z_{L_{S1}} (\Omega)$	$Z_{L_{S2}} (\Omega)$	$Z_{L_{S1}} (\Omega)$
N = 1	137.89	83.94	169.15	126.04
N = 2	107.58	96.34	146.01	137.02
N = 3	103.25	98.40	142.59	138.71
N = 4	101.81	99.11	141.44	139.28
N = 5	101.15	99.43	140.92	139.54

TABLE V

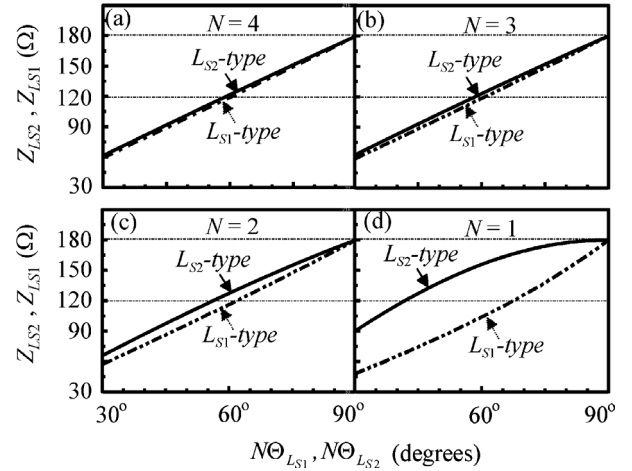
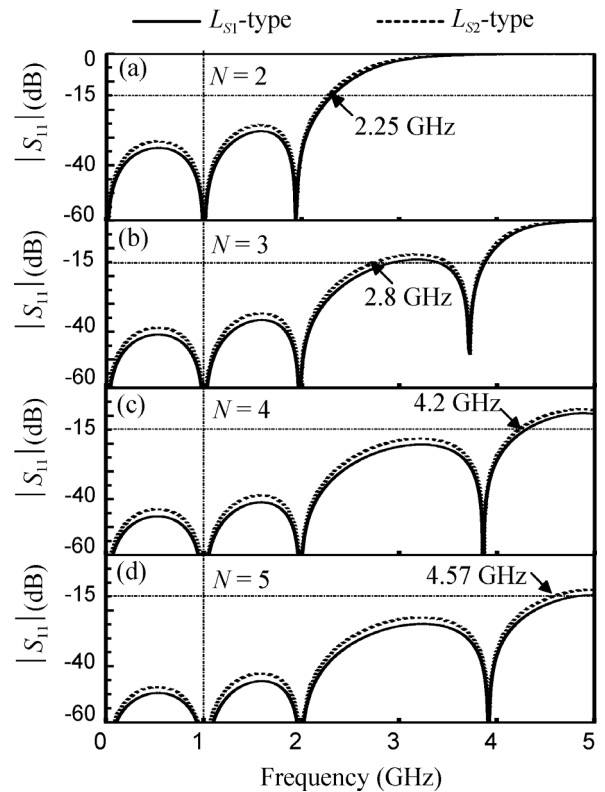
DESIGN DATA OF  $L_{S2}$ - AND  $L_{S1}$ -TYPES FOR  $N\Theta_{L_{S2}} = N\Theta_{L_{S1}} = 60^\circ$ , WHEN THE ORIGINAL TRANSMISSION-LINE SECTION HAVING  $Z_0 = 180 \Omega$  AND  $\Theta = 90^\circ$  AT 1 GHz

N	$N\Theta_{L_{S2}} = N\Theta_{L_{S1}} = 60^\circ$			
	$Z_{L_{S2}} (\Omega)$	$Z_{L_{S1}} (\Omega)$	$L_{S2} (\text{nH})$	$L_{S1} (\text{nH})$
N = 1	155.88	103.92	14.32	19.09
N = 2	127.28	116.44	6.44	11.78
N = 3	123.13	118.45	4.22	8.12
N = 4	121.74	119.14	3.15	6.16
N = 5	121.11	119.45	2.51	4.95

growing, the two values of  $Z_{L_{S2}}$  and  $Z_{L_{S1}}$  become about the same.

If it is assumed that a characteristic impedance of  $120 \Omega$  is realized without any difficulty, all the cases of the  $L_{S1}$ -type are possible with  $N\Theta_{L_{S1}} = 50^\circ$ , while only four cases are possible for the  $L_{S2}$ -type from the data in Fig. 12. With the data for  $N\Theta_{L_{S1}} = N\Theta_{L_{S2}} = 60^\circ$  in Table V, the  $L_{S1}$ - and  $L_{S2}$ -types were simulated, and the simulation results of  $|S_{11}|$  are plotted in Fig. 13 where the design center frequency is 1 GHz. If a bandwidth is defined as a frequency region where return loss is greater than 15 dB, the bandwidth of  $L_{S1}$ - and  $L_{S2}$ -types for  $N = 2$  is about 225%, that for  $N = 3$  is about 280%, while those for  $N = 4$  and 5 are about 420% and 450%, respectively. Even better, the resulting transmission-line sections are  $60^\circ$  long, which is shorter than the original  $90^\circ$  transmission-line section.

d) *Measurements of  $L_{S1}$ -Type With  $N = 2$* : For the verification of  $L_{S1}$ -type, one with  $Z_0 = 180 \Omega$  and  $\Theta = 90^\circ$  was

