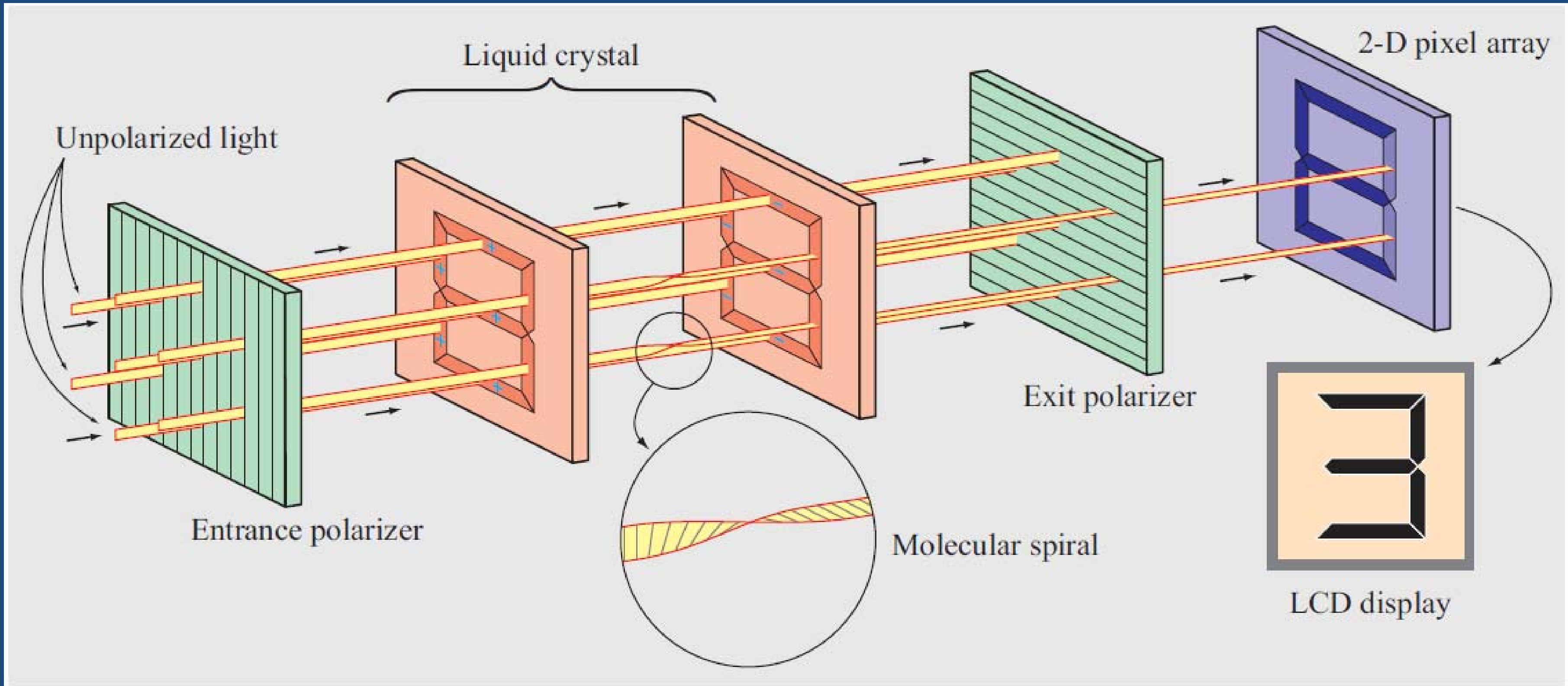


. ¬ 2-D Array of a Liquid Crystal Display

1. WAVES & PHASORS

7e Applied EM by Ulaby and Ravaioli

Chapter 1 Overview

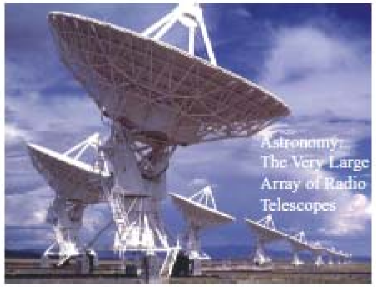
Chapter Contents

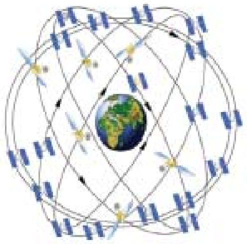
Objectives

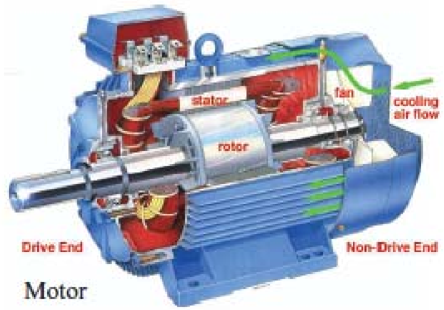
Upon learning the material presented in this chapter, you should be able to:

1. Describe the basic properties of electric and magnetic forces.
2. Ascribe mathematical formulations to sinusoidal waves traveling in both lossless and lossy media.
3. Apply complex algebra in rectangular and polar forms.
4. Apply the phasor-domain technique to analyze circuits driven by sinusoidal sources.

Examples of EM Applications

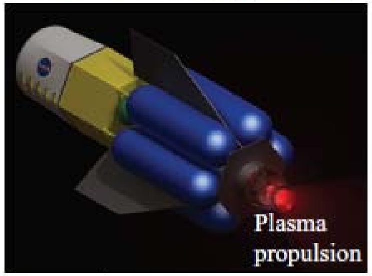

Astronomy: The Very Large Array of Radio Telescopes

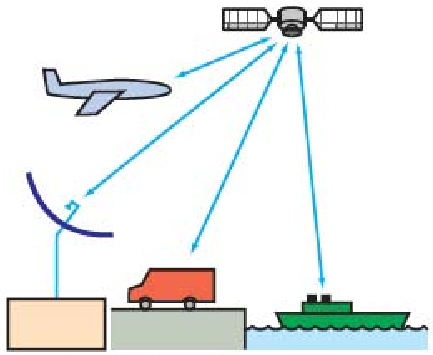

Global Positioning System (GPS)



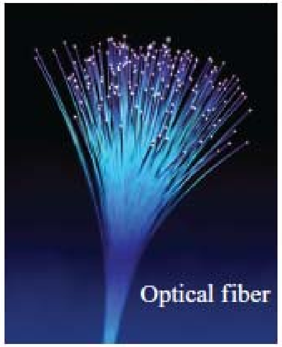
Motor

LCD Screen



Plasma propulsion

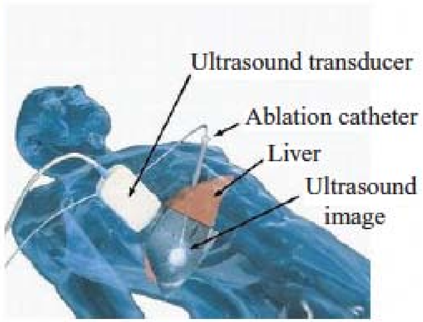

Telecommunication


Radar


Optical fiber


Electromagnetic sensors

Cell phone

Microwave ablation for liver cancer treatment

Dimensions and Units

Table 1-1: Fundamental SI units.

