Lecture 02: Vector Calculus

1. Vector
2. Vector Addition and Subraction
3. Position Vector and Distance Vector
4. Scalar Product
5. Vector Product
6. Scalar and Vector Triple Products
7. Problems
8. Coding Examples

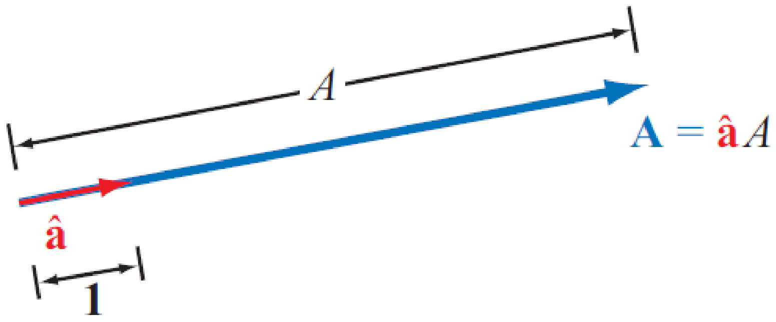
1. Vector

- Vector:

- Magnitude and direction

- Vector notation: roman bold

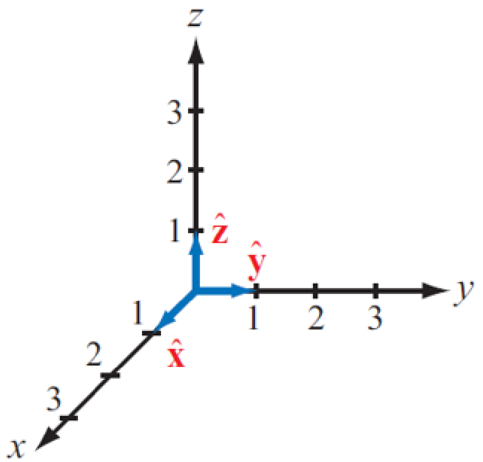
- Magnitude notation: italic

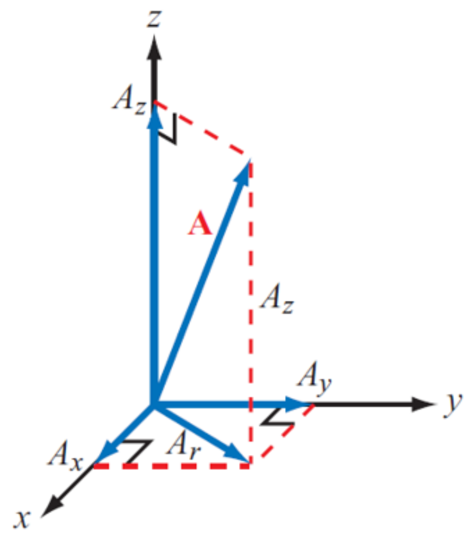


$$\mathbf{A} = \hat{\mathbf{A}}\,|A| = \hat{\mathbf{A}}\,A\,;\;\; \hat{\mathbf{A}} = \frac{\mathbf{A}}{A}$$

$\hat{\mathbf{A}}$: unit vector

A : magnitude





$$\mathbf{A} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}} = (A_x, A_y, A_z):\ \text{component notation}$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

1. Vector

- Vector component notation:

성분을 이용한 벡터의 표기

$$\boxed{\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z}$$

B_x, B_y, B_z : 벡터 $\mathbf{B}$의 x, y, z 성분(component)

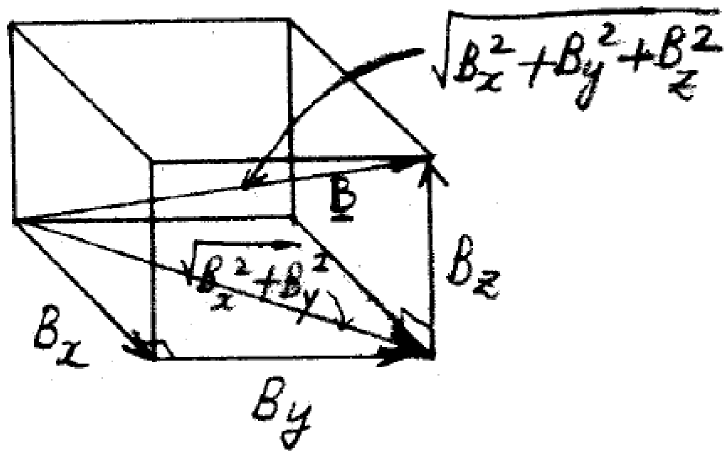
- Unit vector (단위벡터):

\- 벡터 **B**의 단위벡터 = **B** 방향의 단위벡터

$$\boxed{\mathbf{a}_B = \frac{\mathbf{B}}{\sqrt{B_x^{\,2} + B_y^{\,2} + B_z^{\,2}}} = \frac{\mathbf{B}}{|\mathbf{B}|}}$$

- Vector magnitude (벡터크기):

$$|\mathbf{B}| = \sqrt{B_x^{\ 2} + B_y^{\ 2} + B_z^{\ 2}}$$



1. Vector

- Exercise:

Find the magnitude of a vector **A** = (1, 5, 3).

Find the unit vector of a vector **A** = (1, 5, 3).

▪ Exercise: 두 벡터가 평행할 조건

두 벡터 $\mathbf{A} = A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z$, $\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z$가 같은 방향

또는 반대 방향이려면 $\dfrac{A_x}{B_x} = \dfrac{A_y}{B_y} = \dfrac{A_z}{B_z}$를 만족해야 함을 증명하라.

