

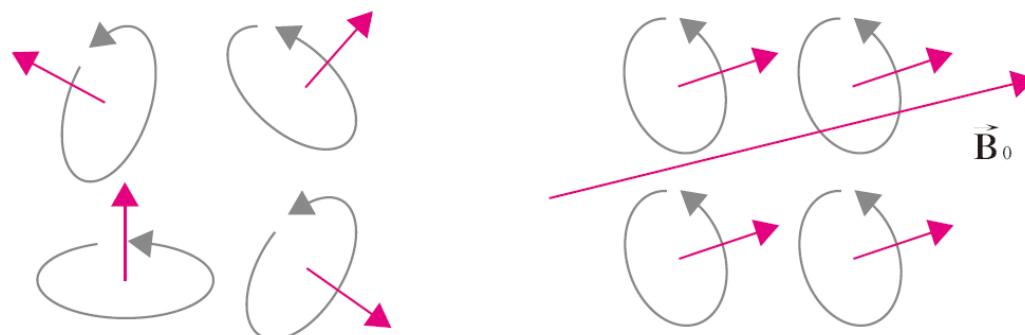
5.5 Magnetic Properties of Materials

1

자화와 투자율(Magnetization and Permeability)

❖ 자화(Magnetization)

- 외부자기장이 인가되면 원자의 자기모멘트가 정렬하여 자석이 되는 현상
- 자발자화: 강자성체가 외부자기장이 없어도 자기구역내에서 자기모멘트를 형성
- 자화원인
 - 전자의 궤도 공전
 - 전자, 원자핵의 회전(spin)
 - 각 원인에 의한 자기장이 동일방향 정렬



자화벡터(Magnetization Vector)

2

자기모멘트의 합을 체적으로 나눔 → 자기모멘트의 체적밀도

$$\vec{M} = \lim_{\Delta v \rightarrow 0} \frac{1}{\Delta v} \sum_{i=1}^{n_{\Delta v}} \vec{m}_i \text{ (A/m)}$$

- 단위가 \vec{H} 와 동일하므로 자기모멘트의 정렬에 의해서 생성된 \vec{H} 로 해석
- 진공매질은 자기모멘트가 없으므로 $\vec{M} = 0$

투자율(Permeability)

3

- 진공은 자화가 일어나지 않음 →

$$\vec{B} = \mu_0 \vec{H}$$

- 자화가 발생한 자성체에서는 →

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

$$\downarrow \quad \vec{M} = \chi_m \vec{H}$$

$$\vec{B} = \mu_0 (1 + \chi_m) \vec{H} = \mu_0 \mu_r \vec{H} = \mu \vec{H} \text{ (T)}$$

표 7-1 몇 가지 물질의 비투자율

매질	비투자율
알루미늄 은, 금	1
구리, 납, 아연	1
코발트	250
니켈	600
강철 (steel)	2,000
순철 (pure iron)	4,000~5,000
뮤합금 (Mumetal)	~100,000
정제철 (purified iron)	~200,000

투자율(permeability): $\mu = \mu_0 \mu_r$

비투자율(relative permeability): $\mu_r = 1 + \chi_m$

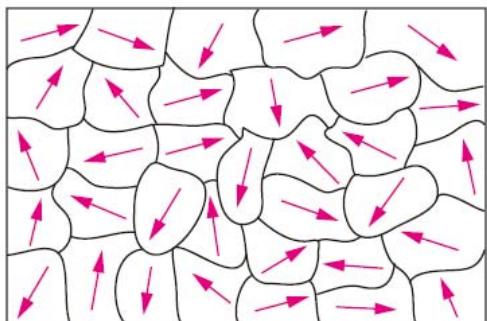
Table 5-2: Properties of magnetic materials.

	Diamagnetism	Paramagnetism	Ferromagnetism
Permanent magnetic dipole moment	No	Yes, but weak	Yes, and strong
Primary magnetization mechanism	Electron orbital magnetic moment	Electron spin magnetic moment	Magnetized domains
Direction of induced magnetic field (relative to external field)	Opposite	Same	Hysteresis (see Fig. 5-22)
Common substances	Bismuth, copper, diamond, gold, lead, mercury, silver, silicon	Aluminum, calcium, chromium, magnesium, niobium, platinum, tungsten	Iron, nickel, cobalt
Typical value of χ_m	$\approx -10^{-5}$	$\approx 10^{-5}$	$ \chi_m \gg 1$ and hysteretic
Typical value of μ_r	≈ 1	≈ 1	$ \mu_r \gg 1$ and hysteretic

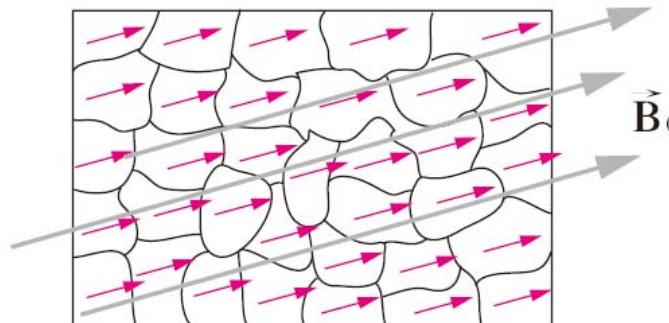
Thus, $\mu_r \approx 1$ or $\mu \approx \mu_0$ for diamagnetic and paramagnetic substances, which include dielectric materials and most metals. In contrast, $|\mu_r| \gg 1$ for ferromagnetic materials; $|\mu_r|$ of purified iron, for example, is on the order of 2×10^5 .