Dimension	Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol

Table 1-2: Multiple and submultiple prefixes.

Prefix	Symbol	Magnitude
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}

Charge: Electrical property of particles

Units: coulomb

One coulomb: amount of charge accumulated in one second by a current of one ampere.

1 coulomb represents the charge on ~ 6.241×10^{18} electrons

The coulomb is named for a French physicist, Charles-Augustin de Coulomb (1736-1806), who was the first to measure accurately the forces exerted between electric charges.

Charge of an electron

$$e = 1.602 \times 10^{-19}\ \mathrm{C}$$

Charge conservation

Cannot create or destroy charge, only transfer

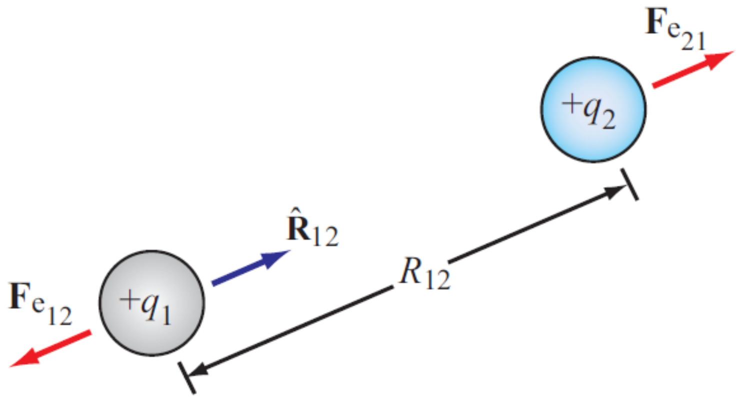
Electrical Force

Coulomb's experiments demonstrated that:

(1) *two like charges repel one another, whereas two charges of opposite polarity attract,*

(2) *the force acts along the line joining the charges, and*

(3) *its strength is proportional to the product of the magnitudes of the two charges and inversely proportional to the square of the distance between them.*



$$\mathbf{F}_{e_{21}} = \hat{\mathbf{R}}_{12} \frac{q_1 q_2}{4\pi \varepsilon_0 R_{12}^2} \quad \text{(N)} \quad \text{(in free space)},$$

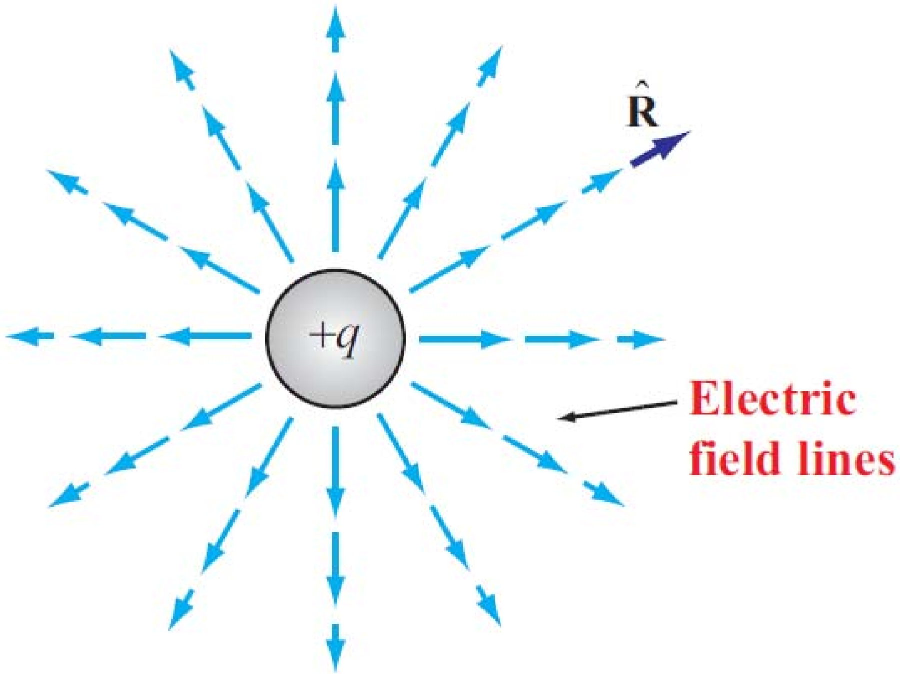
Force exerted on charge 2 by charge 1

Electric Field In Free Space

If any point charge q' is present in an electric field $\mathbf{E}$ *(due to other charges), the point charge will experience a force acting on it equal to* $\mathbf{F}_e = q'\mathbf{E}$.

$$\mathbf{E} = \hat{\mathbf{R}} \frac{q}{4\pi\varepsilon_0 R^2} \quad \text{(V/m)} \quad \text{(in free space)},$$

Permittivity of free space



Magnetic Field

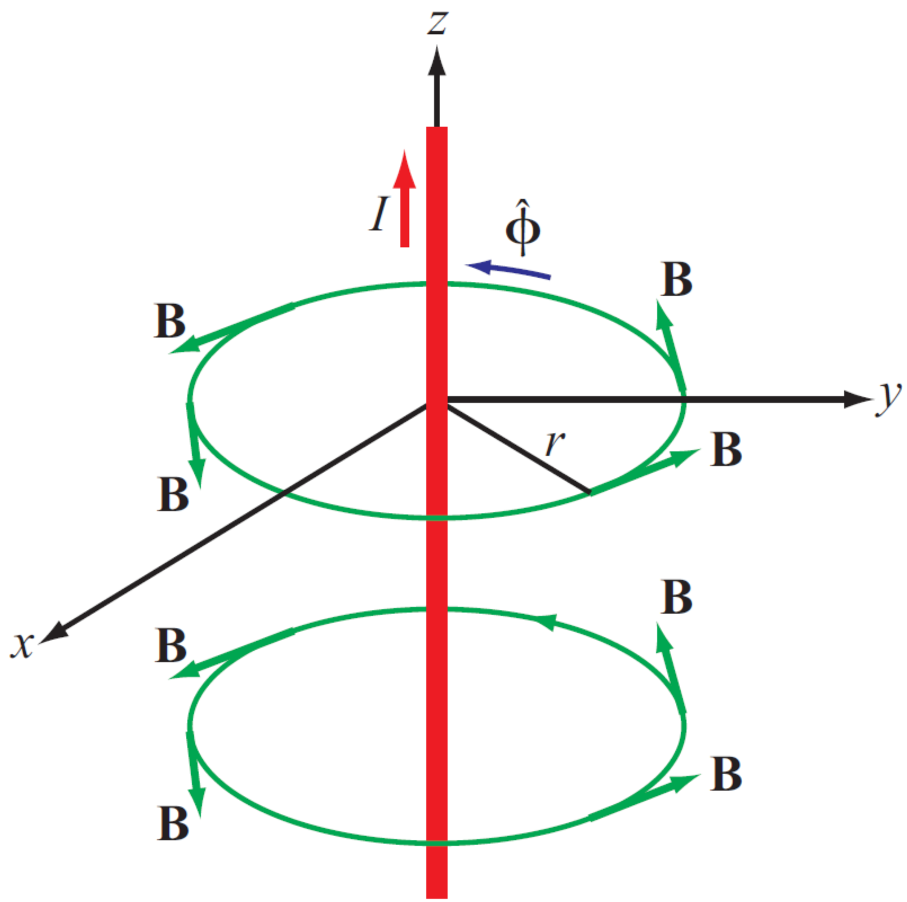
Electric charges can be isolated, but magnetic poles always exist in pairs.

