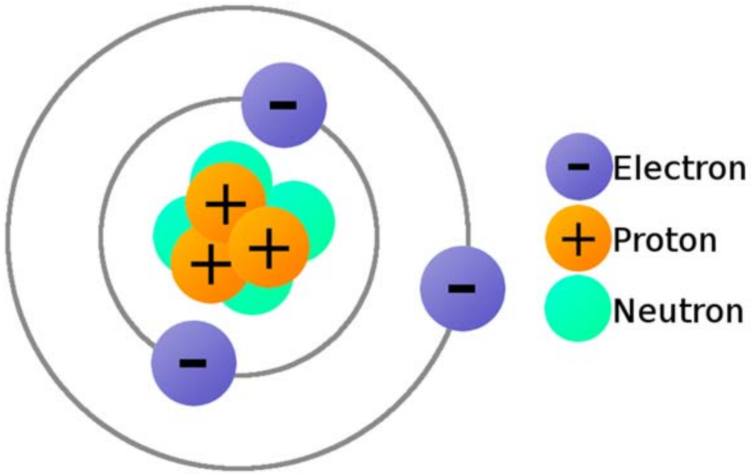
Lecture 4 Electrostatics (1)

1. Electric Charge
2. Electrostatic Force
3. Coulomb's Law
4. Electric Field
5. Coding Examples

- **Sources of Electric Charge**



Ref: learn.sparkfun.com

- All matter is made up of atoms.

- Atoms contain

  - Protons (+)

  - Neutrons (0)

  - Electrons (−)

- Protons and neurons are tightly bound to atoms.

- Electrons can move away from their atoms.

- **Charging / Charged / Ionization**

- An object acquires electrons. → Negatively charged

- An object loses electrons. → Postively charged

- **Charging Mechanisms**

- By contacts: contact electrification, triboelectric effect

- By pressure: piezoelectric effect

- By heat: pyroelectric effect

- By charge or electric field: influence charging, corona charging

- Electrochemical charging: battery

- Diffusion charging: aerosol particles are brought into contact with gas phase ions

▪ Charge of An Electron

- Charge is quantized.
- Elementary charge:

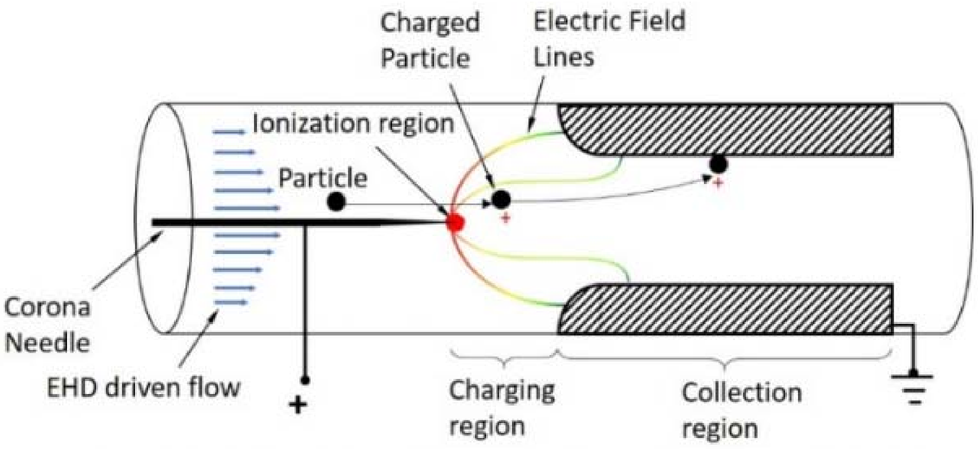
$$e = 1.602\ 176\ 634 \times 10^{-19}\ \mathrm{C}$$

- Unit of charge: C (Coulomb)
- Charge of an electron: $Q_e = -e$
- Charge of a proton: $Q_p = +e$