Fig. 12. Characteristic impedances of  $Z_{L_{S2}}$  and  $Z_{L_{S1}}$  for a transmission-line section with  $Z_0 = 180 \Omega$  and  $\Theta = 90^\circ$ . (a)  $N = 4$ . (b)  $N = 3$ . (c)  $N = 2$ . (d)  $N = 1$ .Fig. 13. Frequency responses of  $L_{S1}$ - and  $L_{S2}$ -types. (a)  $N = 2$ . (b)  $N = 3$ . (c)  $N = 4$ . (d)  $N = 5$ .

designed at a design center frequency of 700 MHz and fabricated on the same substrate. To make use of available inductors with 20 nH (Murata, LQW18AN20NG00), the electrical length of  $N\Theta_{L_{S1}}$  in Fig. 6(d) was determined as  $52.5^\circ$  with  $N = 2$ . Its corresponding characteristic impedance of  $Z_{L_{S1}}$  is then  $101.325 \Omega$ , which can be realized without any problem. The fabricated  $L_{S1}$ -type is displayed in Fig. 14(a), and the measured frequency responses of  $|S_{11}|$  and  $|S_{21}|$  are compared with the predicted ones in Fig. 14 where good agreement is shown with the predicted ones.



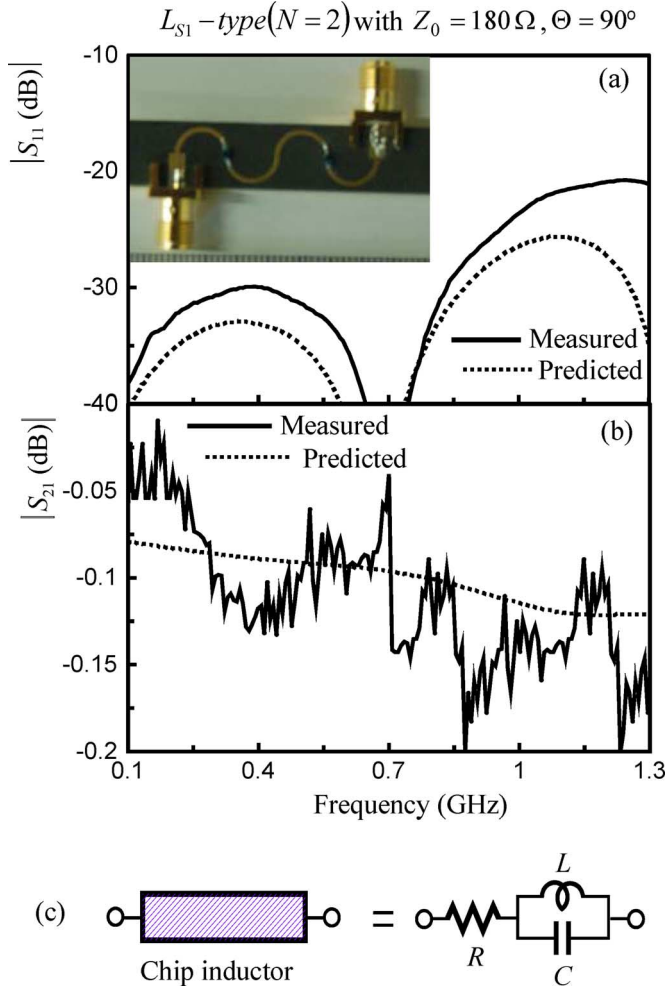


Fig. 14. Measured results of  $L_{S1}$ -type with  $N = 2$ . (a)  $S_{11}$ . (b)  $S_{12}$ . (c) Modeling of a lumped-element inductor.

The lumped-element inductor may be modeled as the circuit in Fig. 14(c) where a resistance of  $R$  is connected to a parallel resonator with a pure inductance of  $L$  and a pure capacitance of  $C$ . The pure inductance represents the value of the lumped-element inductor. The loss produced by the inductor is expressed as the resistance, and the capacitance of  $C$  is a parasitic element, which can determine the self resonance frequency of the lumped-element inductor. The self resonance frequency for the inductor is minimum of 5 GHz specified in the datasheet, from which the capacitance of  $C$  may be computed as 0.05 pF, and from the insertion loss, the resistance may also be extracted as 1.5  $\Omega$ .

#### D. Modified $L_{S2}$ -Type ( $ML_{S2}$ -Type)

The characteristic impedance  $Z_{L_{S2}}$  of the  $L_{S2}$ -type in Fig. 6(c) is definitely smaller than  $Z_0$  of the original transmission-line section if  $\Theta_{L_{S2}}$  is smaller than  $\Theta$  with any  $N$ . Nevertheless, the structure of the  $L_{S2}$ -type itself is not suitable for the coupled-line ring hybrids due to the inductances of  $L_{S2}$  at both ends. The value of  $L_{S2}$  is inversely proportional to the operating frequencies, but proportional to the characteristic impedance of  $Z_0$ , as indicated in Fig. 5(d). If the operating