Magnetic field induced by a current in a long wire

$$\mathbf{B} = \hat{\boldsymbol{\phi}} \, \frac{\mu_0 I}{2\pi r} \qquad \text{(T)}$$

$$\mu_0 = 4\pi \times 10^{-7} \ \text{henry per meter (H/m)}$$

Magnetic permeability of free space



Electric and magnetic fields are connected through the speed of light:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \qquad \text{(m/s)}$$

Static vs. Dynamic

Static conditions: charges are stationary or moving, but if moving, they do so at a constant velocity.

Table 1-3: The three branches of electromagnetics.

Branch	Condition	Field Quantities (Units)
Electrostatics	Stationary charges $(\partial q/\partial t = 0)$	Electric field intensity $\mathbf{E}$ (V/m) Electric flux density $\mathbf{D}$ (C/m^2) $\mathbf{D} = \varepsilon \mathbf{E}$
Magnetostatics	Steady currents $(\partial I/\partial t = 0)$	Magnetic flux density $\mathbf{B}$ (T) Magnetic field intensity $\mathbf{H}$ (A/m) $\mathbf{B} = \mu \mathbf{H}$
Dynamics (Time-varying fields)	Time-varying currents $(\partial I/\partial t \neq 0)$	$\mathbf{E}$, $\mathbf{D}$, $\mathbf{B}$, and $\mathbf{H}$ $(\mathbf{E}, \mathbf{D})$ coupled to $(\mathbf{B}, \mathbf{H})$

Under static conditions, electric and magnetic fields are independent, but under dynamic conditions, they become coupled.

Material Properties

Table 1-4: Constitutive parameters of materials.

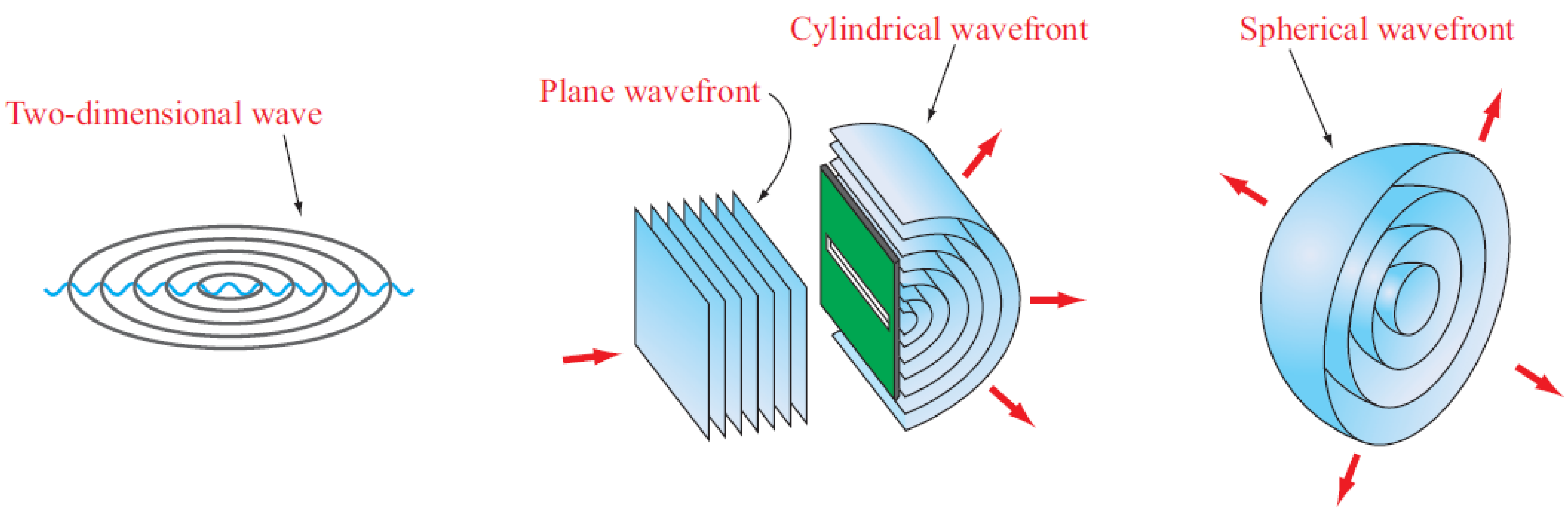
Parameter	Units	Free-space Value
Electrical permittivity ε	F/m	$\varepsilon_0 = 8.854 \times 10^{-12}$ (F/m) $\simeq \dfrac{1}{36\pi} \times 10^{-9}$ (F/m)
Magnetic permeability μ	H/m	$\mu_0 = 4\pi \times 10^{-7}$ (H/m)
Conductivity σ	S/m	0

Traveling Waves

- Waves carry energy
- Waves have velocity
- Many waves are linear: they do not affect the passage of other waves; they can pass right through them

- Transient waves: caused by sudden disturbance
- Continuous periodic waves: repetitive source

Types of Waves



(a) Circular waves (b) Plane and cylindrical waves (c) Spherical wave

Sinusoidal Waves in Lossless Media

A medium is said to be ***lossless*** *if it does not attenuate the amplitude of the wave traveling within it or on its surface.*

y = height of water surface
x = distance

$$y(x,t) = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0\right) \quad \text{(m)}, \qquad (1.17)$$

where A is the ***amplitude*** of the wave, T is its ***time period***, λ is its ***spatial wavelength***, and ϕ_0 is a ***reference phase***.