(풀이)

두 벡터가 같은 방향일 경우:

$$\mathbf{a}_A = \mathbf{a}_B \rightarrow \frac{A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z}{A} = \frac{B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z}{B}$$

그러므로

$$\frac{A_x}{A} = \frac{B_x}{B},\ \frac{A_y}{A} = \frac{B_y}{B},\ \frac{A_z}{A} = \frac{B_z}{B} \rightarrow \frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z} = \frac{A}{B}$$

두 벡터가 반대 방향일 경우: $\mathbf{a}_A = -\mathbf{a}_B$

이 경우

$$\frac{A_x}{A} = \frac{-B_x}{B},\ \frac{A_y}{A} = \frac{-B_y}{B},\ \frac{A_z}{A} = \frac{-B_z}{B} \rightarrow \frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z} = -\frac{A}{B}$$

그러므로 두 벡터 $\mathbf{A}, \mathbf{B}$가 동일 방향 또는 반대 방향일 경우 (즉 평행할 경우)

$$\boxed{\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}}$$

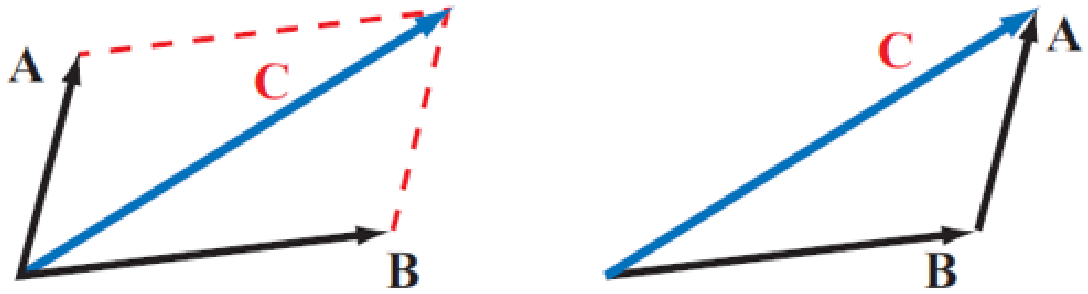
가 성립한다.

2. Vector Addition and Subtraction

- Vector Addition:

$$\mathbf{A} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}} \; ; \; \mathbf{B} = B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}}$$

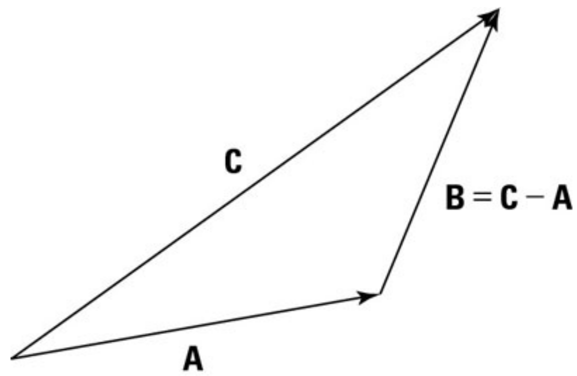
$$\mathbf{C} = \mathbf{A} + \mathbf{B} = (A_x + B_x)\hat{\mathbf{x}} + (A_y + B_y)\hat{\mathbf{y}} + (A_z + B_z)\hat{\mathbf{z}}$$



- Vector Subraction:

$$\mathbf{A} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}} \; ; \; \mathbf{C} = C_x\hat{\mathbf{x}} + C_y\hat{\mathbf{y}} + C_z\hat{\mathbf{z}}$$

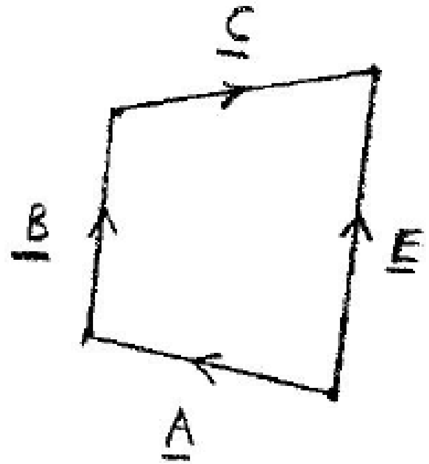
$$\mathbf{B} = \mathbf{C} - \mathbf{A} = (C_x - A_x)\hat{\mathbf{x}} + (C_y - A_y)\hat{\mathbf{y}} + (C_z - A_z)\hat{\mathbf{z}}$$



- Exercise:

Express the vector **C** using vectors **A**, **B**, and **E**.

Express the vector **E** using vectors **A**, **B**, and **C**.

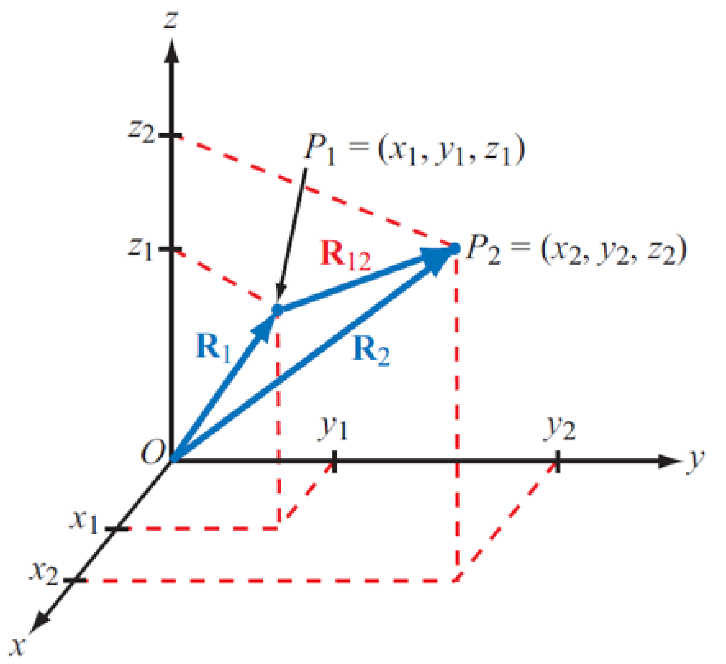


3. Position Vector and Distance Vector

- Position vector:

$$\mathbf{R}_1 = \overrightarrow{OP_1} = \hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$$

$$\mathbf{R}_2 = \overrightarrow{OP_2} = \hat{\mathbf{x}}x_2 + \hat{\mathbf{y}}y_2 + \hat{\mathbf{z}}z_2$$



- Distance vector:

$$\begin{aligned}\mathbf{R}_{12} &= \overrightarrow{P_1P_2} \\ &= \mathbf{R}_2 - \mathbf{R}_1 \\ &= \hat{\mathbf{x}}(x_2 - x_1) + \hat{\mathbf{y}}(y_2 - y_1) + \hat{\mathbf{z}}(z_2 - z_1)\end{aligned}$$

- Exercise:

Find the position vector $\mathbf{R}_P$ of a point $P(1, 5, 3)$.