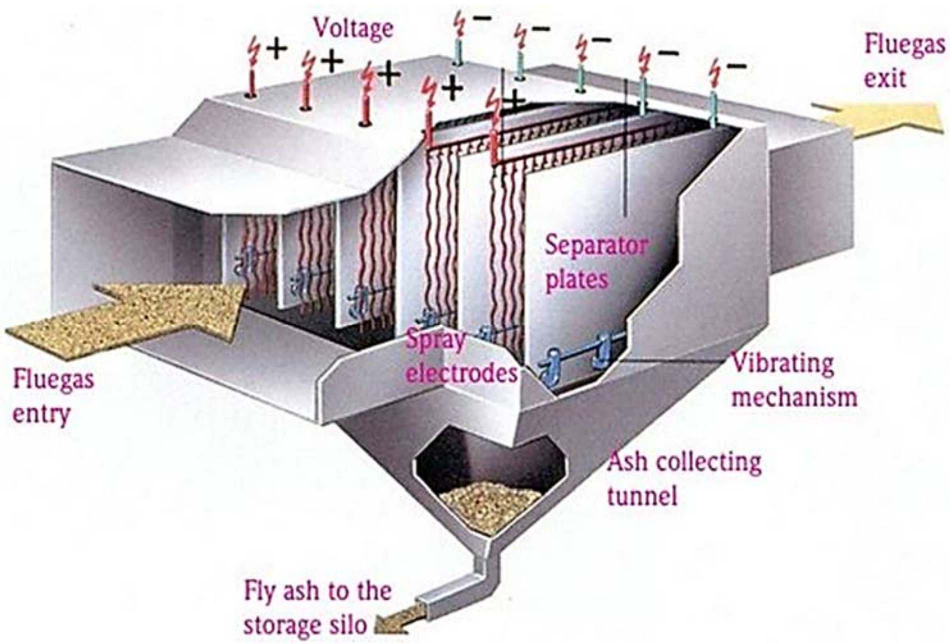
▪ Charge Conservation

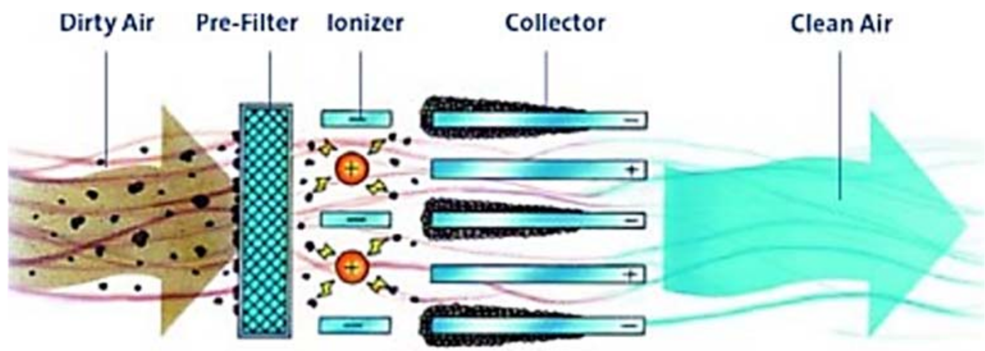
- The total electric charge in an isolated system never changes.
- The net quantity of electric charge, the amount of positive charge minus the amount of negative charge in the universe, is always *conserved*.