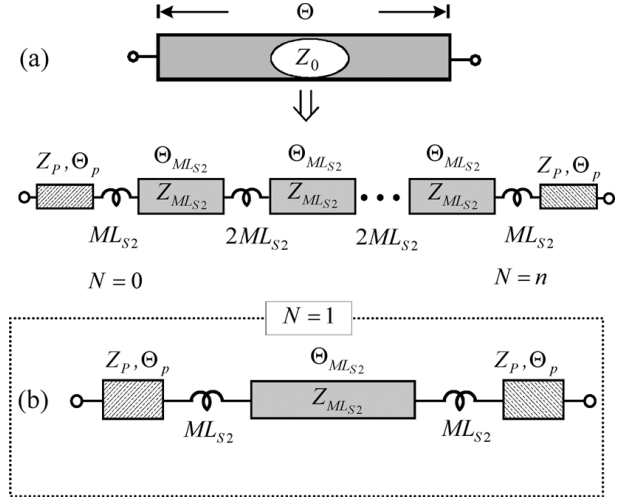


Fig. 15.  $ML_{S2}$ -type. (a)  $ML_{S2}$ -type with  $N$ . (b)  $ML_{S2}$ -type with  $N = 1$ .

frequencies are lower than 5 GHz, and the characteristic impedance of  $Z_0$  is higher than 180  $\Omega$ , the inductance of  $L_{S2}$  cannot be realized with distributed elements and should be implemented with lumped-element inductors.

If it is desired to realize the inductance of the  $L_{S2}$ -type with a lumped element, for example, a 0402-sized chip inductor, half of the width of a 50- $\Omega$  transmission-line section on a substrate (RT/Duroid 5880,  $\epsilon_r = 2.2$ ,  $H = 31$  mil) is wider than the length of the chip inductor. Therefore, it is almost impossible to form a Y-junction with the  $L_{S2}$ -type, i.e., to connect a third transmission-line section to make a port or to connect another transmission-line section, which results in incorrect operation of the  $L_{S2}$ -type.

To overcome the difficulty, modified  $L_{S2}$ -type ( $ML_{S2}$ -type), being equivalent to a transmission-line section with the characteristic impedance of  $Z_0$  and arbitrary electrical length of  $\Theta$ , is suggested in Fig. 15, where two identical transmission-line sections with the characteristic impedance of  $Z_P$  and the electrical length of  $\Theta_P$  are connected to the inductances of the problem, as depicted in Fig. 15(a). The characteristic impedance of  $Z_{ML_{S2}}$ , the electrical length of  $\Theta_{ML_{S2}}$ , and the inductance of  $L_{S2}$  in the  $L_{S2}$ -type in Figs. 5(d) and 6(c) are changed to  $Z_{ML_{S2}}$ ,  $\Theta_{ML_{S2}}$ , and  $ML_{S2}$ . The relations among  $Z_{ML_{S2}}$ ,  $\Theta_{ML_{S2}}$ , and  $ML_{S2}$  are

$$Z_{ML_{S2}} = Z_Q \frac{\sin \Theta_{ML_{S2}}}{\sin \frac{\Theta_Q}{N}} \quad (9a)$$

$$\omega \cdot ML_{S2} = Z_Q \frac{\cos \Theta_{ML_{S2}} - \cos \frac{\Theta_Q}{N}}{\sin \frac{\Theta_Q}{N}} \quad (9b)$$

where the design formulas for  $Z_Q$  and  $\Theta_Q$  are shown in (9c) and (9d) at the bottom of the following page.

There are many solutions to  $Z_{ML_{S2}}$ ,  $\Theta_{ML_{S2}}$ , and  $ML_{S2}$  by varying  $Z_P$ ,  $\Theta_P$ , and  $N$ . In the view of engineering perspective that the simplest is the most preferable, the  $ML_{S2}$ -type will be discussed intensively on  $N = 1$ , as displayed in Fig. 15(b). When  $Z_P = 125\ \Omega$  and  $2\Theta_P + \Theta_{ML_{S2}} = 60^\circ$  at 1 GHz are fixed for a 90 $^\circ$  transmission-line section with  $Z_0 = 180\ \Omega$ , the design data are listed in Table VI, and frequency responses of



TABLE VI  
DESIGN DATA OF THE  $ML_{S2}$ -TYPE WITH  $N = 1$  FOR A TRANSMISSION-LINE SECTION HAVING  $Z_0 = 180 \Omega$  AND  $\Theta = 90^\circ$  AT 1 GHz

$\Theta_{ML_{S2}} + 2\Theta_P = 60^\circ, Z_P = 125 \Omega$			
$\Theta_P$	$Z_{ML_{S2}} (\Omega)$	$\Theta_{ML_{S2}}$	$ML_{S2} (nH)$
$13^\circ$	119.19	$34^\circ$	12.25
$14^\circ$	116.31	$32^\circ$	12.12
$15^\circ$	113.34	$30^\circ$	11.99
$16^\circ$	110.25	$28^\circ$	11.86
$16.7^\circ$	108.01	$26.6^\circ$	11.77
$17^\circ$	107.03	$26^\circ$	11.74