Find the distance vector $\mathbf{R}_{PQ}$ from a point $P(1, 5, 3)$ to $Q(2, 4, 1)$.

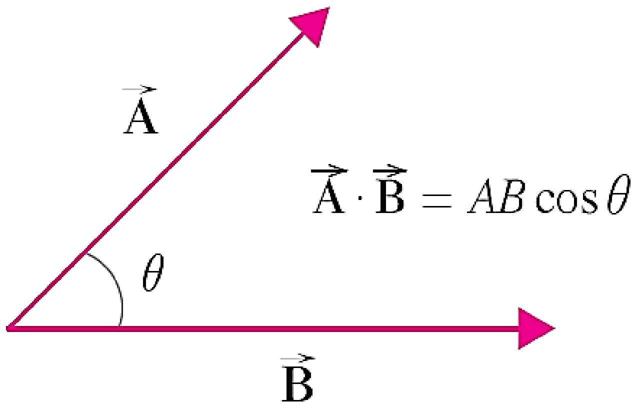
4. Scalar Prouct

- Scalar or dot product:

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$$

$$\theta = \cos^{-1}\frac{\mathbf{A}\cdot\mathbf{B}}{AB}$$

$$\mathbf{A}\cdot\mathbf{B} = 0 \rightarrow \mathbf{A}\perp\mathbf{B}$$



$$\vec{\mathbf{A}}\cdot\vec{\mathbf{A}} = A^2$$

$$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$$

$$\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$$

$$\rightarrow$$

$$\begin{aligned}\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} &= (A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}})\cdot(B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}})\\ &= A_xB_x + A_yB_y + A_zB_z\end{aligned}$$

4. Scalar Prouct

- Scalar product의 활용:

- 직선과 점 사이의 거리

- 두 벡터의 사이각

- 두 벡터의 수직 여부

- 한 벡터를 다른 벡터에 수직인 성분과 수평인 성분으로 분해

- Exercise:

A triangle has vertices $A(3, 2, 2)$, $B(5, 4, 3)$, and $C(-1, 6, 5)$

Find the base length $R_{AB} = |\mathbf{R}_{AB}|$.

Find the height with R_{AB} as the base.

Find the interior angle at the vertex A.

5. Vector Prouct

- Vector or Cross Product:

\- Right-hand rule

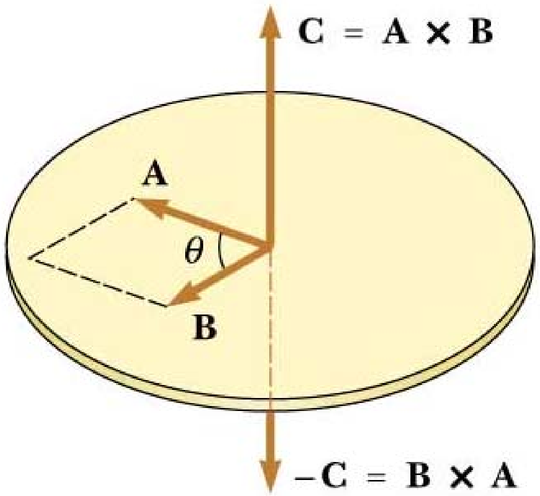
$$\mathbf{A}\times\mathbf{B}=\hat{\mathbf{n}}\,AB\sin\theta$$

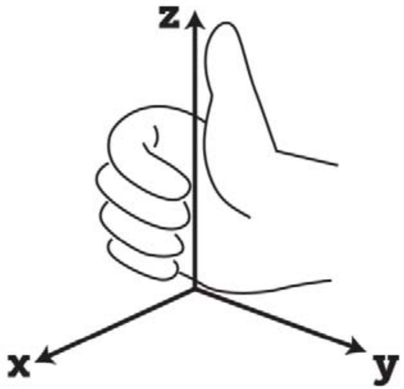
$$\mathbf{B}\times\mathbf{A}=-\mathbf{A}\times\mathbf{B}$$

$$\mathbf{A}\times\mathbf{A}=0$$

$$\hat{\mathbf{x}}\times\hat{\mathbf{x}}=0,\ \hat{\mathbf{y}}\times\hat{\mathbf{y}}=0,\ \hat{\mathbf{z}}\times\hat{\mathbf{z}}=0$$

$$\hat{\mathbf{x}}\times\hat{\mathbf{y}}=\hat{\mathbf{z}},\ \hat{\mathbf{y}}\times\hat{\mathbf{z}}=\hat{\mathbf{x}},\ \hat{\mathbf{z}}\times\hat{\mathbf{x}}=\hat{\mathbf{y}}$$





$$\hat{\mathbf{x}}\times\hat{\mathbf{y}}=\hat{\mathbf{z}}$$

$$\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

$$=\hat{\mathbf{x}}(A_yB_z-A_zB_{y-})+\hat{\mathbf{y}}(A_zB_x-A_xB_z)+\hat{\mathbf{z}}(A_xB_y-A_yB_x)$$