- Corona (= air ionization with bluish voilet light) Charging



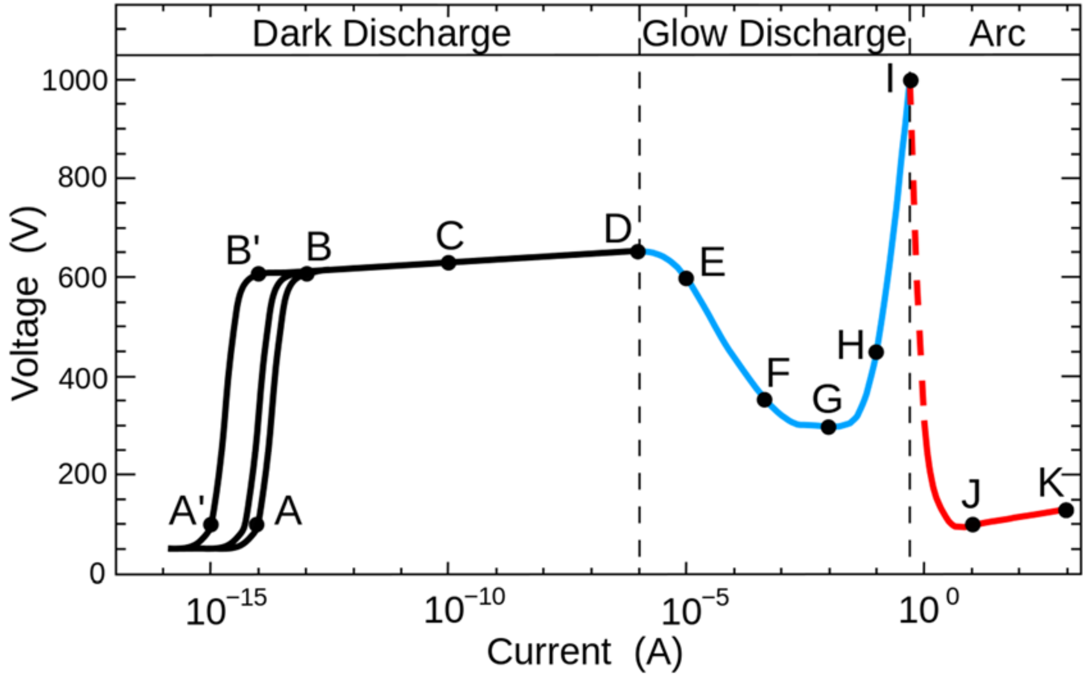
- Electrostatic Precipitator, Air Purifier

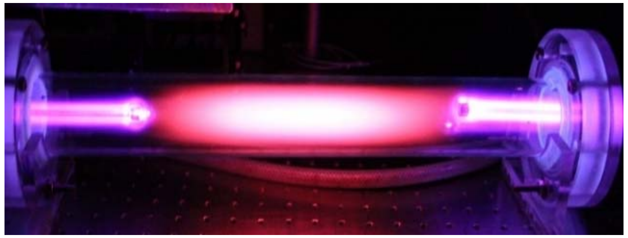




▪ Discharge

- An object loses all of its charges either by regaining all of its lost electrons or by losing all of its acquired electrons.
- Electrical discharge in gases





A: cosmic radiation random pulses

C: Townsend discharge

E: corona discharge = air breakdown by high electric field

G: glow discharge = passage of electric current through a gas

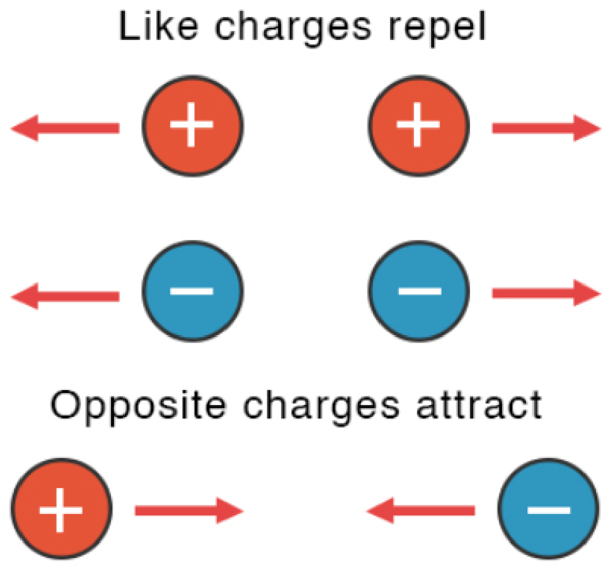
J: arc discharge = with continuous current

2. Electrostatic Force

- **Electrostactic Force in Materials**

- In atoms, protons & electrons exert force on each other.