강자성체(Ferromagnetic Material)

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$$\vec{B}_0 = 0$$



1. 강한 전자스핀에 의한 강력한 영구 자기모멘트가 존재
2. 원자의 자기모멘트가 정렬 \rightarrow 자기구역
3. 큰 범위의 자기모멘트 평균은 0
4. 외부자기장이 가해지면 자기구역의 자기모멘트 정렬
5. 외부자기장보다 매우 큰 자기장 생성

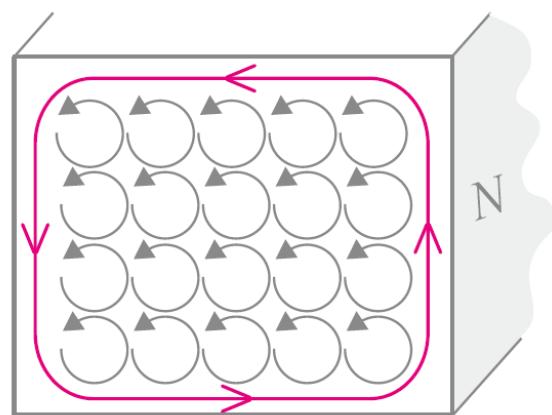
$$|\vec{m}_s| \gg |\vec{m}_o| \quad (\text{when } \vec{B}_0 = 0),$$

$$|\vec{B}| \gg |\vec{B}_0| \quad (\text{when } \vec{B}_0 \neq 0),$$

$$\mu_r \gg 1$$

영구자석(Permanent Magnet)

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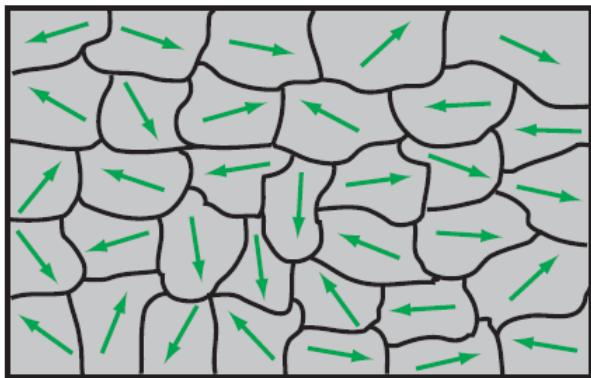