- Lagrange's Identity:

$$|\mathbf{A}\times\mathbf{B}|^2=(AB)^2-(\mathbf{A}\cdot\mathbf{B})^2$$

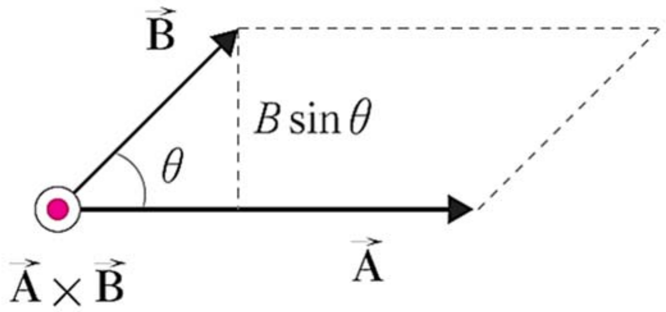
(Proof)

$$|\mathbf{A}\times\mathbf{B}|^2=(AB)^2\sin^2\theta=(AB)^2(1-\cos^2\theta)=(AB)^2-(\mathbf{A}\cdot\mathbf{B})^2$$

5. Vector Prouct

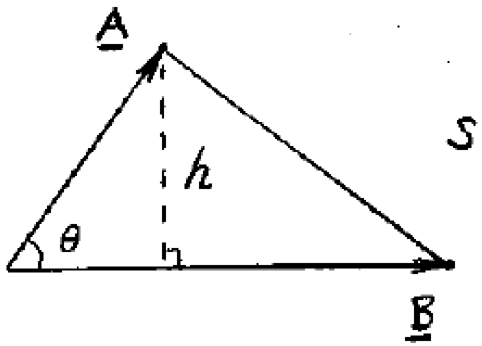
- Area of a parallelepiped:

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$



- Area of a triangle:

· 삼각형의 면적계산



$$S = \frac{1}{2}Bh = \frac{1}{2}BA\sin\theta$$

$$= \frac{1}{2}|\underline{A}\times\underline{B}|$$

$$\boxed{S = \frac{1}{2}\left|\mathbf{A}\times\mathbf{B}\right|}$$

- Exercise: 곡선으로 둘러 싸인 영역의 면적

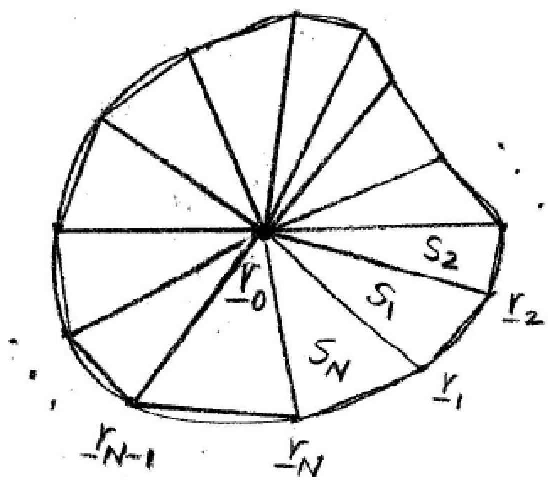
- 곡선을 선분을 사용하여 근사화

- 선분 시점과 종점의 위치벡터: $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N$

- 좌표원점을 점 **P**에 설정: $\mathbf{P} = (\mathbf{r}_1 + \mathbf{r}_2 + \ldots + \mathbf{r}_N) / N$

- 면적: $S = S_1 + S_2 + \ldots + S_N$

$$S_n = \frac{1}{2}\left|(\mathbf{r}_{n+1} - \mathbf{r}_0) \times (\mathbf{r}_n - \mathbf{r}_0)\right|$$