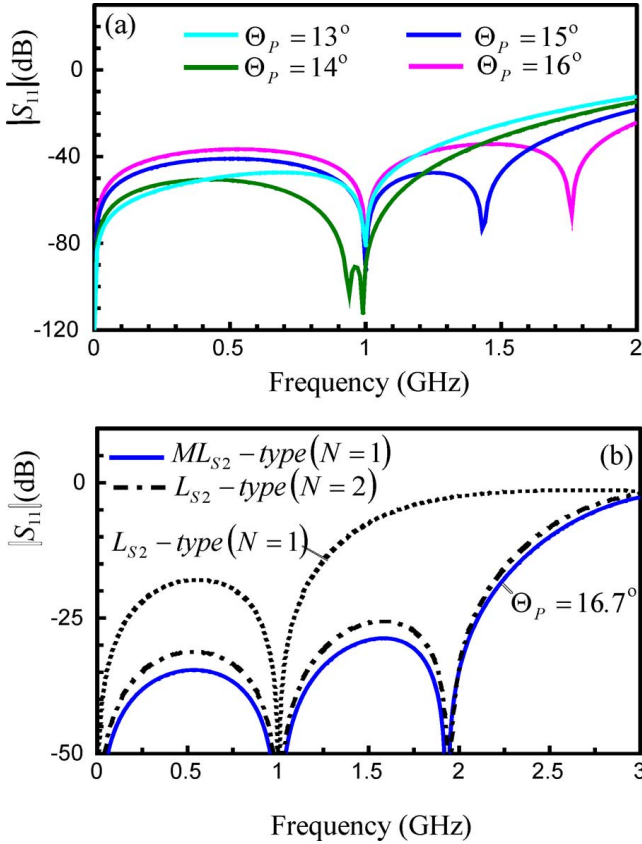


Fig. 16. Frequency responses. (a)  $ML_{S2}$ -types. (b) Comparisons between one  $ML_{S2}$ -type and  $L_{S2}$ -types with  $N = 1$  and  $N = 2$ .

the several cases are plotted in Fig. 16(a). When  $\Theta_P = 13^\circ$  in Fig. 16(a), perfect matching occurs only at the design center frequency of 1 GHz. When  $\Theta_P = 14^\circ$ , two perfect matching frequencies are possible; one is 1 GHz and another less than

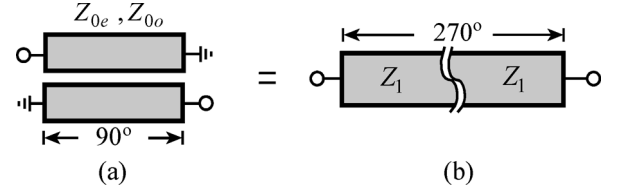


Fig. 17. (a) Set of coupled transmission-line sections. (b) Equivalent circuit.

1 GHz. When  $\Theta_P = 15^\circ$  or  $16^\circ$ , two poles exist for each case by which the bandwidths can be increased. This kind of frequency performance is not possible with the  $L_{S2}$ - and  $L_{S1}$ -types, as shown in Fig. 13.

The  $ML_{S2}$ -type with  $\Theta_P = 16.7^\circ$  is compared with  $L_{S2}$ -types with  $N = 1$  and 2 in Fig. 16(b) where the frequency performance with the  $L_{S2}$ -type with  $N = 1$  in [19] is the worst, even with the same number of inductances, while the  $ML_{S2}$ -type is better than the  $L_{S2}$ -type with  $N = 2$ , even with a less number of inductances. That much, the  $ML_{S2}$ -types have definite advantages over the  $L_{S2}$ -types, in terms of the substitutes of the high-impedance transmission line sections and better frequency performance.

#### IV. $90^\circ$ COUPLED-LINE TRANSMISSION-LINE SECTIONS

##### A. Design Formulas

A set of coupled transmission-line sections with two short circuits in a diagonal direction and its equivalent circuit are depicted in Fig. 17 where the even- and odd-mode impedances are  $Z_{0e}$  and  $Z_{0o}$ , and the effective characteristic impedance of the equivalent transmission-line section in Fig. 17(b) is  $Z_1$ , as mentioned above in Fig. 1. As is well known, the electrical length of the equivalent transmission-line section is  $270^\circ$  if the coupled transmission-line sections are  $90^\circ$  long. In this case, the design formulas of the even- and odd-mode impedances  $Z_{0e}$  and  $Z_{0o}$  [3], [13], [16], [17] in Fig. 17(a) are

$$Z_{0e} = Z_1 \frac{10^{\frac{C_{dB}}{20}}}{1 - 10^{\frac{C_{dB}}{20}}} \quad (10a)$$

$$Z_{0o} = Z_1 \frac{10^{\frac{C_{dB}}{20}}}{1 + 10^{\frac{C_{dB}}{20}}} \quad (10b)$$

where  $C_{dB}$  is the coupling coefficient in dB.

##### B. Feasibility of Coupled Transmission-Line Sections

A pair of coupled transmission-line sections is a four-port circuit, and the circuit in Fig. 17(a) is obtained by placing short

$$Z_Q = Z_P \sqrt{\left( \frac{Z_0 \cot \frac{\Theta}{2} + Z_P \tan \Theta_P}{Z_P - Z_0 \cot \frac{\Theta}{2} \cdot \tan \Theta_P} \right) \left( \frac{Z_0 \tan \frac{\Theta}{2} - Z_P \tan \Theta_P}{Z_0 \tan \frac{\Theta}{2} \cdot \tan \Theta_P + Z_P} \right)} \quad (9c)$$

$$\tan \frac{\Theta_Q}{2} = \sqrt{\left( \frac{Z_P - Z_0 \cot \frac{\Theta}{2} \cdot \tan \Theta_P}{Z_P + Z_0 \tan \frac{\Theta}{2} \cdot \tan \Theta_P} \right) \left( \frac{Z_0 \tan \frac{\Theta}{2} - Z_P \tan \Theta_P}{Z_0 \cot \frac{\Theta}{2} + Z_P \tan \Theta_P} \right)} \quad (9d)$$