각 원자의 자기모멘트가 정렬 \rightarrow 전체 자기모멘트 $\neq 0$

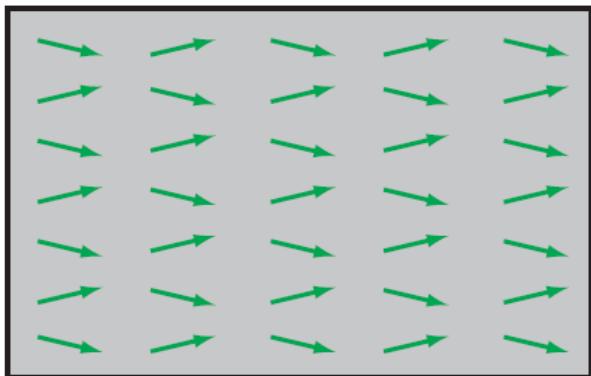
영구자석도 전류에 의해서 자기장을 발생

Magnetic Hysteresis

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(a) Unmagnetized domains



(b) Magnetized domains

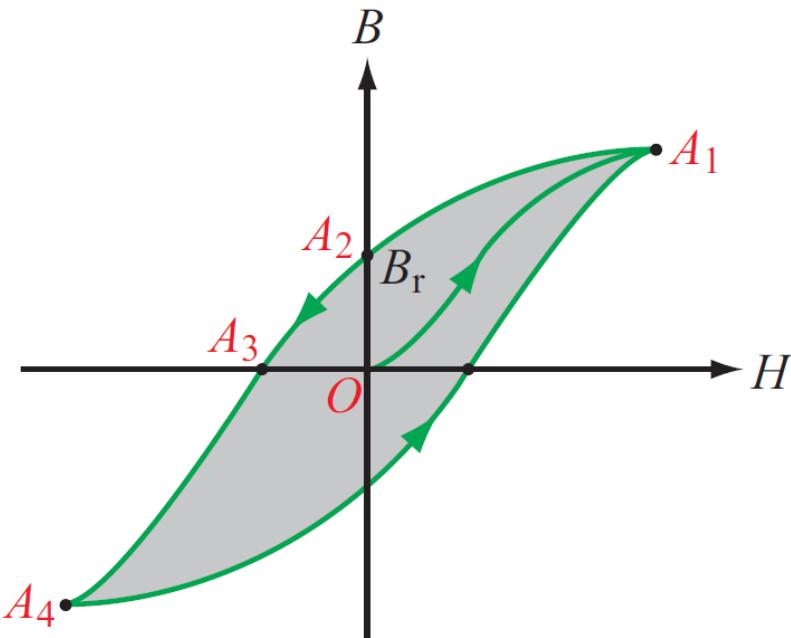
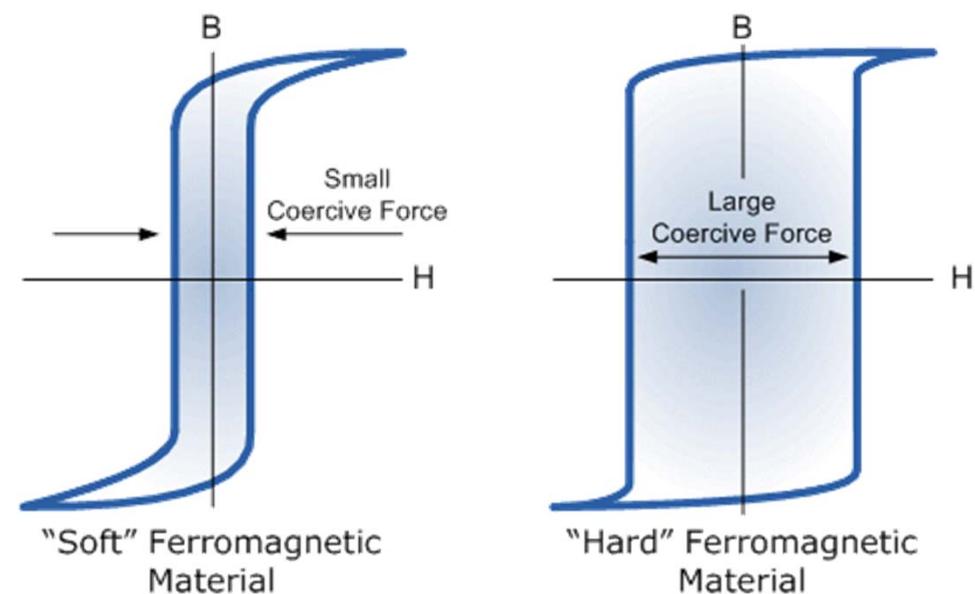
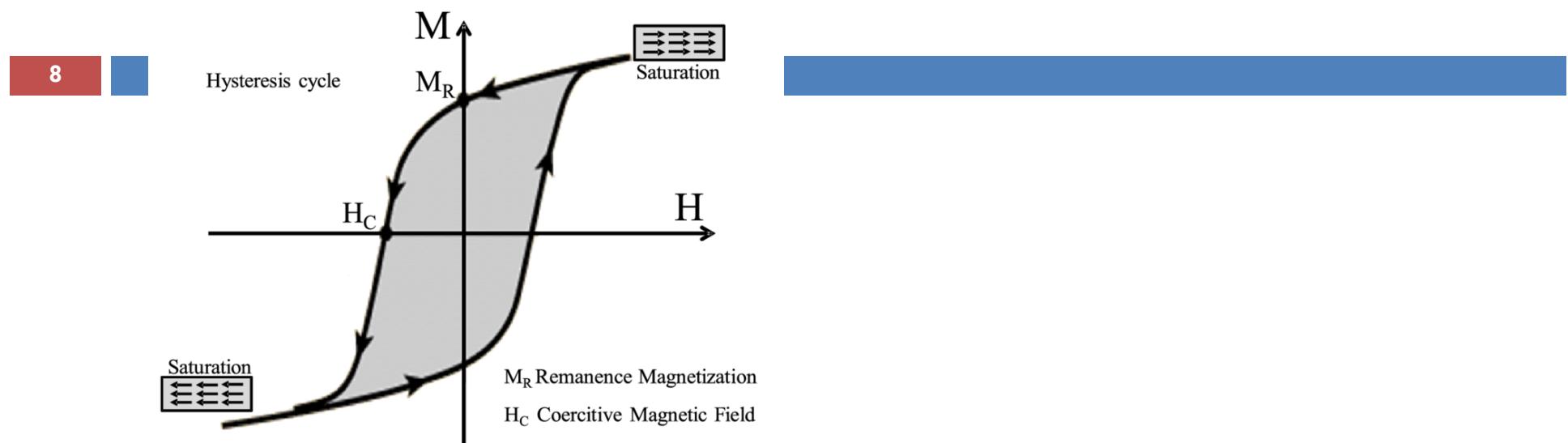