- Forces between molecules



- **Electrosatic Force in Industrial Applications**

- Printers: laser printer, ink jet printers

- Air purifier, electrostatic precipitator

- Electrostatic spray gun (coating)

- Forensics: electrostatic dust print lifting

- Particle separation and sizing

- Electrostatic atomization: liquids to uniformly dispersed aerosol

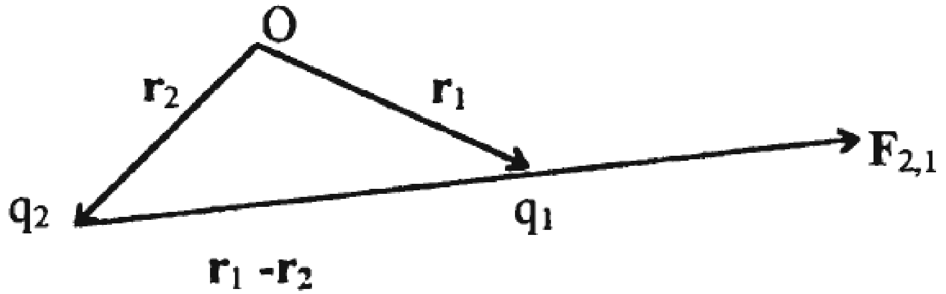
- Plastics identification for recycling

- Powder coating

3. Coulomb's Law

Coulomb's Law

$$\mathbf{F}_{2,1} = \frac{q_2 q_1 (\mathbf{r}_1 - \mathbf{r}_2)}{4\pi\varepsilon_0 \left| \mathbf{r}_1 - \mathbf{r}_2 \right|^3}$$



$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m : permittivity of vacuum

$1/(4\pi\varepsilon_0) \simeq 9 \times 10^9$ m/F

$$\mathbf{F}_1 = \sum_{i=2}^{n} \mathbf{F}_{i,1} = \frac{q_1}{4\pi\varepsilon_0} \sum_{i=2}^{n} \frac{q_i (\mathbf{r}_1 - \mathbf{r}_i)}{\left| \mathbf{r}_1 - \mathbf{r}_i \right|^3}$$

Example:

- Coulomb force in the hyrdogen atom

$Q_e = -1.6 \times 10^{-19}$ C , $Q_p = 1.6 \times 10^{-19}$ C, $R = 5.3 \times 10^{-11}$ m

$$F = 9 \times 10^9 \frac{(1.6 \times 10^{-19})^2}{(5.3 \times 10^{-11})^2} = 8.2 \times 10^{-8} \text{ N}$$

- **Electric Field and Force**

$$\mathbf{F}_1 = q_1 \mathbf{E}$$

- **Electric Field**

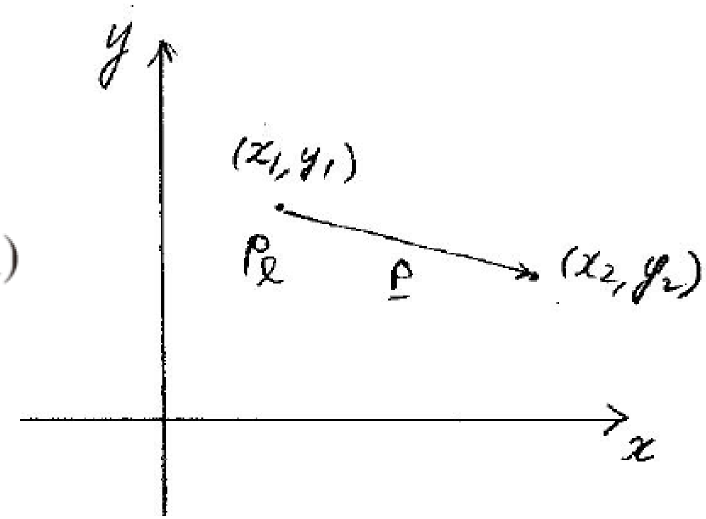
$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=2}^{n}\frac{q_i(\mathbf{r}-\mathbf{r}_i)}{|\mathbf{r}-\mathbf{r}_i|^3}$$