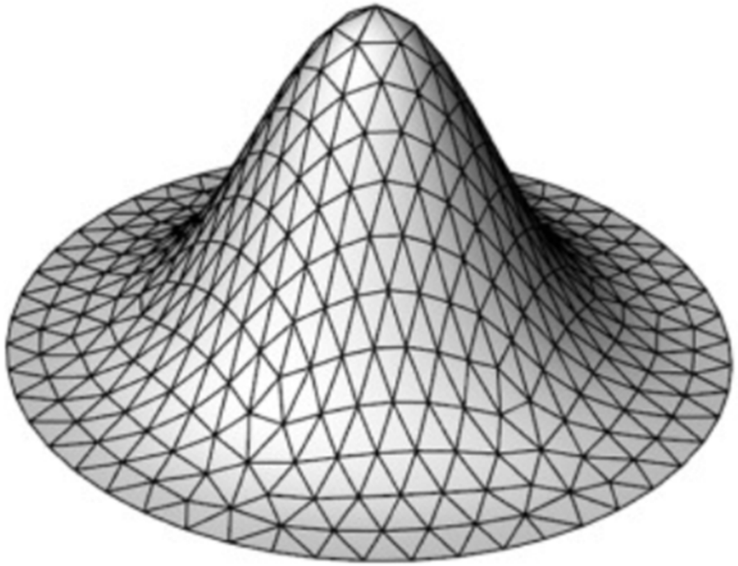


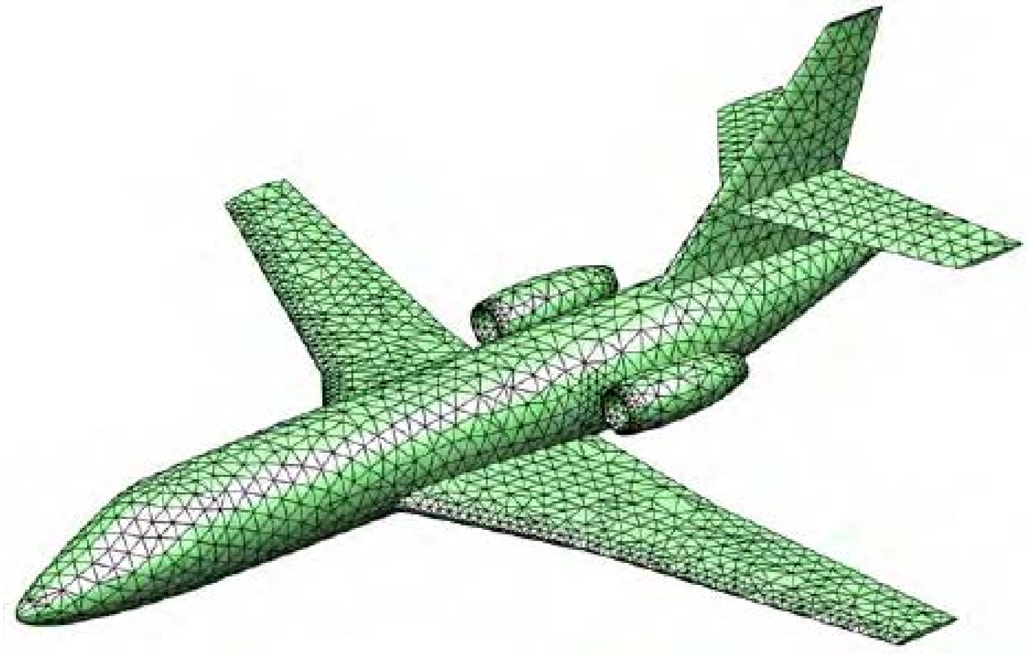
5. Vector Prouct

- Exercise: 곡면의 면적

- 곡면 triangular meshing

- 각 삼각형의 면적을 구한 후 합한다.





5. Vector Prouct

- Applications of Vector Product:

- A vector perpendicular to two vectors

- A vector perpendicular to a plane

- Torque calculation: $\mathbf{T} = \mathbf{R} \times \mathbf{F}$

- Check if two vectors are parallel to each other.

- Area of a triangle

- Exercise:

두 벡터 $\mathbf{A} = 2\mathbf{a}_x + \mathbf{a}_y + 3\mathbf{a}_z$, $\mathbf{B} = 5\mathbf{a}_x - 2\mathbf{a}_y + 4\mathbf{a}_z$ 에 수직인 단위벡터를 구하라.

(풀이)

두 벡터에 수직인 단위벡터를 $\mathbf{a}_n$ 라 하면 공식으로부터

$$\mathbf{a}_n = \pm \frac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|}$$

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 2 & 1 & 3 \\ 5 & -2 & 4 \end{vmatrix} = \mathbf{a}_x(4+6) - \mathbf{a}_y(8-15) + \mathbf{a}_z(-4-5)$$

$$\mathbf{a}_n = \pm \frac{1}{\sqrt{230}}(10\mathbf{a}_x + 7\mathbf{a}_y - 9\mathbf{a}_z)$$

- Exercise:

두 벡터 $\mathbf{A} = 2\mathbf{a}_x + \mathbf{a}_y + 3\mathbf{a}_z$, $\mathbf{B} = 5\mathbf{a}_x - 2\mathbf{a}_y + 4\mathbf{a}_z$ 가 서로 평행한지 확인하라.

(풀이)

두 벡터가 평행한 경우 두 벡터 사이의 각도는 0°이므로 $\mathbf{A} \times \mathbf{B} = 0$ 가 성립한다.

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 2 & 1 & 3 \\ 5 & -2 & 4 \end{vmatrix} = \mathbf{a}_x(4+6) - \mathbf{a}_y(8-15) + \mathbf{a}_z(-4-5)$$

으로부터 $\mathbf{A} \times \mathbf{B} \neq 0$ 이므로 두 벡터는 평행하지 않음을 알 수 있다.

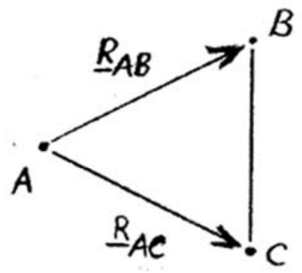
- Exercise:

세 점 $A(6,-1,2), B(-2,3,-4), C(-3,1,5)$ 에 의해 정의된 삼각형의 면적과 삼각형면에 수직인 단위벡터를 구하라.

(풀이)

아래 그림과 같이 세 점 A,B,C 를 표시한 후 점 A 에서 점 B 로 향하는 벡터 $\mathbf{R}_{AB}$ 와 점 A 에서 점 C 로 향하는 벡터 $\mathbf{R}_{AC}$ 를 표시한다. 벡터 $\mathbf{R}_{AB}$ 와 $\mathbf{R}_{AC}$ 는 점 A,B,C 의 위치벡터 $\mathbf{r}_A, \mathbf{r}_B, \mathbf{r}_C$ 로부터 구한다. 즉,

$$\mathbf{R}_{AB} = \mathbf{r}_B - \mathbf{r}_A = (-2,3,-4) - (6,-1,2) = (-8,4,-6)$$
$$\mathbf{R}_{AC} = \mathbf{r}_C - \mathbf{r}_A = (-3,1,5) - (6,-1,2) = (-9,2,3)$$



삼각형 ABC 의 면적은 다음과 같이 공식을 사용하여 구한다.

$$S = \frac{1}{2}|\mathbf{R}_{AB} \times \mathbf{R}_{AC}| = \frac{1}{2}\begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ -8 & 4 & -6 \\ -9 & 2 & 3 \end{vmatrix} = \frac{1}{2}|\mathbf{a}_x 24 - \mathbf{a}_y(-78) + \mathbf{a}_z 20|$$

$$= \frac{1}{2}\sqrt{24^2 + (-78)^2 + 20^2} \simeq 42$$

5. Vector Prouct

- Exercise:

삼각형에 수직인 단위벡터는 벡터 $\mathbf{R}_{AB}$와 $\mathbf{R}_{AC}$에 모두 수직인 단위벡터와 같으며 이 것은 다음과 같이 공식을 사용하여 구한다.

$$\mathbf{a}_n = \pm\frac{\mathbf{R}_{AB}\times\mathbf{R}_{AC}}{|\mathbf{R}_{AB}\times\mathbf{R}_{AC}|} = \pm\frac{1}{84}(24\mathbf{a}_x + 78\mathbf{a}_y + 20\mathbf{a}_z)$$

6. Scalar and Vector Tripole Proucts

Scalar Triple Product

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B}).$$

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

Vector Triple Product

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B}),$$

6. Scalar and Vector Tripole Proucts

스케일러 3 중적(scalar triple product)

· 계산 공식

$$\mathbf{A}\times\mathbf{B} = \mathbf{a}_x(A_yB_z - A_zB_y) + \mathbf{a}_y(A_zB_x - A_xB_z) + \mathbf{a}_z(A_xB_y - A_yB_x)$$