TABLE VII

PHYSICAL GAP SIZE OF THE COUPLED-LINE SECTIONS ON A SUBSTRATE (RT/DUROID 5870,  $\epsilon_r = 2.33$ ,  $H = 0.787$  mm) WITH A FIXED COUPLING COEFFICIENT OF  $-10$  dB AND A CENTER FREQUENCY OF 3 GHz

$Z_1$ ( $\Omega$ )	$(Z_{0e}, Z_{0o})$ ( $\Omega$ )	$C_m$ (pF)	$s$ (mm)
100 $\Omega$	(46.2, 24.0)	46.27	0.039
200 $\Omega$	(92.5, 48.1)	22.61	0.200
250 $\Omega$	(115.6, 60.1)	17.91	0.272
300 $\Omega$	(138.7, 72.1)	14.82	0.326
350 $\Omega$	(161.9, 84.1)	12.61	0.365

circuits on two diagonal terminals. Since a set of the coupled transmission-line sections with an appropriate combination of short and open terminations produces special performance that a single transmission section is unable to provide, the usage of the coupled transmission-line sections is indispensable. The problem is, however, the feasibility of the even- and odd-mode impedances required. To realize a set of coupled transmission-line sections, the width of each transmission-line section and the gap size between two coupled transmission-line sections should be realizable. The width is related mainly with the even-mode impedance and the gap size with the mutual capacitance. To see the feasibility of the coupled transmission-line sections in Fig. 17(a), the relation between the mutual capacitance and the gap distance is investigated further.

The mutual capacitance  $C_m$  of a set of coupled transmission-line sections is expressed as

$$C_m = \frac{\sqrt{\mu\epsilon_{\text{eff}}}}{2} \left( \frac{1}{Z_{0o}} - \frac{1}{Z_{0e}} \right) \quad (11)$$

where  $C_m = C_{12}$  in [23], and  $\epsilon_{\text{eff}}$  and  $\mu$  are effective permittivity and permeability of a substrate. The relation between the characteristic impedance of  $Z_1$  in (10) of the coupled transmission-line sections and the mutual capacitance is

$$C_m = \frac{\sqrt{\mu\epsilon_{\text{eff}}}}{Z_1}. \quad (12)$$

Equation (12) expresses that the mutual capacitance  $C_m$  is inversely proportional to the characteristic impedance  $Z_1$  of the coupled transmission-line sections. Due to this property, the easier fabrication is possible with the higher characteristic impedance of the coupled transmission-line sections. In the case of a substrate (RT/Duroid 5870,  $\epsilon_r = 2.33$ ,  $H = 0.787$  mm) for an operating center frequency of 3 GHz, the mutual capacitance and the gap size are calculated (Table VII), and the results are plotted in Fig. 18. For the calculations, ADS (LineCalc) was used, and the coupling coefficient was fixed at  $-10$  dB. For the characteristic impedance of  $Z_1 = 100 \Omega$ , the even- and odd-mode impedances are 46.2 and 24.0  $\Omega$ , while the effective even- and odd-mode dielectric constants are 2.11 and 1.74. The mutual capacitance  $C_m$  is therefore 46.27 pF, as in Table VII. In this case, the required gap size is  $s = 0.039$  mm, which is relatively difficult to fabricate in a planar structure of microstrip lines with a standard printed circuit board (PCB) technology. On the other hand, for  $Z_1 = 250 \Omega$ , the effective even- and odd-mode dielectric constants are 1.94 and 1.67,

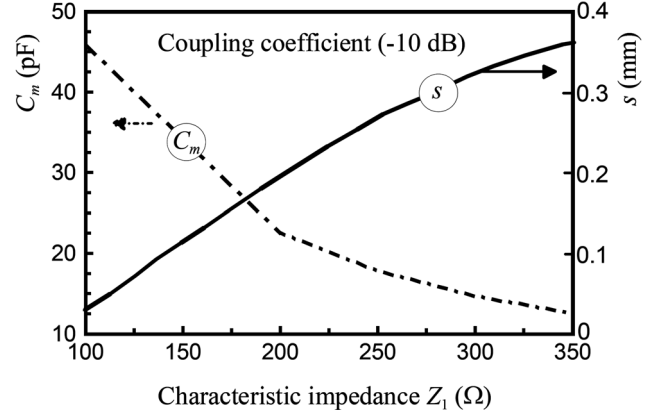


Fig. 18. Relation between mutual capacitance, gap size, and characteristic impedance of the coupled transmission-line sections.