$$\vec{\mathrm{E}}(\vec{\mathrm{r}}) = \lim_{i\to\infty}\sum_i \frac{1}{4\pi\varepsilon_0}\frac{\rho_l(\vec{\mathrm{r}}_i)\Delta l_i}{|\vec{\mathrm{r}}-\vec{\mathrm{r}}_i|^3}(\vec{\mathrm{r}}-\vec{\mathrm{r}}_i) = \frac{1}{4\pi\varepsilon_0}\int_{l'}\frac{\rho_l(\vec{\mathrm{r}}')(\vec{\mathrm{r}}-\vec{\mathrm{r}}')}{|\vec{\mathrm{r}}-\vec{\mathrm{r}}'|^3}dl'$$

$$\vec{\mathrm{E}}(\vec{\mathrm{r}}) = \frac{1}{4\pi\varepsilon_0}\int_{S'}\frac{\rho_S(\vec{\mathrm{r}}')(\vec{\mathrm{r}}-\vec{\mathrm{r}}')}{|\vec{\mathrm{r}}-\vec{\mathrm{r}}'|^3}dS'$$

$$\vec{\mathrm{E}}(\vec{\mathrm{r}}) = \frac{1}{4\pi\varepsilon_0}\int_{v'}\frac{\rho_v(\vec{\mathrm{r}}')(\vec{\mathrm{r}}-\vec{\mathrm{r}}')}{|\vec{\mathrm{r}}-\vec{\mathrm{r}}'|^3}dv'$$

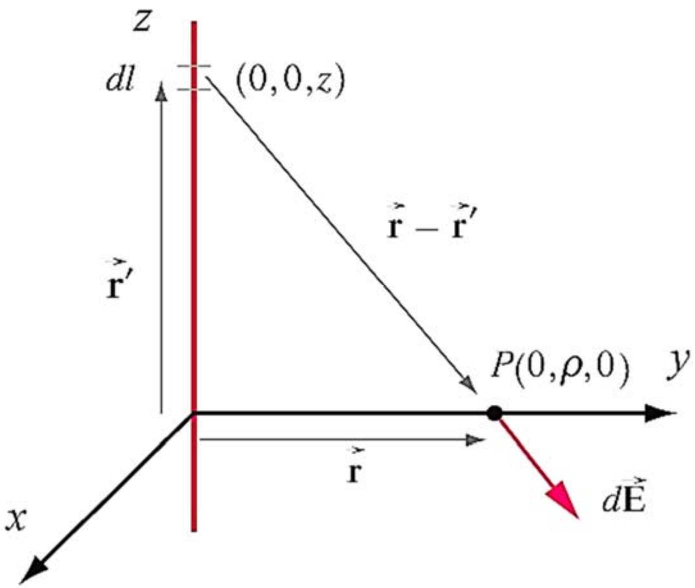
Example: Electric field of an infinite line charge



$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{z=-\infty}^{\infty} \frac{\rho_l \rho\, dz}{(\rho^2 + z^2)^{3/2}}\hat{\boldsymbol{\rho}} = \frac{\rho_l}{2\pi\varepsilon_0 \rho}\hat{\boldsymbol{\rho}} \ \text{(V/m)}$$

$$\boldsymbol{\rho} = (x_2, y_2) - (x_1, y_1) = (x_2 - x_1)\hat{\mathbf{x}} + (y_2 - y_1)\hat{\mathbf{y}}$$