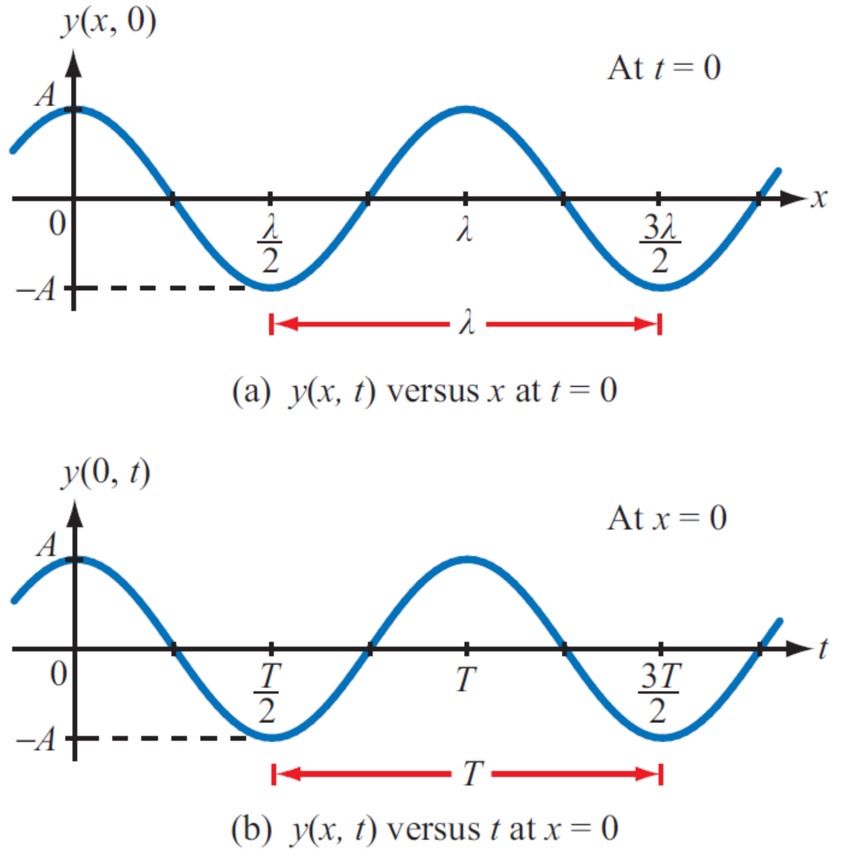


Figure 1-12: Plots of $y(x,t) = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$ as a function of (a) x at $t = 0$ and (b) t at $x = 0$.

Phase velocity

$$y(x,t) = A\cos\phi(x,t) \qquad \text{(m)},$$

where

$$\phi(x,t) = \left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0\right) \qquad \text{(rad)}.$$

If we select a fixed height y_0 and follow its progress, then

$$y_0 = y(x,t) = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$$

$$\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} = \cos^{-1}\left(\frac{y_0}{A}\right) = \text{constant}$$

$$\frac{2\pi}{T} - \frac{2\pi}{\lambda}\frac{dx}{dt} = 0$$

$$\boxed{u_p = \frac{dx}{dt} = \frac{\lambda}{T} \qquad \text{(m/s)}}$$

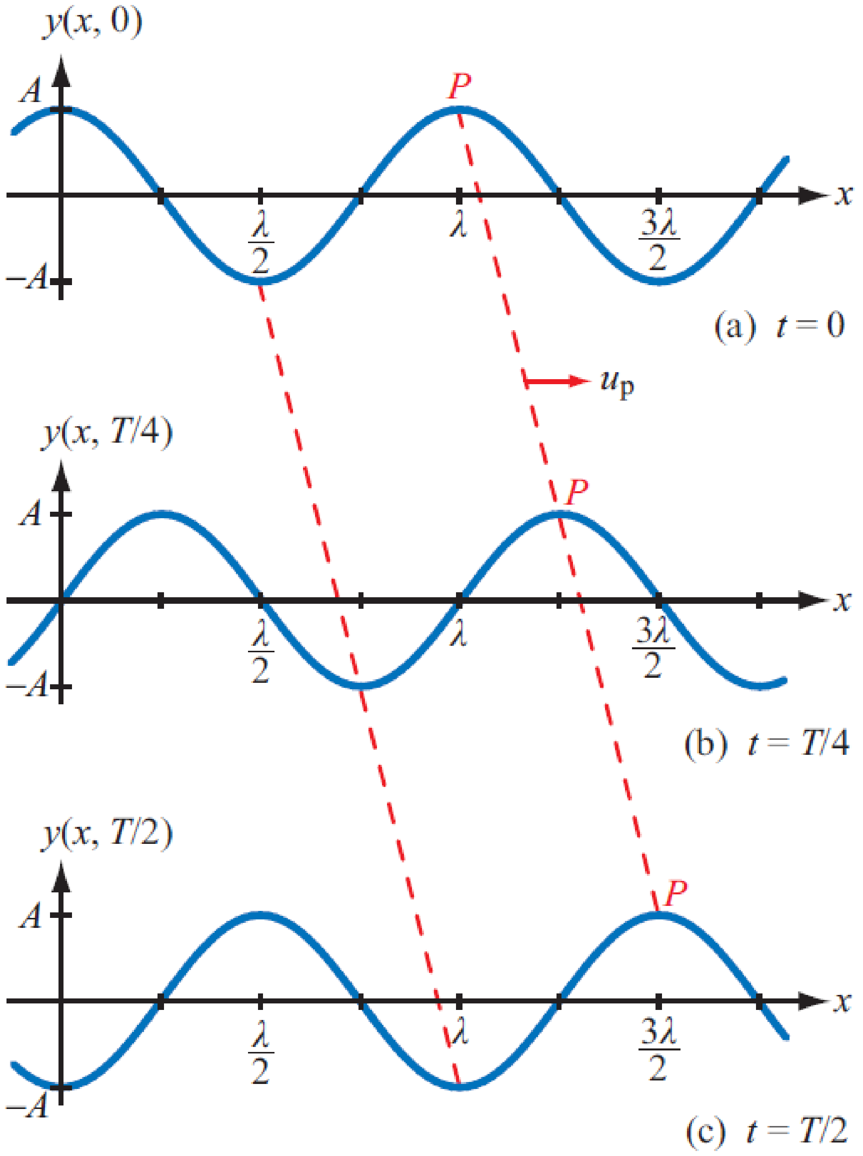


Figure 1-13: Plots of $y(x,t) = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$ as a function of x at (a) $t = 0$, (b) $t = T/4$, and (c) $t = T/2$. Note that the wave moves in the $+x$-direction with a velocity $u_p = \lambda/T$.

Wave Frequency and Period

The ***frequency*** of a sinusoidal wave, f, is the reciprocal of its time period T:

$$f = \frac{1}{T} \qquad \text{(Hz).} \qquad (1.26)$$

Combining the preceding two equations yields

$$u_{\mathrm{p}} = f\lambda \qquad \text{(m/s).} \qquad (1.27)$$

Using Eq. (1.26), Eq. (1.20) can be rewritten in a more compact form as

$$y(x,t) = A\cos\left(2\pi f t - \frac{2\pi}{\lambda}x\right) = A\cos(\omega t - \beta x), \quad (1.28)$$

where ω is the ***angular velocity*** of the wave and β is its ***phase constant*** (or ***wavenumber***), defined as

$$\omega = 2\pi f \qquad \text{(rad/s),} \qquad (1.29a)$$

$$\beta = \frac{2\pi}{\lambda} \qquad \text{(rad/m).} \qquad (1.29b)$$

Direction of Wave Travel

$y(x,t) = A\cos(\omega t - \beta x) \longleftarrow$ Wave travelling in +x direction

$y(x,t) = A\cos(\omega t + \beta x) \longleftarrow$ Wave travelling in −x direction