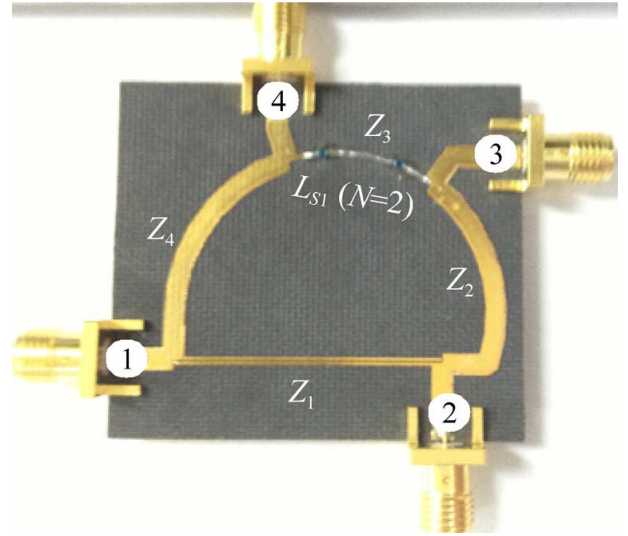


Fig. 19. Photograph of a fabricated ring hybrid with  $P_{\text{dB}} = 13$  dB.

and therefore,  $C_m = 17.91$  pF. The physical gap size becomes 0.27 mm, which is easier for the fabrication. In this way, the mutual capacitance is inversely proportional to the characteristic impedances of the coupled transmission-line sections with the fixed coupling coefficient. Thus, if the characteristic impedance of the coupled transmission-line sections becomes higher, it is easier to realize in terms of gap distance.

## V. COUPLED-LINE RING HYBRIDS

To verify the suggested theory, two different coupled-line ring hybrids with the power-division ratios of 13 and 11 dB were fabricated on a substrate (RT/Duroid 5880,  $\epsilon_r = 2.2$ ,  $H = 30$  mil).

### A. $P_{\text{dB}} = 13$ dB

To implement a coupled-line ring hybrid, a design center frequency of 2 GHz was determined. The power-division ratio of  $P_{\text{dB}}$  was chosen as 13 dB, under the assumption of equal termination impedances of 50  $\Omega$ . In this case,  $Z_1 = Z_3 = 228.9 \Omega$  and  $Z_2 = Z_4 = 51.2 \Omega$ , as calculated in Table II. To realize the characteristic impedance of 228.9  $\Omega$ , the coupling coefficient of the coupled transmission-line sections was chosen as  $-8.5$  dB,

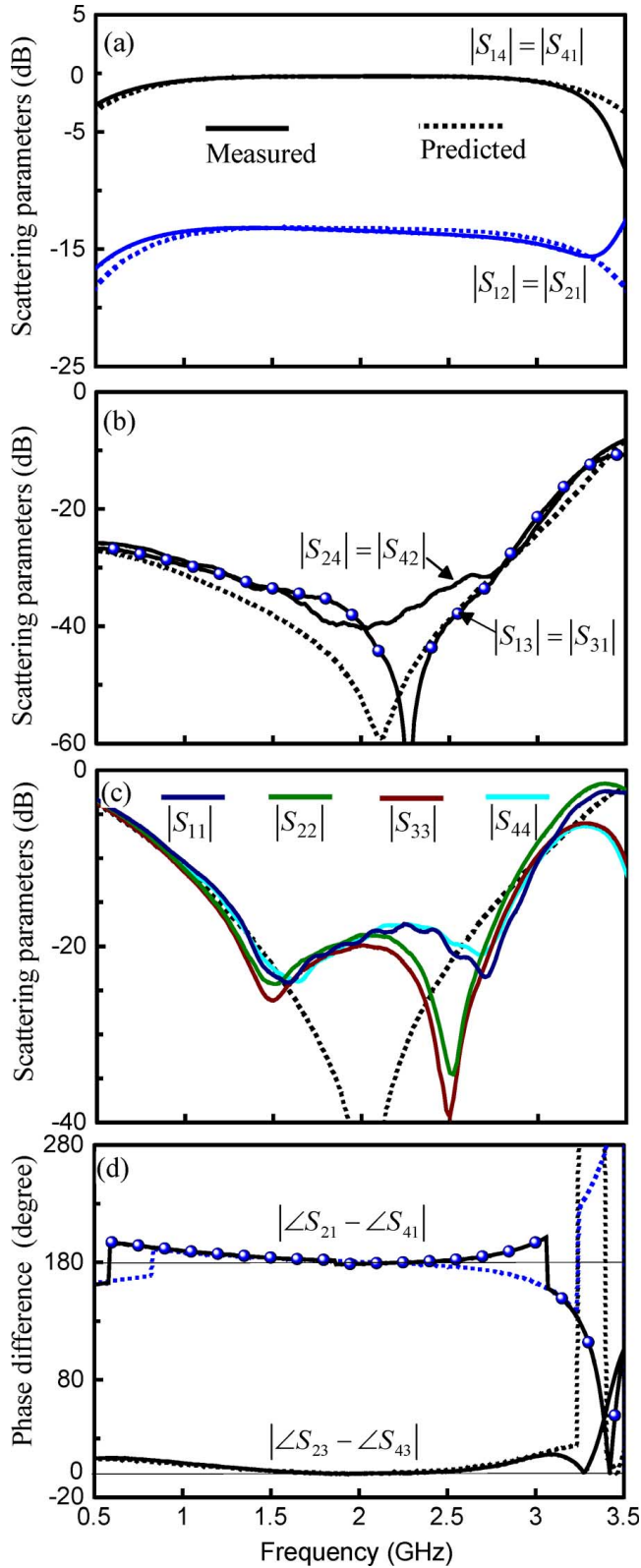


Fig. 20. Measured results with  $P_{dB} = 13$  dB. (a) Power divisions. (b) Isolations. (c) Matching. (d) Phase responses.

and the  $L_{S1}$ -type with  $N = 2$  in Fig. 6(d) was employed for the single transmission-line section.