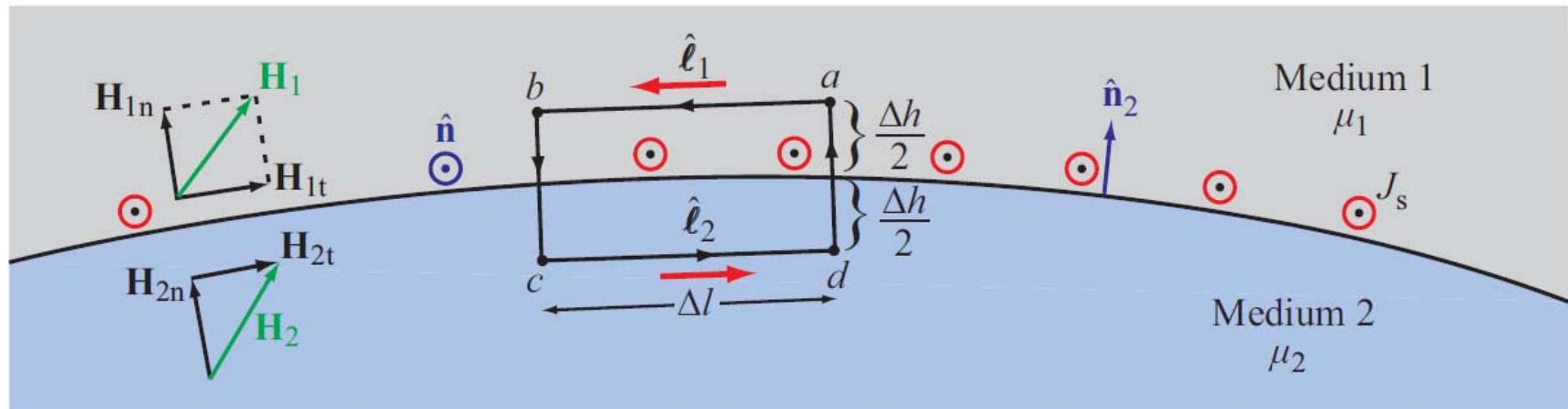
Figure 5-22: Typical hysteresis curve for a ferromagnetic material.

Magnetic Hysteresis Curve



5.6 Magnetic Boundary Conditions

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$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q \rightarrow D_{1n} - D_{2n} = \rho_s. \quad (5.78)$$

$$\hat{\mathbf{n}}_2 \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s.$$

By analogy, application of Gauss's law for magnetism, as expressed by Eq. (5.44), leads to the conclusion that

$$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0 \rightarrow B_{1n} = B_{2n}. \quad (5.79)$$

Surface currents can exist only on the surfaces of perfect conductors and superconductors. Hence, *at the interface between media with finite conductivities*, $\mathbf{J}_s = 0$ and

$$H_{1t} = H_{2t}. \quad (5.85)$$

Thus the normal component of \mathbf{B} is continuous across the boundary between two adjacent media.

5.6b 자기회로(The Magnetic Circuit)

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코일, 영구자석, 강자성체 철심코어, 공극 등으로 구성

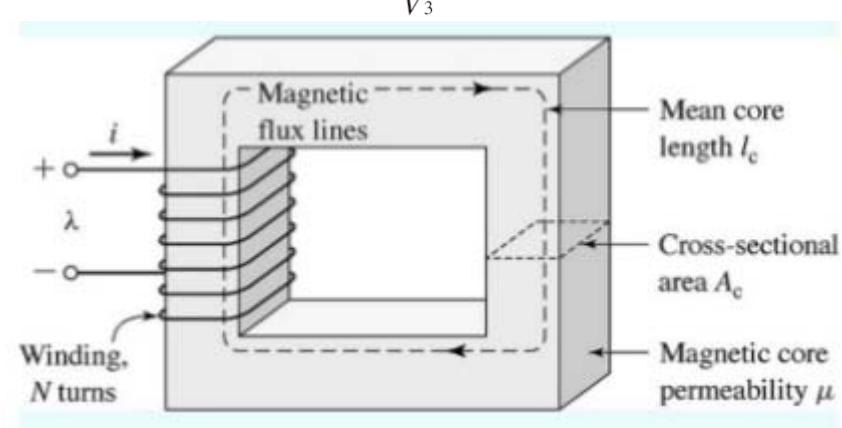
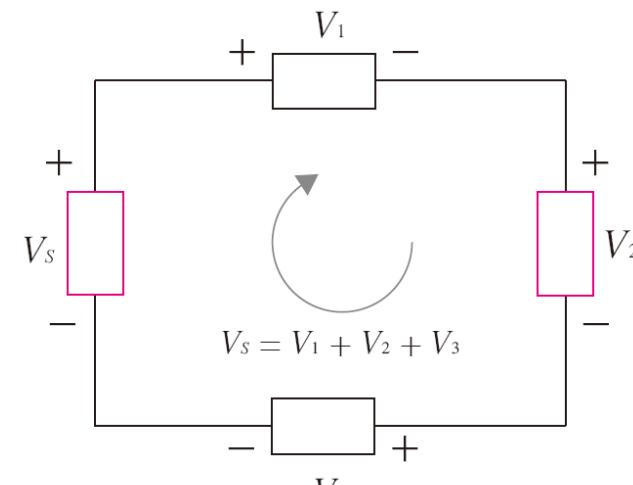
→ 회로 각 부분의 자속과 자계강도를 구하는 것이 자기회로 해석의 목표