+x direction: if coefficients of *t* and *x* have opposite signs

−x direction: if coefficients of *t* and *x* have same sign (both positive or both negative)

Phase Lead & Lag

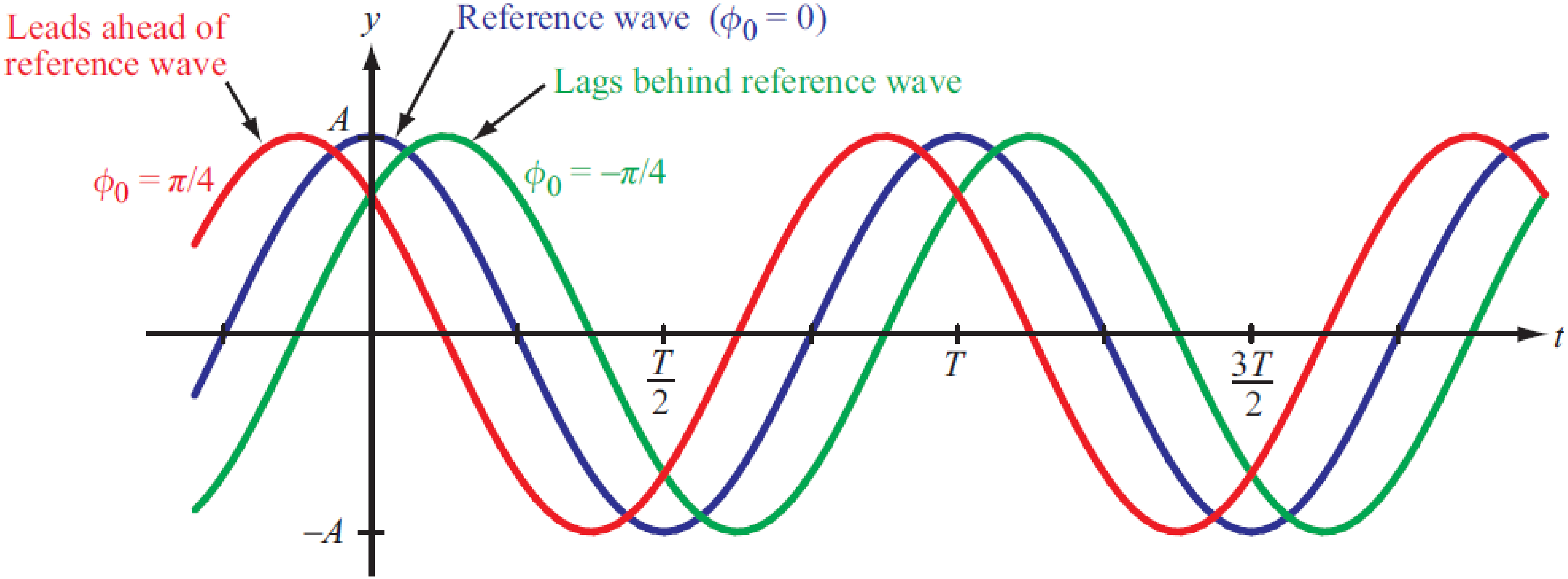


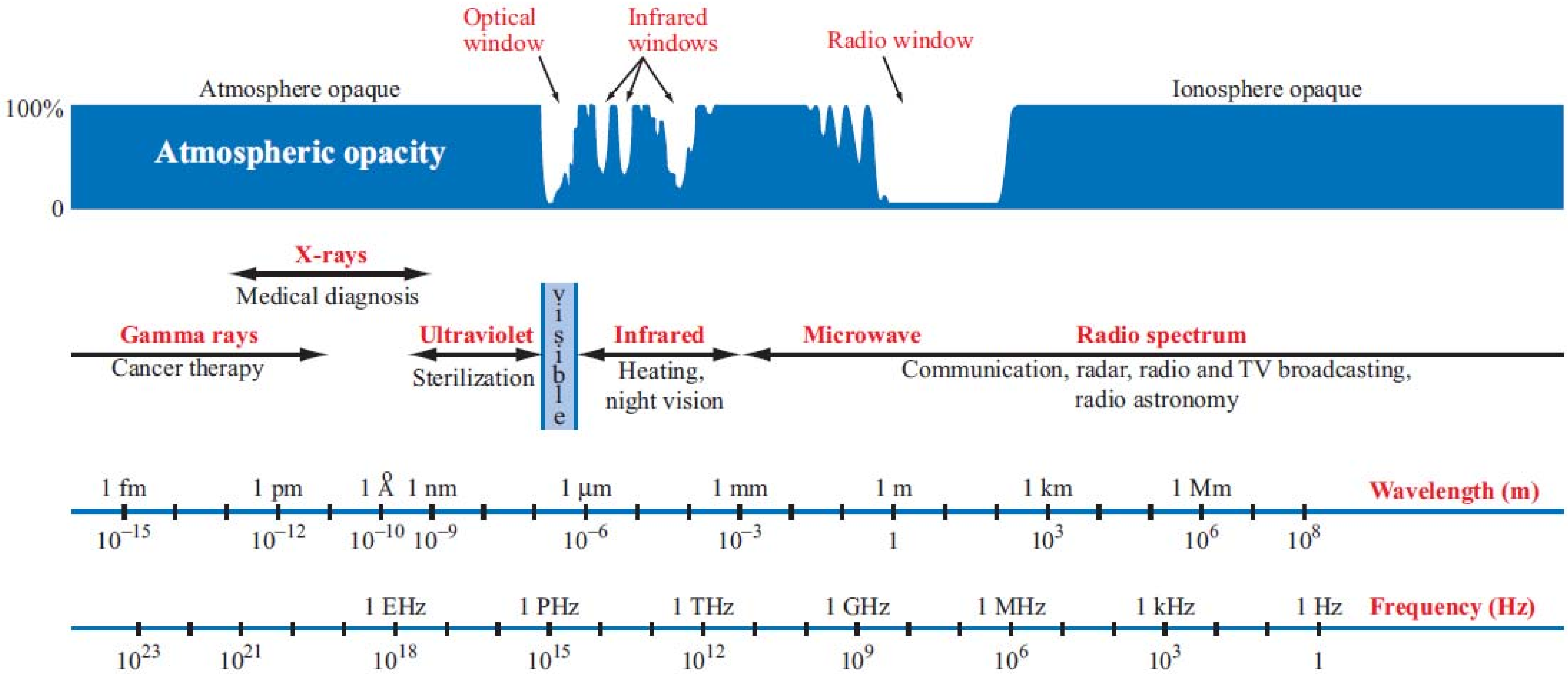
Figure 1-14: Plots of $y(0, t) = A\cos[(2\pi t/T) + \phi_0]$ for three different values of the reference phase ϕ_0.

$$y(x, t) = A\cos(\omega t - \beta x + \phi_0)$$

When its value is positive, ϕ_0 signifies a phase lead in time, and when it is negative, it signifies a phase lag.