The corresponding  $Z_{0e}$  and  $Z_{0o}$  are 137.8 and 62.53  $\Omega$  for the coupled transmission-line sections, and the physical line

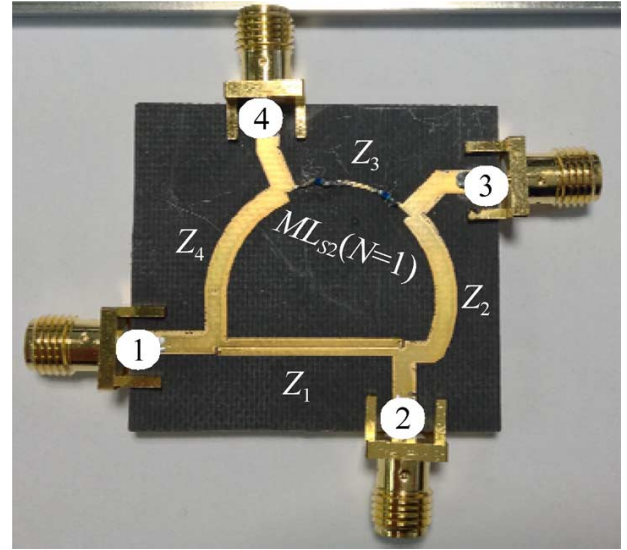


Fig. 21. Photograph of a fabricated ring hybrid with  $P_{dB} = 11$  dB.

width  $w$  and gap size  $s$  are  $w = 0.319$  mm and  $s = 0.176$  mm on the substrate. For the single transmission-line section with the characteristic impedance of  $Z_3 = 228.9 \Omega$  in Fig. 1, to make use of available inductors with 9.1 nH (Murata, LQW15A series), the electrical length of  $\Theta_{L_{S1}}$  in Fig. 6(d) is calculated as  $25.29^\circ$ , leading to  $Z_{L_{S1}} = 123.98 \Omega$ . The fabricated ring hybrid is displayed in Fig. 19, and the measured results are in Fig. 20 where solid and dotted lines are the measured and predicted results.

The power-division responses are in Fig. 20(a), isolations in Fig. 20(b), matching responses in Fig. 20(c), and phase responses in Fig. 20(d). Since the ring hybrid in Fig. 19 was designed for the power-division ratio of 13 dB, or  $20 \log(d_2/d_1) = 13$ , the scattering parameter of  $S_{21}$  is calculated as  $10 \log\{d_1^2/(d_1^2 + d_2^2)\}$ , i.e.,  $-13.21$  dB. Due to the difference of 13 dB between  $S_{21}$  and  $S_{41}$ , the scattering parameter of  $S_{41}$  is  $-0.21$  dB. The measured  $S_{21}$  and  $S_{41}$  are  $-13.732$  and  $-0.245$  dB at 2 GHz in Fig. 20(a), and the measured power-division ratio is 13.487 dB. In Fig. 20(b), the isolations of  $S_{31}$  and  $S_{24}$  are greater than 40 dB at 2 GHz and greater than 15 dB from 0 to 3.2 GHz. Since the simulated matching results at all ports have about the same responses, they are expressed with only a single dotted line in Fig. 20(c) where the measured return losses at all ports are greater than 20 dB at the design center frequency of 2 GHz, and the frequency bandwidth with 15-dB return loss is 1.15–2.9 GHz. The fact of the desirable isolations in Fig. 20(b) indicates that good in-phase and out-of-phase responses are possible. In Fig. 20(d) of the phase responses, the measured  $|\angle S_{21} - \angle S_{41}|$  and  $|\angle S_{23} - \angle S_{43}|$  at 2 GHz are obtained as  $178.9^\circ$  and  $0.244^\circ$ . The measured results are in good agreement with the predicted ones.

#### B. $P_{dB} = 11$ dB

A coupled-line ring hybrid with  $P_{dB} = 11$  dB was fabricated on the same substrate, and the design center frequency