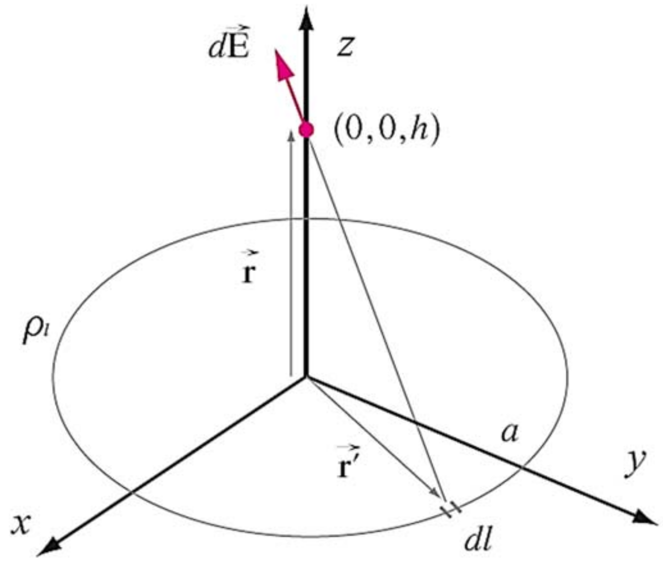
Example: Electric field of a finite line charge



$$d\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{\rho_\ell dz}{(\rho^2 + z^2)^{3/2}}(\rho\hat{\boldsymbol{\rho}} - z\hat{\mathbf{z}})$$

$$\mathbf{E} = \frac{\rho_\ell}{4\pi\varepsilon_0}\int_{z_1}^{z_2}\frac{\rho\hat{\boldsymbol{\rho}} - z\hat{\mathbf{z}}}{(\rho^2 + z^2)^{3/2}}dz = \frac{\rho_\ell}{4\pi\varepsilon_0}\left[\frac{z\hat{\boldsymbol{\rho}}}{\rho R} + \frac{\hat{\mathbf{z}}}{R}\right]_{z_1}^{z_2} = \frac{\rho_\ell}{4\pi\varepsilon_0\rho}\left[\left(\frac{z_2}{R_2} - \frac{z_1}{R_1}\right)\hat{\boldsymbol{\rho}} + \left(\frac{\rho}{R_2} - \frac{\rho}{R_1}\right)\hat{\mathbf{z}}\right]$$

Example: Electric field of a loop charge



$$d\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{\rho_\ell a d\varphi}{(h^2+a^2)^{3/2}}(h\hat{\mathbf{z}} - a\hat{\boldsymbol{\rho}}) = \frac{\rho_\ell a}{4\pi\varepsilon_0 R^3}(h\hat{\mathbf{z}} - a\cos\varphi\hat{\mathbf{x}} - a\sin\varphi\hat{\mathbf{y}})d\varphi$$

$$\mathbf{E} = \frac{\rho_\ell a}{4\pi\varepsilon_0 R^3}\int_{\varphi_1}^{\varphi_2}(h\hat{\mathbf{z}} - a\cos\varphi\hat{\mathbf{x}} - a\sin\varphi\hat{\mathbf{y}})d\varphi$$

$$= \frac{\rho_\ell a}{4\pi\varepsilon_0 R^3}\left[h(\varphi_2-\varphi_1)\hat{\mathbf{z}} - a(\sin\varphi_2 - \sin\varphi_1)\hat{\mathbf{x}} + a(\cos\varphi_2 - \cos\varphi_1)\hat{\mathbf{y}}\right]$$

$$R = \sqrt{a^2+h^2}$$

$$\mathbf{E} = \frac{\rho_\ell(2\pi a)h}{4\pi\varepsilon_0 R^3}\hat{\mathbf{z}} \quad \text{(full loop)}$$