전기회로의 해석: KVL을 이용

$$\sum_i V_i = 0: \text{키르히호프의 전압법칙(KVL)}$$

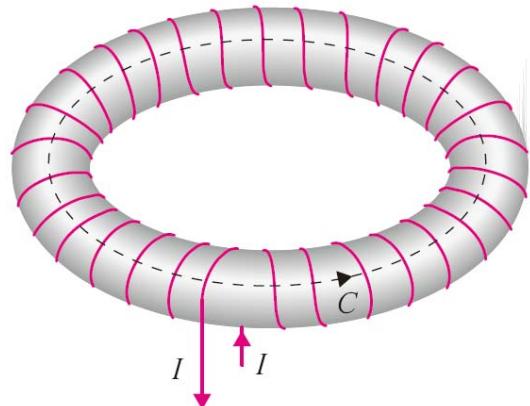
자기회로의 해석: 암페어의 법칙을 이용

$$\oint_C \vec{H} \cdot d\vec{l} = NI = V_m (\text{A} \cdot \text{t})$$



토로이드 자기회로의 해석

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$$\oint_C \vec{H} \cdot d\vec{l} = \frac{B}{\mu} l = NI = V_m$$

$$B = \frac{\mu NI}{l} = \frac{\mu V_m}{l}$$

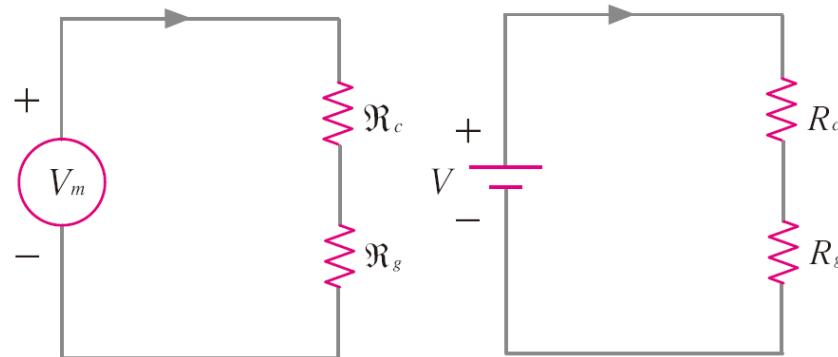
$$\downarrow \quad \Phi \simeq BS$$

$$\boxed{\Phi = \frac{\mu S}{l} V_m = \frac{V_m}{\mathfrak{R}}}$$



$$\boxed{\mathfrak{R} = \frac{l}{\mu S} (\text{A} \cdot \text{t}/\text{Wb})} : \text{자기저항(Reluctance)}$$

전기, 자기회로의 유사성



자기회로

전기회로

기자력

$$V_m = NI$$

기전력

$$V$$

자속

$$\Phi$$

전류

$$I$$

자기저항

$$\mathfrak{R}$$

저항

$$R$$

투자율

$$\mu$$

전도율

$$\sigma$$

토로이드내의 철심에 그림 E7-13과와 같이 공극(air gap)이 존재할 때,

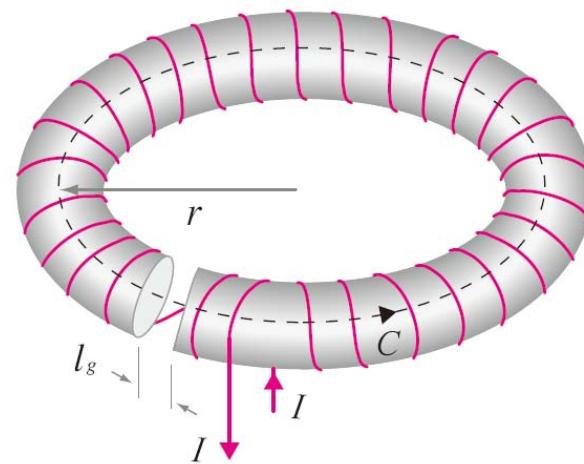
- a) 철심과 공극에서 자속밀도 \vec{B}_c, \vec{B}_g
- b) 철심과 공극의 자기저항 R_c, R_g 를 구하고,
- c) 철심과 공극에서의 자계강도의 크기를 비교하라.

a) $\vec{B}_c = B\hat{\phi}$ (철심내부의 자속밀도)

$$B_{1n} = B_{2n}$$

$$\vec{B}_g = \vec{B}_c = B\hat{\phi} \text{ (T)}$$

$$\vec{H}_c = \frac{B}{\mu}\hat{\phi}, \quad \vec{H}_g = \frac{B}{\mu_0}\hat{\phi}$$



$$\oint \vec{H} \cdot d\vec{l} = \int_c \vec{H}_c \cdot d\vec{l} + \int_g \vec{H}_g \cdot d\vec{l} = \frac{B}{\mu}(2\pi r - l_g) + \frac{B}{\mu_0}l_g = NI_0 \quad \rightarrow \quad B = \frac{\mu_0 \mu N I_0}{\mu_0(2\pi r - l_g) + \mu l_g}$$

$$\vec{B}_g = \vec{B}_c = B\hat{\phi} = \frac{\mu_0 \mu N I_0}{\mu_0(2\pi r - l_g) + \mu l_g} \hat{\phi} \text{ (T)} : \text{철심과 공극의 자속밀도}$$

b) 철심과 공극의 자기저항 \mathfrak{R}_c , \mathfrak{R}_g 를 구하고,

$$\Phi \simeq BS$$

$$\Phi = \frac{\mu_0 \mu N I_0 S}{\mu_0 (2\pi r - l_g) + \mu l_g}$$

$$= \frac{NI_0}{(2\pi r - l_g)/\mu S + l_g/\mu_0 S}$$

$$= \frac{V_m}{\mathfrak{R}_c + \mathfrak{R}_g} \quad \longrightarrow$$

$$\mathfrak{R}_c = \frac{2\pi r - l_g}{\mu S} = \frac{l_c}{\mu S} \text{ (A · t/Wb)}$$

$$\mathfrak{R}_g = \frac{l_g}{\mu_0 S} \text{ (A · t/Wb)}$$

c) 철심과 공극에서의 자계강도의 크기를 비교

$$H_g = \frac{\mu}{\mu_0} H_c = \mu_r H_c$$

공극의 자계강도는 철심내부에 비하여 μ_r 배 크다.