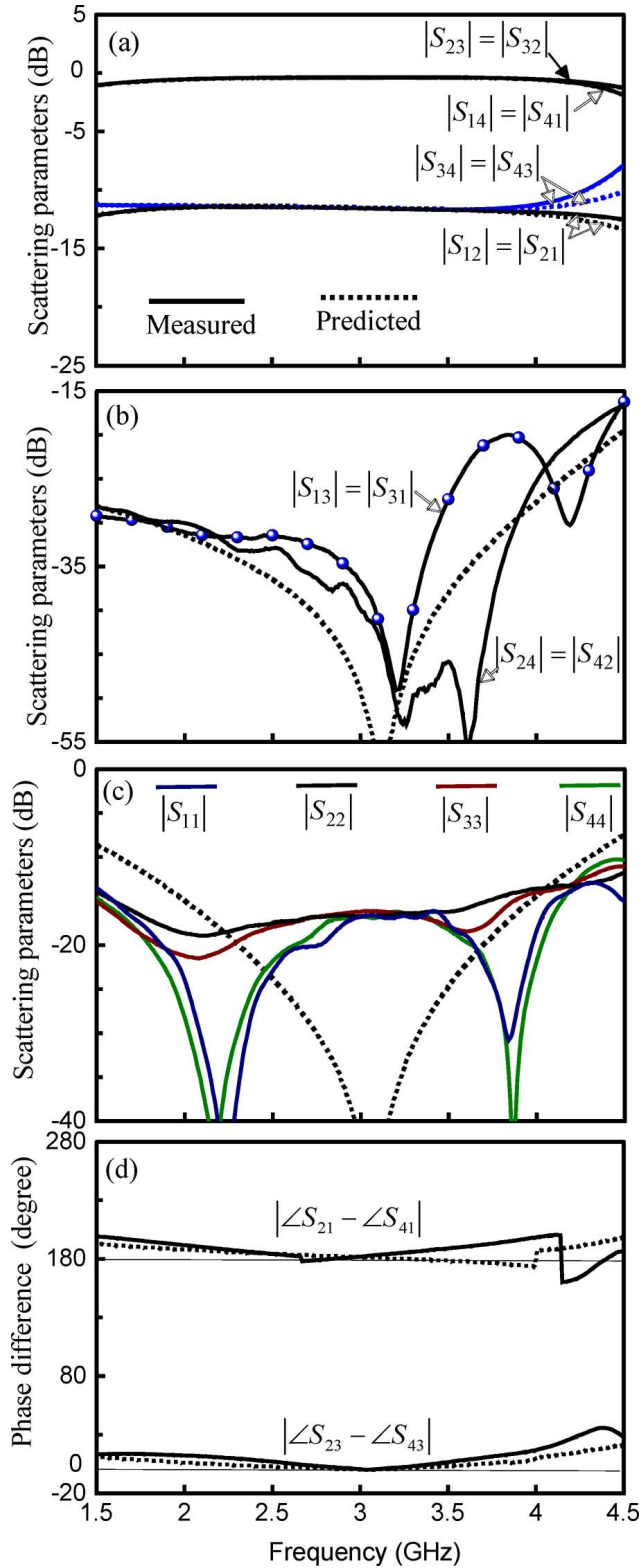


Fig. 22. Measured results with  $P_{dB} = 11$  dB. (a) Power divisions. (b) Isolations. (c) Matching. (d) Phase responses.

was chosen as 3 GHz. In this case, if all the termination impedances are assumed to be  $50 \Omega$ ,  $Z_1 = Z_3 = 184.32 \Omega$  and  $Z_2 = Z_4 = 51.95 \Omega$ , as calculated in Table II. To realize the characteristic impedance of  $184.32 \Omega$  for the single transmission-line section, the  $ML_{S_2}$ -type with  $N = 1$  in Fig. 15(b) was employed.

To make use of available inductors of  $4.1$  nH at  $3$  GHz,  $Z_P$  and  $\Theta_P$  in Fig. 15(b) were predetermined as  $125 \Omega$  and  $14.3^\circ$ .  $Z_{ML_{S_2}} = 119.1 \Omega$  and  $\Theta_{ML_{S_2}} = 31.4^\circ$  were then calculated. For the coupled transmission-line sections, the coupling coefficient was chosen as  $-8$  dB and its corresponding  $Z_{0e}$  and  $Z_{0o}$  are  $121.9$  and  $52.48 \Omega$ . The fabricated ring hybrid is in Fig. 21 where the  $ML_{S_2}$ -type with  $N = 1$  is indicated.

The measured and predicted results are compared in Fig. 22 where the measured power-division ratio is  $11.26$  dB in Fig. 22(a), the measured isolations are more than  $15$  dB in a frequency range of  $1.5$ – $4.5$  GHz in Fig. 22(b), and return losses at all ports are more than  $18$  dB in a frequency range of  $1.5$ – $3.5$  GHz in Fig. 22(c). The phase differences of  $|\angle S_{21} - \angle S_{41}|$  are measured as  $180^\circ \pm 5^\circ$  in the frequency range of  $2.45$ – $3.26$  GHz, and those of  $|\angle S_{23} - \angle S_{43}|$  are tested as  $\pm 5^\circ$  in  $2.62$ – $3.42$  GHz in Fig. 22(d). The measured results, in general, agree with the predicted ones, given fabrication errors.

## VI. CONCLUSION

In this paper, a wideband coupled-line ring hybrid has been suggested for high power-division ratios. For this, a  $90^\circ$  single transmission-line section and a set of  $90^\circ$  coupled transmission-line sections with high characteristic impedances are indispensable. To make the high values of characteristic impedances feasible for the single transmission-line sections, four types ( $L_{\Pi^-}$ ,  $L_{T^-}$ ,  $L_{S_2^-}$ , and  $L_{S_1}$ -types) of equivalent circuits have been discussed, and an integer  $N$  has been introduced for wideband characteristics and design flexibilities. Even though all the characteristic impedances of the transmission-line sections in the four types are lower than that of the original transmission-line section, the  $L_{S_2}$ -type cannot be utilized for the coupled-line ring hybrids with high power-division ratios. Therefore, the  $ML_{S_2}$ -types were suggested, and  $ML_{S_2}$ -types with  $N = 1$  were intensively discussed.

Since the high-impedance transmission-line sections for both single and coupled transmission-line sections are needed for various applications including the ring hybrids demonstrated in this paper, many other applications are expected in microwave circuits.

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