Example: Electric field of a finite rectangular surface charge

Field point $\mathbf{r} = (0, 0, 0)$

Source point $\mathbf{r}' = (x, y, z)$, $x_1 \le x \le x_2, y_1 \le y \le y_2$

$\mathbf{R} = \mathbf{r} - \mathbf{r}' = (-x, -y, -z)$

$$d\mathbf{E} = \frac{\rho_s}{4\pi\varepsilon_0}\frac{\mathbf{R}dxdy}{R^3} = -\frac{1}{4\pi\varepsilon_0}\frac{x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}}{R^3}dxdy, \quad R = \sqrt{x^2 + y^2 + z^2}$$

$$\mathbf{E} = -\frac{\rho_s}{4\pi\varepsilon_0}\int_{x_1}^{x_2}\int_{y_1}^{y_2}\frac{x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}}{R^3}dxdy = \frac{\rho_s}{4\pi\varepsilon_0}\left[\left[\log(R+y)\hat{\mathbf{x}} + \log(R+x)\hat{\mathbf{y}} - \tan^{-1}\frac{xy}{zR}\hat{\mathbf{z}}\right]_{x_1}^{x_2}\right]_{y_1}^{y_2}$$

$$\mathbf{E} = -\frac{\rho_s}{2\varepsilon_0}\hat{\mathbf{z}} \quad \text{(infinite plate)}$$

Example: Surface ring charge

Field point $\mathbf{r} = (0, 0, h)$

Source point $\mathbf{r}' = (\rho, \varphi, 0)$, $\rho_1 \le \rho \le \rho_2, \varphi_1 \le \varphi \le \varphi_2$

$$\mathbf{R} = \mathbf{r} - \mathbf{r}' = h\hat{\mathbf{z}} - \rho\hat{\boldsymbol{\rho}}$$

$$d\mathbf{E} = \frac{\rho_s}{4\pi\varepsilon_0}\frac{\mathbf{R}\rho d\rho d\varphi}{R^3} = -\frac{1}{4\pi\varepsilon_0}\frac{x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}}{R^3}dxdy, \quad R = \sqrt{x^2 + y^2 + z^2} = \sqrt{\rho^2 + z^2}$$

$$\mathbf{E} = \frac{\rho_s}{4\pi\varepsilon_0}\int_0^{2\pi} d\varphi \int_{\rho_1}^{\rho_2} d\rho \frac{h\hat{\mathbf{z}} - \rho\hat{\boldsymbol{\rho}}}{R^3} = \frac{\rho_s h}{2\varepsilon_0}\hat{\mathbf{z}}\left[\frac{-1}{R}\right]_{\rho_1}^{\rho_2} = \frac{\rho_s h}{2\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\hat{\mathbf{z}}$$

$$R_1 = \sqrt{\rho_1^2 + h^2},\ R_2 = \sqrt{\rho_2^2 + h^2}$$

$$\mathbf{E} = \frac{\rho_s}{2\varepsilon_0}\frac{h}{|h|}\hat{\mathbf{z}} \quad \text{(infinite plate)}$$

Example 1:

Infinite line charge: charge density σ (C/m), location (x_s, y_s) m

Field point: $P(x, y)$ m

Find the electric field **E** at P.

(Solution)

- Formulas

$$\mathbf{E} = \frac{\sigma \boldsymbol{\rho}}{2\pi\varepsilon_0\rho^2} \;\;;\;\; \boldsymbol{\rho} = (x - x_s)\hat{\mathbf{x}} + (y - y_s)\hat{\mathbf{y}}$$

- Code

```
# EM-Lec4-Example 1: Single line charge
# Given a line charge at (xs,ys) m with charge density s (C/m)and
# the field point P(x,y) m, find the electric field E at P

from math import *
e0=8.853e-12;pi=3.141593
while True:
  s=float(input('Charge density s (C/m) ='))
  xs,ys=map(float,input('Line charge position xs,ys (m) =').split())
  x,y=map(float,input('Field point x,y (m) =').split())
```

6. Coding Examples

```
  rx=x-xs;ry=y-ys
  r=sqrt(rx**2+ry**2)
  ex=s/(2*pi*e0)*rx/r**2
  ey=s/(2*pi*e0)*ry/r**2

  print('  E (V/m) = (',ex,',',ey,')')

'''
Charge density s (C/m) =
-2e-9
Line charge position xs,ys (m) =
3 4
Field point x,y (m) =
1 2
  E (V/m) = ( 8.988756666810751 , 8.988756666810751 )
Charge density s (C/m) =
'''
```

Example 2:

1) 문제

n infinite line charge:

Charge 1: density σ_1 (C/m), location (x_1, y_1) m

...