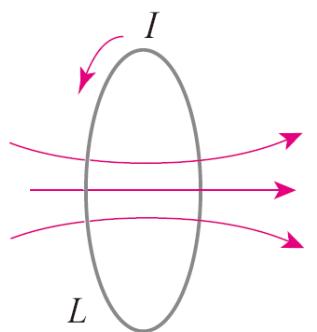
5.7 인덕턴스(Inductance)

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- 커패시터: 전기장 에너지 저장. 마주 보는 금속판 구조
- 인덕터: 자기장 에너지 저장. 금속도선 루프(다중권선) 구조

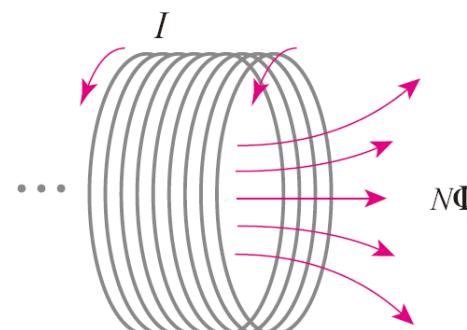
인덕터의 자체 인덕턴스 (Self inductance)

- 인덕터의 에너지 저장 효율
- 인덕턴스가 크면 적은 전류로 큰 자기에너지를 저장



$$L = \frac{\Phi}{I} \text{ (H)}$$

(루프의 인덕턴스)



$$L = \frac{N\Phi}{I} = \frac{\Lambda}{I} \text{ (H)}$$

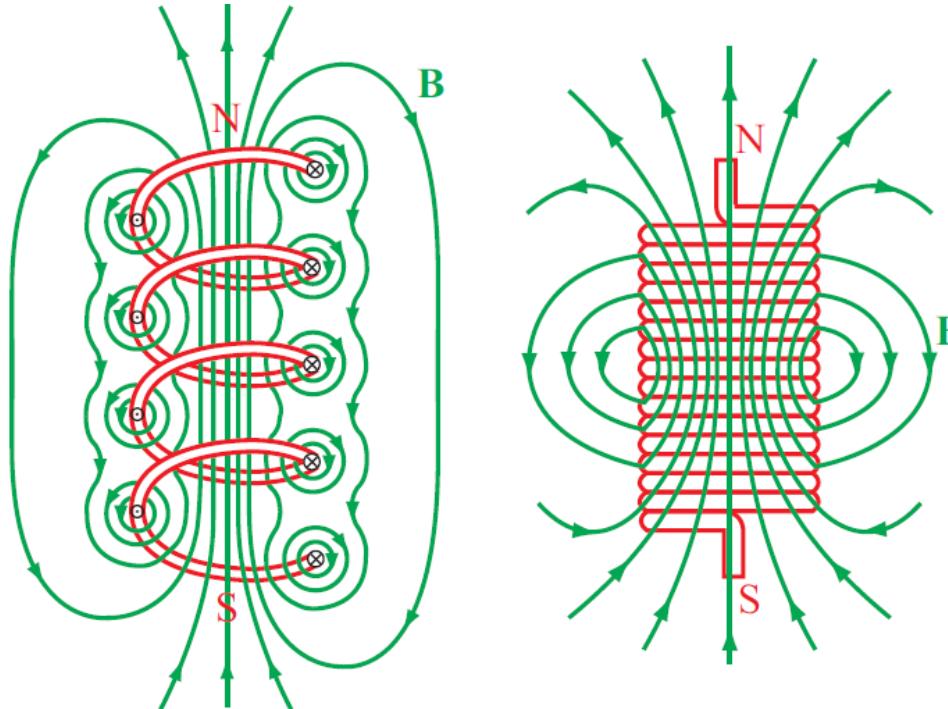
(N 턴 코일의 인덕턴스)

인덕턴스를 구하는 방법

- i) 암페어의 법칙이나 비오-사바르의 법칙으로 \vec{B} 를 구한다.
- ii) 자속 $\Phi = \int_S \vec{B} \cdot d\vec{S}$ 를 구한다.
- iii) 턴수 N 을 곱하여 자속쇄교수 $\Lambda = N\Phi$ 를 구한다.
- iv) $L = \frac{\Lambda}{I}$ 로 인덕턴스를 구한다.