이므로

$$\mathbf{C}\cdot\mathbf{A}\times\mathbf{B} = C_x(A_yB_z - A_zB_y) + C_y(A_zB_x - A_xB_z) + C_z(A_xB_y - A_yB_x)$$

그러므로 $\mathbf{A}\times\mathbf{B}$ 를 계산하기 위한 행렬식에서 $(\mathbf{a}_x, \mathbf{a}_y, \mathbf{a}_z)$ 대신 (C_x, C_y, C_z) 를 사용하면 된다. 즉,

$$\boxed{\mathbf{C}\cdot(\mathbf{A}\times\mathbf{B}) = \begin{vmatrix} C_x & C_y & C_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}}$$

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) \equiv (\mathbf{A},\mathbf{B},\mathbf{C})$$

6. Scalar and Vector Tripole Proucts

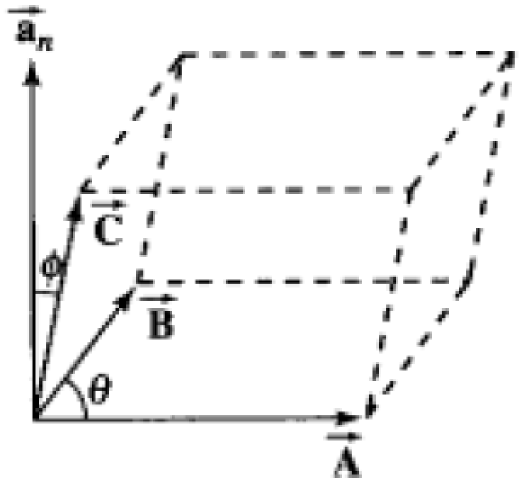
- Volume of a parallelepiped (평행육면체):

평행육면체의 체적

$$V = |A(B\sin\theta)|\,|C\cos\phi| = |\mathbf{A}\times\mathbf{B}|\,|C\cos\phi|$$

그러므로

$$\boxed{V = |(\mathbf{A}\times\mathbf{B})\bullet\mathbf{C}|}$$



$S = |\mathbf{A}\times\mathbf{B}|$: area of parallelogram

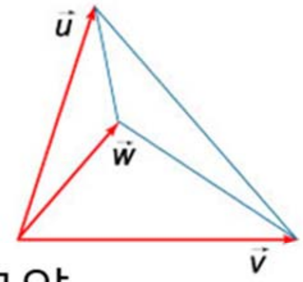
$H = \dfrac{\mathbf{A}\times\mathbf{B}}{|\mathbf{A}\times\mathbf{B}|}\cdot\mathbf{C}$: height of parallelepiped

$$V = SH = \mathbf{A}\times\mathbf{B}\cdot\mathbf{C}$$

6. Scalar and Vector Tripole Proucts

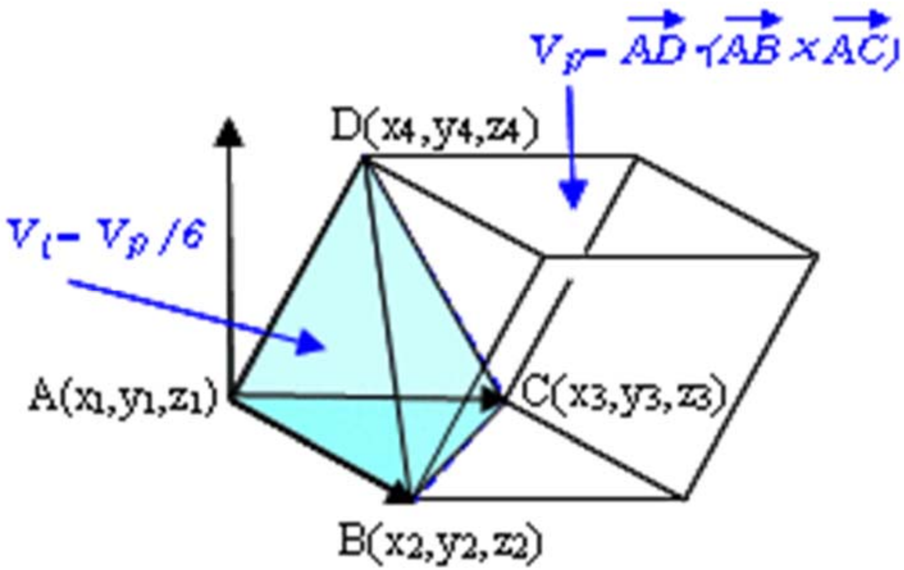
- Volume of a tetrahedron (사면체):

$$V = \frac{1}{6}\left|(\vec{u}, \vec{v}, \vec{w})\right| = \frac{1}{6}\left|\vec{u} \cdot (\vec{v} \times \vec{w})\right|$$



- 1/6의 근원: 1/2 삼각형, 1/3 뿔모양
- 피러미드의 체적

$$V = \frac{1}{3}\left|(\mathbf{u}, \mathbf{v}, \mathbf{w})\right|$$



- Exercise:

Find the volume of a tetrahedron whose vertices are given by $A(0, 4, 1)$, $B(4, 0, 0)$, $C(3, 5, 2)$, $D(2, 2, 5)$

(Answer)

Take the point $D(2, 2, 5)$ as a new coordinate origin. Then vectors **A**, **B**, and **C** define the tetrahedron.