Charge n: density σ_n (C/m), location (x_n, y_n) m

Field point: $P(x, y)$ m

Find the electric field $\mathbf{E}$ at P.

2) 수식

$$\mathbf{E} = \frac{1}{2\pi\varepsilon_0}\sum_{i=1}^{n}\frac{\sigma_i \boldsymbol{\rho}_i}{\rho_i^2},\quad \boldsymbol{\rho}_i = (x - x_i)\hat{\mathbf{x}} + (y - y_i)\hat{\mathbf{y}}$$

3) 코드

```
# EM-04-Python-Ex2: Multiple line charges
# Given line charges at (xi,yi) m with charge density si (C/m), i=1,2,...
# (선전류 개수는 코드실행시 결정)
# and the field point P(x,y) m, find the electric field E at P
```

```
from math import *
e0=8.853e-12;pi=3.141593
xs=[0]*100; ys=[0]*100; s=[0]*100
# xs, ys, s는 모두 갯수가 100인 list이며 0의 값으로 초기화
# xs[0] 부터 xs[99]까지 총 100개 list 변수 사용 가능
# 변수 뒤에 [ ]는 list라 하면 변경가능하며 데이터형도 모두 수용가능한 배열. 지금의 경우 ]
# 1차원 배열. 2차원 배열은 s[i][j]로 표현. List를 사용하려면 사용하기 전에 초기화 해야 함.
while True: # while loop 1
  i=0; ex=0; ey=0
  x,y=map(float,input('Field point x,y (m) =').split())
  while True: # while loop 2
    print('Line charge no.=',i)
    s[i]=float(input('Charge density s (C/m)(0 to finish defining line charges) ='))
    if s[i]==0:
      break # get out of while loop 2
    xs[i],ys[i]=map(float,input('Line charge position xs,ys (m) =').split())
    rx=x-xs[i];ry=y-ys[i]
    r=sqrt(rx**2+ry**2)
    ex=ex+s[i]/(2*pi*e0)*rx/r**2
    ey=ey+s[i]/(2*pi*e0)*ry/r**2
    i=i+1
  print('E (V/m) = (',ex,',',ey,')')
```

4) 코드 수행결과

```
Field point x,y (m) =
1 2
Line charge no.= 0
Charge density s (C/m)(0 to finish defining line charges) =
3e-9
Line charge position xs,ys (m) =
3 4
Line charge no.= 1
Charge density s (C/m)(0 to finish defining line charges) =
-1e-10
Line charge position xs,ys (m) =
4 6
Line charge no.= 2
Charge density s (C/m)(0 to finish defining line charges) =
0
E (V/m) = ( -13.267404840212667 , -13.19549478687818 )
Field point x,y (m) =** Process Stopped **

Press Enter to exit terminal
```

Example 3:

A point charges: charge q C, location (x_s, y_s, z_s) m

Field point: $P(x, y, z)$

Find the electric field **E** at P.

(Solution)

- Formulas

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{q\mathbf{R}}{R^3}, \quad \mathbf{R} = (x - x_s)\hat{\mathbf{x}} + (y - y_s)\hat{\mathbf{y}} + (z - z_s)\hat{\mathbf{z}}$$

- Codes

Example 4:

n point charges:

Charge 1: charge q_1 C, location (x_1, y_1, z_1) m

...

Charge n: density q_n C, location (x_n, y_n, z_n) m

Field point: $P(x, y, z)$

Find the electric field **E** at P.

(Solution)

- Formulas

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i\mathbf{R}_i}{R_i^3}, \quad \mathbf{R}_i = (x - x_i)\hat{\mathbf{x}} + (y - y_i)\hat{\mathbf{y}} + (z - z_i)\hat{\mathbf{z}}$$

- Codes

Fin

(End)