Solenoid

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Inside the solenoid:

(a) Loosely wound
solenoid

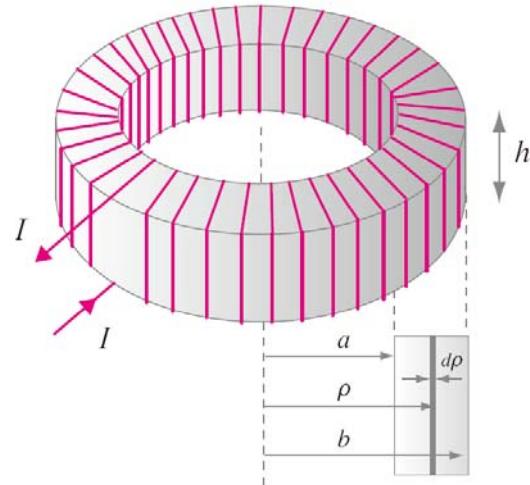
(b) Tightly wound
solenoid

$$\mathbf{B} \simeq \hat{\mathbf{z}}\mu n I = \frac{\hat{\mathbf{z}}\mu N I}{l} \quad (\text{long solenoid with } l/a \gg 1)$$

$$\Phi = BA = \frac{\mu NI}{l} A, \Lambda = N\Phi, L = \frac{\Lambda}{I} = \mu N^2 \frac{A}{l}$$

사각형 단면을 가진 강자성체 코어(투자율이 μ)에 코일을 N 번 감아 토로이드를 만들었다. 인덕턴스를 구하라.

$$\oint_C \vec{B} \cdot d\vec{l} = \int_0^{2\pi} B_\phi \hat{\phi} \cdot \rho d\phi \hat{\phi} = 2\pi \rho B_\phi = \mu NI$$



$$\rightarrow \vec{B} = \frac{\mu NI}{2\pi\rho} \hat{\phi} \text{ (T)}$$

$$\begin{aligned} \Phi &= \int_S \vec{B} \cdot d\vec{S} = \int_S \left(\frac{\mu NI}{2\pi\rho} \hat{\phi} \right) \cdot (hd\rho \hat{\phi}) = \frac{\mu NIh}{2\pi} \int_{\rho=a}^b \frac{1}{\rho} d\rho \\ &= \frac{\mu NIh}{2\pi} \ln\left(\frac{b}{a}\right) \text{ (Wb)} \end{aligned}$$

$$L = \frac{\Lambda}{I} = \frac{N\Phi}{I} = \frac{\mu N^2 h}{2\pi} \ln\left(\frac{b}{a}\right) \text{ (H)}$$

적분을 사용하지 않은 근사적 계산: Torroid내 자기장이 균일하다고 가정

$$B_B = \mu \frac{NI}{2\pi R_0} = \mu \frac{NI}{2\pi(a+b)/2}, A \approx h(b-a) \rightarrow \Phi = \mu \frac{NIh(b-a)}{\pi(b+a)} \rightarrow L = \frac{N\Phi}{I} = \mu N^2 \frac{h(b-a)}{\pi(b+a)}$$

Uniform cylindrical structure: capacitance-inductance relation

	COAX	TWO-WIRE	PARALLEL PLATE
L	$\frac{\mu}{2\pi} \ln \frac{b}{a}$	$\frac{\mu}{\pi} \cosh^{-1} \left(\frac{D}{2a} \right)$	$\frac{\mu d}{w}$
C	$\frac{2\pi\epsilon'}{\ln b/a}$	$\frac{\pi\epsilon'}{\cosh^{-1}(D/2a)}$	$\frac{\epsilon' w}{d}$
R	$\frac{R_s}{2\pi} \left(\frac{1}{a} + \frac{1}{b} \right)$	$\frac{R_s}{\pi a}$	$\frac{2R_s}{w}$
G	$\frac{2\pi\omega\epsilon''}{\ln b/a}$	$\frac{\pi\omega\epsilon''}{\cosh^{-1}(D/2a)}$	$\frac{\omega\epsilon'' w}{d}$

$$R_s = \sqrt{\pi f \mu / \sigma} \text{ (surface resistance)}$$

$$\sigma \leftrightarrow \omega \epsilon''$$

$$LC = \mu \epsilon$$

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$$\frac{G}{C} = \frac{\sigma}{\epsilon}$$

Example 5-7: Inductance of Coaxial Cable

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The magnetic field in the region S between the two conductors is approximately

$$\mathbf{B} = \hat{\phi} \frac{\mu I}{2\pi r}$$

Total magnetic flux through S :

$$\Phi = l \int_a^b B \, dr = l \int_a^b \frac{\mu I}{2\pi r} \, dr = \frac{\mu Il}{2\pi} \ln \left(\frac{b}{a} \right)$$

Inductance per unit length:

$$L' = \frac{L}{l} = \frac{\Phi}{lI} = \frac{\mu}{2\pi} \ln \left(\frac{b}{a} \right).$$