The EM Spectrum

$$\lambda = \frac{c}{f}$$



Complex Numbers

We will find it is useful to represent sinusoids as complex numbers

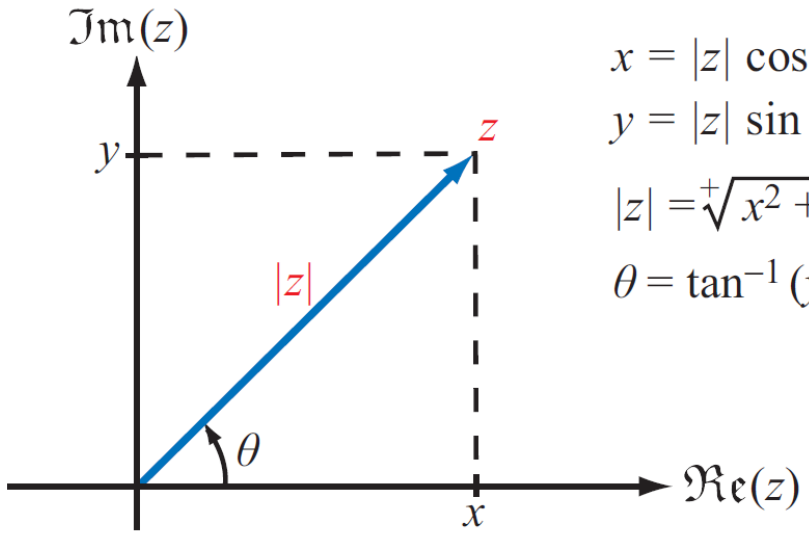
$$j = \sqrt{-1}$$

$z = x + jy$ **Rectangular coordinates**

$z = |z| \angle \theta = |z| e^{j\theta}$ **Polar coordinates**

$\mathrm{Re}(z) = x$

$\mathrm{Im}(z) = y$



$$x = |z| \cos \theta$$
$$y = |z| \sin \theta$$
$$|z| = \sqrt[+]{x^2 + y^2}$$
$$\theta = \tan^{-1}(y/x)$$

Relations based on Euler's Identity

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

Relations for Complex Numbers

Euler's Identity: $e^{j\theta} = \cos\theta + j\sin\theta$	
$\sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$	$\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$
$\mathbf{z} = x + jy = \lvert\mathbf{z}\rvert e^{j\theta}$	$\mathbf{z}^* = x - jy = \lvert\mathbf{z}\rvert e^{-j\theta}$
$x = \Re\mathfrak{e}(\mathbf{z}) = \lvert\mathbf{z}\rvert\cos\theta$	$\lvert\mathbf{z}\rvert = \sqrt[+]{\mathbf{z}\mathbf{z}^*} = \sqrt[+]{x^2 + y^2}$
$y = \Im\mathfrak{m}(\mathbf{z}) = \lvert\mathbf{z}\rvert\sin\theta$	$\theta = \tan^{-1}(y/x)$
$\mathbf{z}^n = \lvert\mathbf{z}\rvert^n e^{jn\theta}$	$\mathbf{z}^{1/2} = \pm\lvert\mathbf{z}\rvert^{1/2} e^{j\theta/2}$
$\mathbf{z}_1 = x_1 + jy_1$	$\mathbf{z}_2 = x_2 + jy_2$
$\mathbf{z}_1 = \mathbf{z}_2$ iff $x_1 = x_2$ and $y_1 = y_2$	$\mathbf{z}_1 + \mathbf{z}_2 = (x_1 + x_2) + j(y_1 + y_2)$
$\mathbf{z}_1\mathbf{z}_2 = \lvert\mathbf{z}_1\rvert\lvert\mathbf{z}_2\rvert e^{j(\theta_1+\theta_2)}$	$\dfrac{\mathbf{z}_1}{\mathbf{z}_2} = \dfrac{\lvert\mathbf{z}_1\rvert}{\lvert\mathbf{z}_2\rvert} e^{j(\theta_1-\theta_2)}$
$-1 = e^{j\pi} = e^{-j\pi} = 1\angle\pm180^\circ$	
$j = e^{j\pi/2} = 1\angle 90^\circ$	$-j = e^{-j\pi/2} = 1\angle -90^\circ$
$\sqrt{j} = \pm e^{j\pi/4} = \pm\dfrac{(1+j)}{\sqrt{2}}$	$\sqrt{-j} = \pm e^{-j\pi/4} = \pm\dfrac{(1-j)}{\sqrt{2}}$

Learn how to perform these with _your_ calculator/computer

Phasor Domain

A ***domain transformation*** is a mathematical process that converts a set of variables from their domain into a corresponding set of variables defined in another domain.

1. The phasor-analysis technique transforms equations from the time domain to the phasor domain.

2. Integro-differential equations get converted into linear equations with no sinusoidal functions.

3. After solving for the desired variable--such as a particular voltage or current-- in the phasor domain, conversion back to the time domain provides the same solution that would have been obtained had the original integro-differential equations been solved entirely in the time domain.

Phasor Domain

$$v(t) = V_0 \cos(\omega t + \phi)$$
$$= \Re\mathfrak{e}[\underbrace{V_0 e^{j\phi}}_{\text{Phasor counterpart of } v(t)} e^{j\omega t}]$$

Time Domain		**Phasor Domain**
$v(t) = V_0 \cos \omega t$	$\leftrightarrow$	$\mathbf{V} = V_0$
$v(t) = V_0 \cos(\omega t + \phi)$	$\leftrightarrow$	$\mathbf{V} = V_0 e^{j\phi}$.

If $\phi = -\pi/2$,

$$v(t) = V_0 \cos(\omega t - \pi/2) \quad \leftrightarrow \quad \mathbf{V} = V_0 e^{-j\pi/2}.$$

Time and Phasor Domain

$x(t)$		$\mathbf{X}$
$A\cos\omega t$	$\leftrightarrow$	A
$A\cos(\omega t+\phi)$	$\leftrightarrow$	$Ae^{j\phi}$
$-A\cos(\omega t+\phi)$	$\leftrightarrow$	$Ae^{j(\phi\pm\pi)}$
$A\sin\omega t$	$\leftrightarrow$	$Ae^{-j\pi/2}=-jA$
$A\sin(\omega t+\phi)$	$\leftrightarrow$	$Ae^{j(\phi-\pi/2)}$
$-A\sin(\omega t+\phi)$	$\leftrightarrow$	$Ae^{j(\phi+\pi/2)}$
$\frac{d}{dt}(x(t))$	$\leftrightarrow$	$j\omega\mathbf{X}$
$\frac{d}{dt}[A\cos(\omega t+\phi)]$	$\leftrightarrow$	$j\omega Ae^{j\phi}$
$\int x(t)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}\mathbf{X}$
$\int A\cos(\omega t+\phi)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}Ae^{j\phi}$

It is much easier to deal with exponentials in the phasor domain than sinusoidal relations in the time domain