$$\mathbf{A} = \mathbf{R}_{DA} = (0,4,1) - (2,2,5) = (-2,2,-4)$$

$$\mathbf{B} = \mathbf{R}_{DB} = (4,0,0) - (2,2,5) = (2,-2,-5)$$

$$\mathbf{C} = \mathbf{R}_{DC} = (3,5,2) - (2,2,5) = (1,3,-3)$$

$$V = \frac{1}{6}|\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})| = \frac{1}{3}\begin{vmatrix} -2 & 2 & -4 \\ 2 & -2 & -5 \\ 1 & 3 & -3 \end{vmatrix} = \frac{1}{6}\left|[-2(6+15) - 2(-6+5) - 4(6+2)]\right|$$

$$= \frac{1}{6}|-42+2-32| = \frac{72}{6} = 12$$

7. Problems

1. A vector on the xy-plane is given $\mathbf{A} = (1, -1)$. Find a unit vector perpendicular to $\mathbf{A}$ on the xy-plane.
2. A line A is defined by $y = x+1$. Find the minium distance from $P(3, 2)$ to A.
3. Find the area of a triangle whose vertices are defined by $A(1, -1)$, $B(4, 0)$, $C(3, 3)$.
4. Find the area of a quadrangle whose vertices are defined by $A(1, -1)$, $B(4, 0)$, $C(3, 3)$, $D(0, 1)$.
5. Find a unit vector perpendicular to $\mathbf{A} = (2, 1, 3)$ and $\mathbf{B} = (-1, 2, 4)$.
6. Find the volume of a tetrahedron whose vertices are given by $A(1, 1, 2)$, $B(3, -2, 4)$, $C(-2, 1, 3)$, $D(5, 3, -5)$.

8. Coding Examples

1) Given a line *A*: $y = ax + b$, and, find a unit vector **u** parallel to *A* and a unit vector **v** normal to *A*.

(Answer)

Two points on *A*: $P(0,b)$, $Q(a,\ a+b)$

$$\mathbf{R}_{PQ} = (a, a+b) - (0,b) = (a,b)$$

$$\mathbf{u} = \frac{\mathbf{R}_{PQ}}{R_{PQ}} = \frac{1}{\sqrt{a^2+b^2}}(a,b)$$

$$\mathbf{v} = \hat{\mathbf{z}} \times \mathbf{u} = \frac{1}{\sqrt{a^2+b^2}}(-b,a)$$

8. Coding Examples

```
# EM-Lec1-Vecor_Calculus
# unit vector parallel to a line and unit vector normal to a line

import math
while True:
  a=float(input('Line:y=ax+b; a='))
  b=float(input('Line:y=ax+b; b='))
  mag=math.sqrt(a**2+b**2)
  ux=a/mag; uy=b/mag
  vx=-b/mag; vy=a/mag
  print('ux,uy=',ux,uy)
  print('vx,vy=',vx,vy)

'''
Line:y=ax+b; a=
1
Line:y=ax+b; b=
2
ux,uy= 0.4472135954999579 0.8944271909999159
vx,vy= -0.8944271909999159 0.4472135954999579
Line:y=ax+b; a=
'''
```

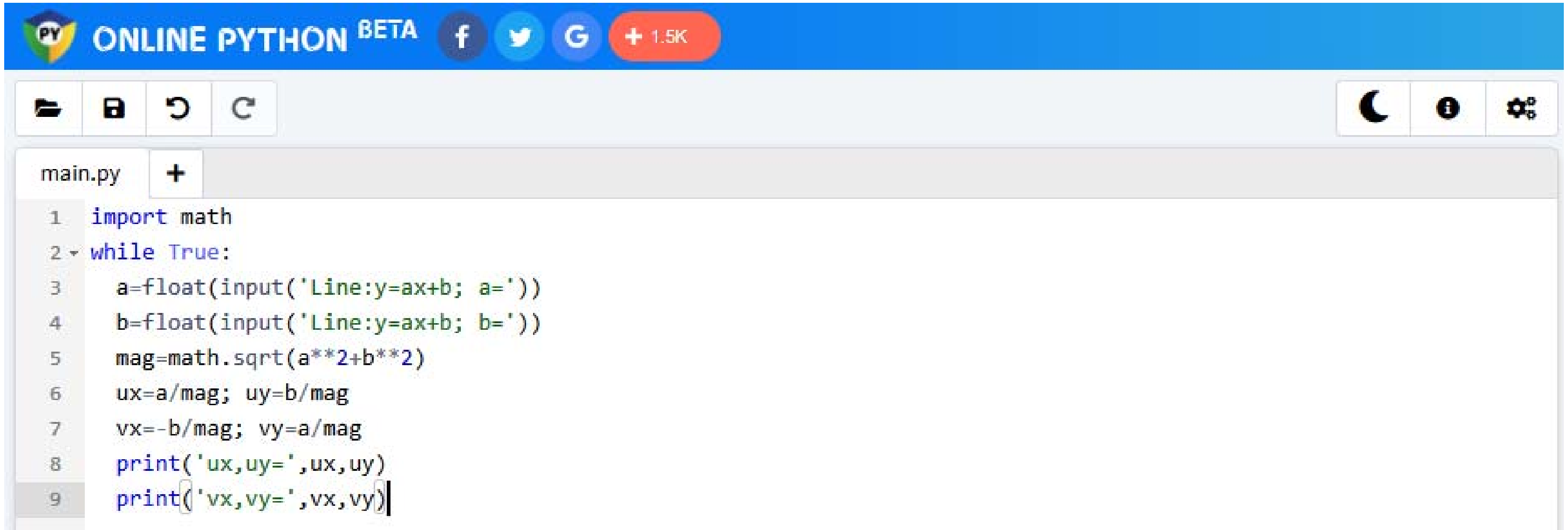
https://www.online-python.com/ 에 접속하여 소스코드를 위창에 copy

[Run]아이콘 클릭

아래창에 데이터 입력 prompt가 표시되면 키보드로 데이터 입력

출력은 아래창에 표시됨.