Just need to track magnitude/phase, knowing that everything is at frequency ω

Phasor Relation for Resistors

Current through resistor

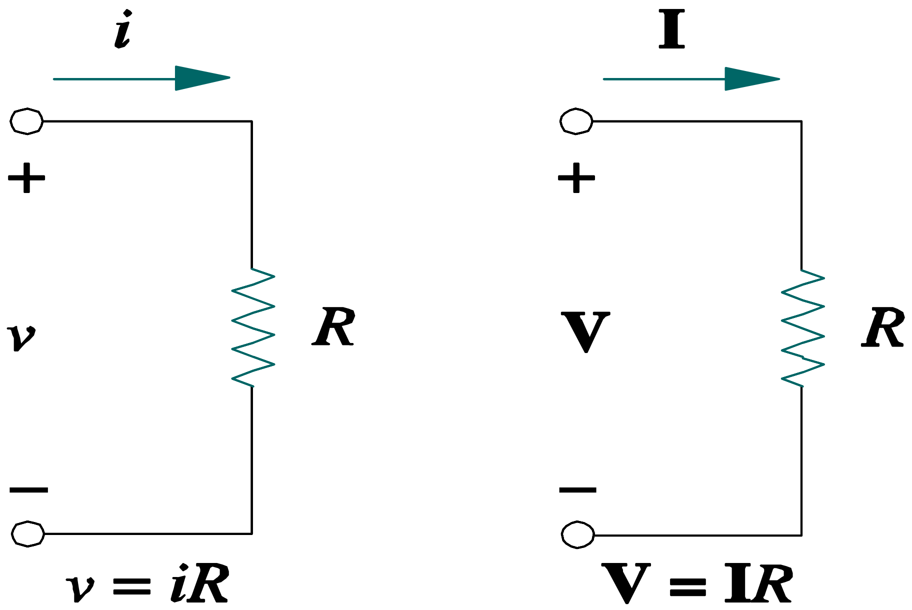
Time domain

$$i = I_m \cos(\omega t + \phi)$$

$$v = iR = RI_m \cos(\omega t + \phi)$$

Time Domain

Frequency Domain

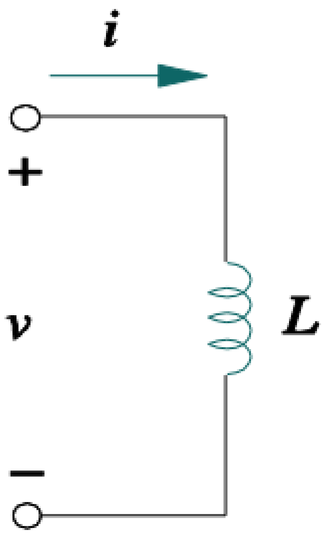


Phasor Domain

$$\mathbf{V} = R\mathbf{I}$$
$$= RI_m \angle \phi$$

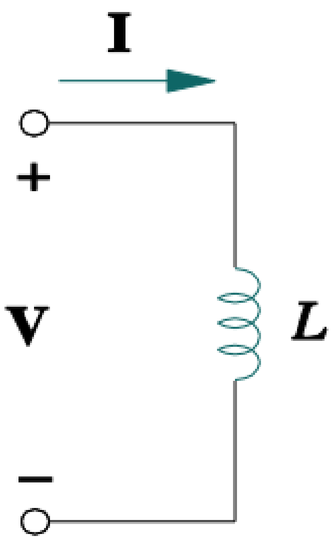
Phasor Relation for Inductors

Time Domain



$$v = L\frac{di}{dt}$$

Frequency Domain



$$\mathbf{V} = j\omega L\mathbf{I}$$

Time domain $\quad v = L\,\dfrac{di}{dt}$

Phasor Domain $\quad v_{\mathrm{L}} = \Re\mathfrak{e}[\mathbf{V}_{\mathrm{L}} e^{j\omega t}]$

and

$$i_{\mathrm{L}} = \Re\mathfrak{e}[\mathbf{I}_{\mathrm{L}} e^{j\omega t}].$$

Consequently,

$$\begin{aligned}\Re\mathfrak{e}[\mathbf{V}_{\mathrm{L}} e^{j\omega t}] &= L\,\frac{d}{dt}[\Re\mathfrak{e}(\mathbf{I}_{\mathrm{L}} e^{j\omega t})] \\ &= \Re\mathfrak{e}[j\omega L\mathbf{I}_{\mathrm{L}} e^{j\omega t}],\end{aligned}$$

which leads to

$$\mathbf{V}_{\mathrm{L}} = j\omega L\mathbf{I}_{\mathrm{L}}$$

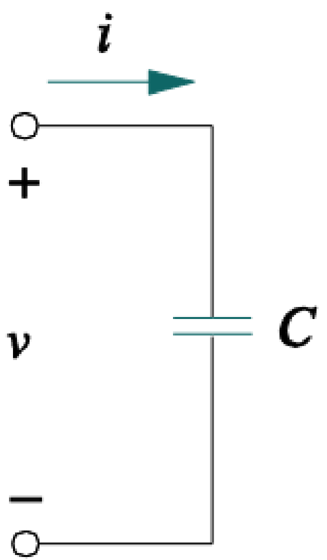
and

$$\boxed{\mathbf{Z}_{\mathrm{L}} = \frac{\mathbf{V}_{\mathrm{L}}}{\mathbf{I}_{\mathrm{L}}} = j\omega L.}$$

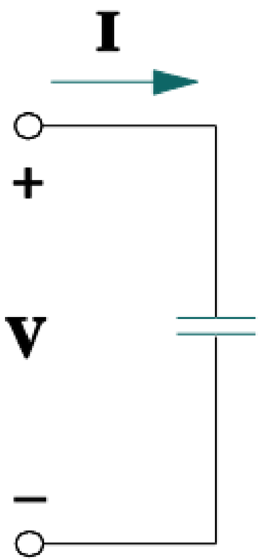
Phasor Relation for Capacitors

Time Domain

Frequency Domain



$$i = C\frac{dv}{dt}$$



$$\mathbf{I} = j\omega C\mathbf{V}$$

Time domain

$$i = C\,\frac{dv}{dt}$$

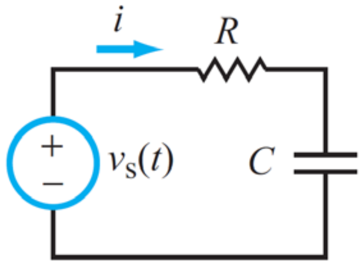
Phasor Domain

$$\mathbf{I}_C = j\omega C\mathbf{V}_C$$

$$\mathbf{Z}_C = \frac{\mathbf{V}_C}{\mathbf{I}_C} = \frac{1}{j\omega C}.$$

ac Phasor Analysis: General Procedure

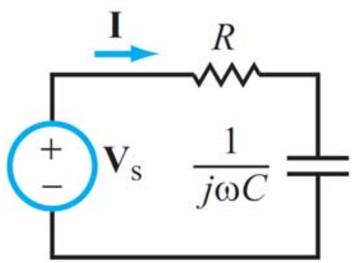
Step 1
Adopt Cosine Reference
(Time Domain)