- 소스코드 입력창(위창)



- 상단 아이콘을 사용하여 필요한 작업

Open file From Disk

Save File to Disk

Undo

Redo

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About Site

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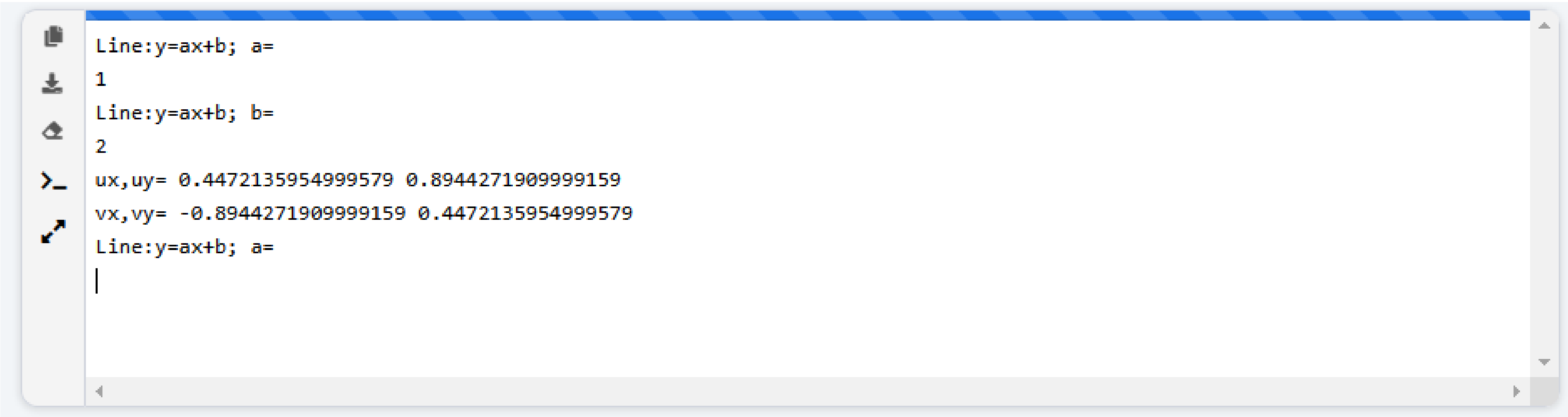
- 입출력창 (아래 창)



- 좌측 아이콘을 사용하여 필요한 작업

Copy to clipboard: 출력창 내용을 클립보드에 copy (다음에 Ctrl+v로 문서에 삽입)

Download: 출력창 내용을 txt 파일로 컴퓨터로 다운로드

Clear: 출력창 내용 지움

Smart terminal: >>>가 표시되고 interpreter 모드. Python를 1줄씩 실행. help(print) 와 같이 help 표시

Expand/Collapse: Expand = 출력창만 전체화면에 표시, Collapse: 위창, 아래창 동시표시

2) Given three points with position vectors $\mathbf{r}_1 = (x1, y_1, z_1)$, $\mathbf{r}_2 = (x_2, y_2, z_2)$ and $\mathbf{r}_3 = (x3, y_3, z_3)$, find 1) a vector $\mathbf{c} = (\mathbf{r}_2 - \mathbf{r}_1) \times (\mathbf{r}_3 - \mathbf{r}_1)$, 2) the area of a triangle whose vertices are $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$, and 3) unit vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ which are normal to the triangle.

(Solution)

$$\mathbf{a} = \mathbf{r}_2 - \mathbf{r}_1 = (x_2 - x_1, y_2 - y_1, z_2 - z_1) = (a_x, a_y, a_z)$$

$$\mathbf{b} = \mathbf{r}_3 - \mathbf{r}_1 = (x_3 - x_2, y_3 - y_2, z_3 - z_2) = (b_x, b_y, b_z)$$

$$\mathbf{c} = \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \hat{\mathbf{x}}(a_y b_z - a_z b_y) + \hat{\mathbf{y}}(a_z b_x - a_x b_z) + \hat{\mathbf{z}}(a_x b_y - a_y b_z)$$

$$= (c_x, c_y, c_z)$$

$$S = \frac{1}{2} | \mathbf{a} \times \mathbf{b} | = \frac{1}{2} \sqrt{c_x^2 + c_y^2 + c_z^2}; \ \mathbf{n}_1 = \frac{\mathbf{a} \times \mathbf{b}}{| \mathbf{a} \times \mathbf{b} |} = \frac{1}{\sqrt{c_x^2 + c_y^2 + c_z^2}} (c_x, c_y, c_z) \ ; \ \mathbf{n}_2 = -\mathbf{n}_1$$

8. Coding Examples

```
# EM-Lec1-Vecor_Calculus
# Three points r1=(x1,y1,z1), r2=(x2,y2,z2) and r3=(x3,y3,z3)
# are given. Find 1) a vector c=(r2-r1)x(r3-r1), 2) the area of
# a triangle whose vertices are r1, r2, and r3, and 3) unit
# vectors n1 and n2 perpendicular to the triangle.

import math
while True:
  x1,y1,z1=map(float, input('x1,y1,z1=').split())
  x2,y2,z2=map(float, input('x2,y2,z2=').split())
  x3,y3,z3=map(float, input('x3,y3,z3=').split())
  ax=x2-x1; ay=y2-y1; az=z2-z1
  bx=x3-x1; by=y3-y1; bz=z3-z1
  cx=ay*bz-az*by; cy=az*bx-ax*bz; cz=ax*by-ay*bx
  print('c=(r2-r1)x(r3-r1)=(',cx,',',cy,',',cz,')')
  cmag=math.sqrt(cx**2+cy**2+cz**2)
  area=cmag/2
  n1x=cx/cmag; n1y=cy/cmag; n1z=cz/cmag
  n2x=-n1x; n2y=-n1y; n2z=-n1z
  print('Area of the triangle=',area)
  print('n1x,n1y,n1z=(',n1x,',',n1y,',',n1z,')')
  print('n2x,n2y,n2z=(',n2x,',',n2y,',',n2z,')')

""
x1,y1,z1=
0 4 3
x2,y2,z2=
2 6 1
x3,y3,z2=
3 1 9
c=(r2-r1)x(r3-r1)=( 6.0 , -18.0 , -12.0 )
Area of the triangle= 11.224972160321824
n1=( 0.2672612419124244 , -0.8017837257372732 , -0.5345224838248488 )
n2=( -0.2672612419124244 , 0.8017837257372732 , 0.5345224838248488 )
```

Lecture 01: Vector Calculus

Fin

(End)