$v_s(t) = 12\sin(\omega t - 45°)$ (V)

Step 2
Transfer to Phasor Domain

$i \longrightarrow \mathbf{I}$
$v \longrightarrow \mathbf{V}$
$R \longrightarrow \mathbf{Z}_R = R$
$L \longrightarrow \mathbf{Z}_L = j\omega L$
$C \longrightarrow \mathbf{Z}_C = 1/j\omega C$



$\mathbf{V}_s = 12e^{-j135°}$ (V)

Step 3
Cast Equations in
Phasor Form

$$\mathbf{I}\left(R + \frac{1}{j\omega C}\right) = \mathbf{V}_s$$

Step 4
Solve for Unknown Variable
(Phasor Domain)

$$\mathbf{I} = \frac{\mathbf{V}_s}{R + \frac{1}{j\omega C}}$$

Step 5
Transform Solution
Back to Time Domain

$$i(t) = \mathfrak{Re}[\mathbf{I}e^{j\omega t}] = 6\cos(\omega t - 105°) \text{ (mA)}$$

Example 1-4: RL Circuit

The voltage source of the circuit shown in Fig. 7-8(a) is given by

$$v_s(t) = 15\sin(4 \times 10^4 t - 30°) \text{ V}.$$

Also, $R = 3\ \Omega$ and $L = 0.1$ mH. Obtain an expression for the voltage across the inductor.

Solution:

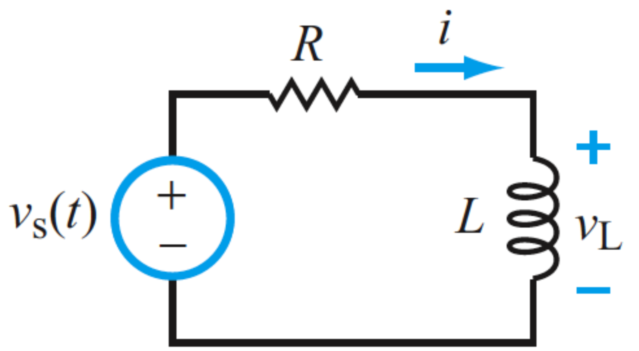
Step 1: Convert $\boldsymbol{v}_{\mathbf{s}}(\boldsymbol{t})$ to the cosine reference

$$\begin{aligned} v_s(t) &= 15\sin(4 \times 10^4 t - 30°) \\ &= 15\cos(4 \times 10^4 t - 30° - 90°) \\ &= 15\cos(4 \times 10^4 t - 120°) \text{ V}, \end{aligned}$$

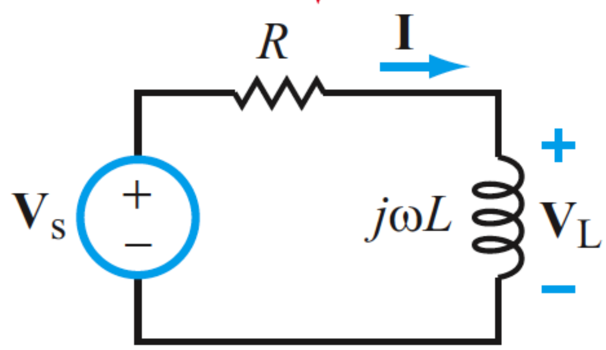
and its corresponding phasor $\mathbf{V}_s$ is given by

$$\mathbf{V}_s = 15e^{-j120°} \text{ V}.$$

Step 2: Transform circuit to the phasor domain



(a) Time domain



(b) Phasor domain

Cont.

Example 1-4: RL Circuit cont.

Step 3: Cast KVL in phasor domain

$$R\mathbf{I} + j\omega L\mathbf{I} = \mathbf{V}_s.$$

Step 4: Solve for unknown variable

$$\mathbf{I} = \frac{\mathbf{V}_s}{R + j\omega L} = \frac{15e^{-j120^\circ}}{3 + j4 \times 10^4 \times 10^{-4}}$$
$$= \frac{15e^{-j120^\circ}}{3 + j4} = \frac{15e^{-j120^\circ}}{5e^{j53.1^\circ}} = 3e^{-j173.1^\circ} \text{ A.}$$

The phasor voltage across the inductor is related to $\mathbf{I}$ by

$$\begin{aligned}\mathbf{V}_\mathrm{L} &= j\omega L\mathbf{I} \\ &= j4 \times 10^4 \times 10^{-4} \times 3e^{-j173.1^\circ} \\ &= j12e^{-j173.1^\circ} \\ &= 12e^{-j173.1^\circ} \cdot e^{j90^\circ} = 12e^{-j83.1^\circ} \text{ V,}\end{aligned}$$

where we replaced j with e^{j90°.

Step 5: Transform solution to the time domain

The corresponding time-domain voltage is

$$\begin{aligned}v_\mathrm{L}(t) &= \mathfrak{Re}[\mathbf{V}_\mathrm{L}e^{j\omega t}] \\ &= \mathfrak{Re}[12e^{-j83.1^\circ}e^{j4\times 10^4 t}] \\ &= 12\cos(4 \times 10^4 t - 83.1^\circ) \text{ V.}\end{aligned}$$

1-7.2 Traveling Waves in the Phasor Domain

According to Table 1-5, if we set $\phi_0 = 0$, its third entry becomes

$$A\cos(\omega t + \beta x) \quad \longleftrightarrow \quad Ae^{j\beta x}. \tag{1.74}$$

From the discussion associated with Eq. (1.31), we concluded that $A\cos(\omega t + \beta x)$ describes a wave traveling in the negative x-direction.

In the phasor domain, a wave of amplitude A traveling in the positive x-direction in a lossless medium with phase constant β is given by the negative exponential $Ae^{-j\beta x}$, and conversely, a wave traveling in the negative x-direction is given by $Ae^{j\beta x}$. Thus, the sign of x in the exponential is opposite to the direction of travel.

Summary

Chapter 1 Relationships

Electric field due to charge q in free space

$$\mathbf{E} = \hat{\mathbf{R}}\,\frac{q}{4\pi\varepsilon_0 R^2}$$

Magnetic field due to current I in free space

$$\mathbf{B} = \hat{\boldsymbol{\phi}}\,\frac{\mu_0 I}{2\pi r}$$

Plane wave $y(x,t) = Ae^{-\alpha x}\cos(\omega t - \beta x + \phi_0)$

- $\alpha = 0$ in lossless medium
- phase velocity $u_p = f\lambda = \frac{\omega}{\beta}$
- $\omega = 2\pi f;\ \beta = 2\pi/\lambda$
- ϕ_0 = phase reference

Complex numbers

- Euler's identity

  $e^{j\theta} = \cos\theta + j\sin\theta$
- Rectangular-polar relations

  $x = |z|\cos\theta,\qquad y = |z|\sin\theta,$

  $|z| = \sqrt[+]{x^2 + y^2}\,,\qquad \theta = \tan^{-1}(y/x)$

Phasor-domain equivalents

Table 1-5