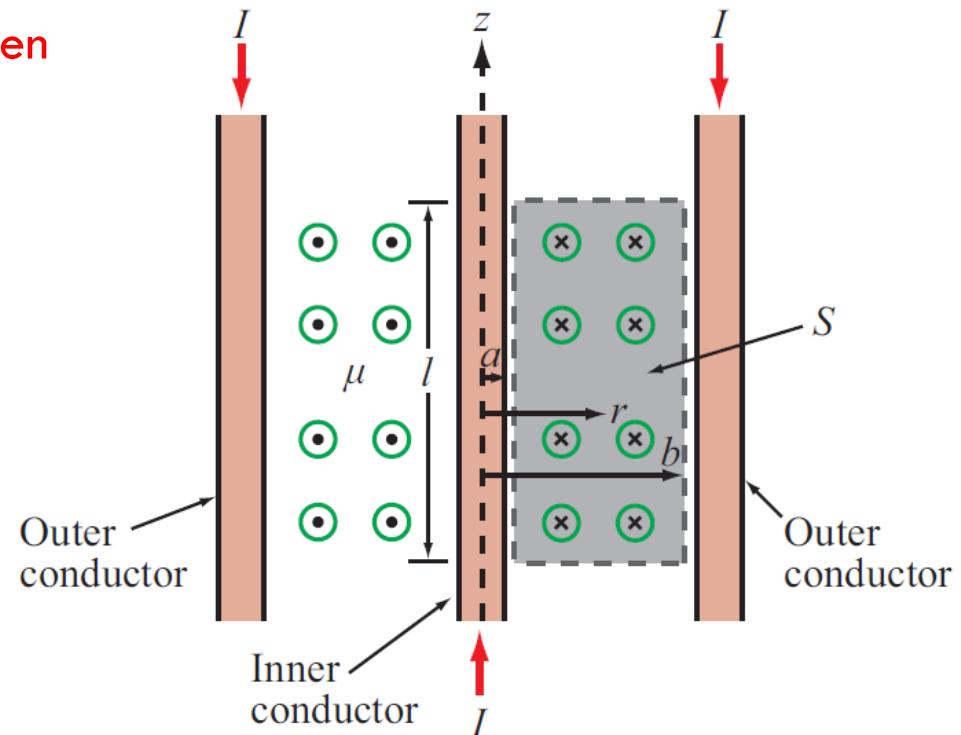


Figure 5-28: Cross-sectional view of coaxial transmission line (Example 5-7).

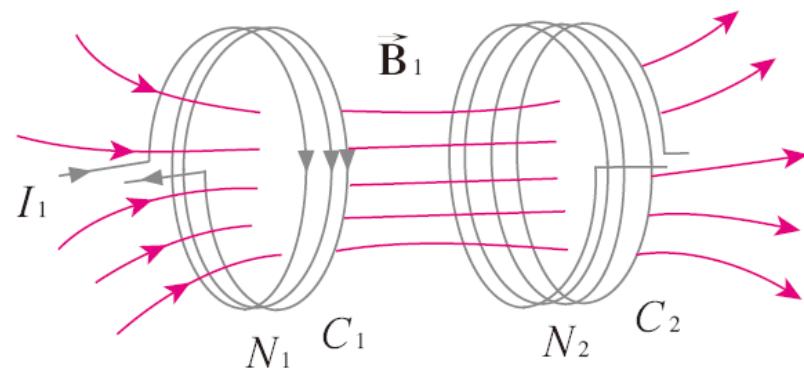
상호 인덕턴스 (mutual inductance)

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상호인덕턴스: 두 인터터 사이의 자기장 결합

C_1 에 의해서 발생하여 C_2 를 통과하는 자속

$$\Phi_{12} = \int_{S_2} \vec{B}_1 \cdot d\vec{S}$$

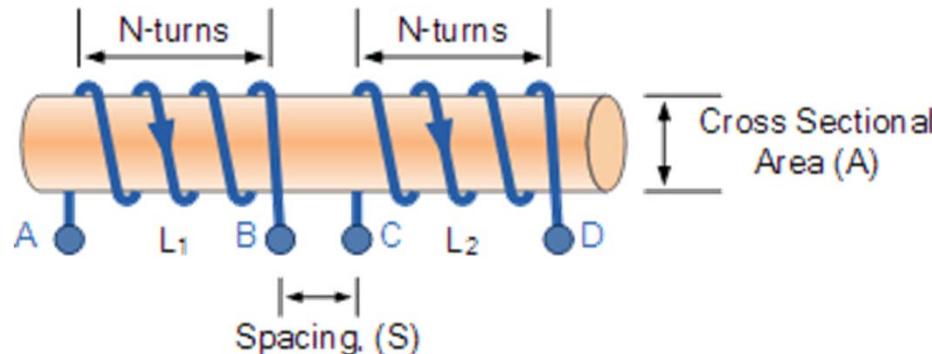


C_1 에 의한 C_2 의 상호인덕턴스

$$M_{12} = \frac{\Lambda_{12}}{I_1} = \frac{N_2 \Phi_{12}}{I_1} = \frac{N_2}{I_1} \int_{S_2} \vec{B}_1 \cdot d\vec{S}$$
 → $M_{12} = M_{21}$

상호 인덕턴스: 솔레노이드 인덕터

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$$M_{12} = \frac{N_2 \Phi_{12}}{l_1}$$

$$L_1 = \frac{\mu_0 \mu_r N_1^2 A}{\ell} \quad \text{and} \quad L_2 = \frac{\mu_0 \mu_r N_2^2 A}{\ell}$$

$$M^2 = L_1 L_2 \quad (\text{Perfect magnetic coupling; Ideal solenoid})$$

$$M = k \sqrt{L_1 L_2} \quad (\text{Imperfect magnetic coupling})$$

상호 인덕턴스: 무선충전 코일 결합계수

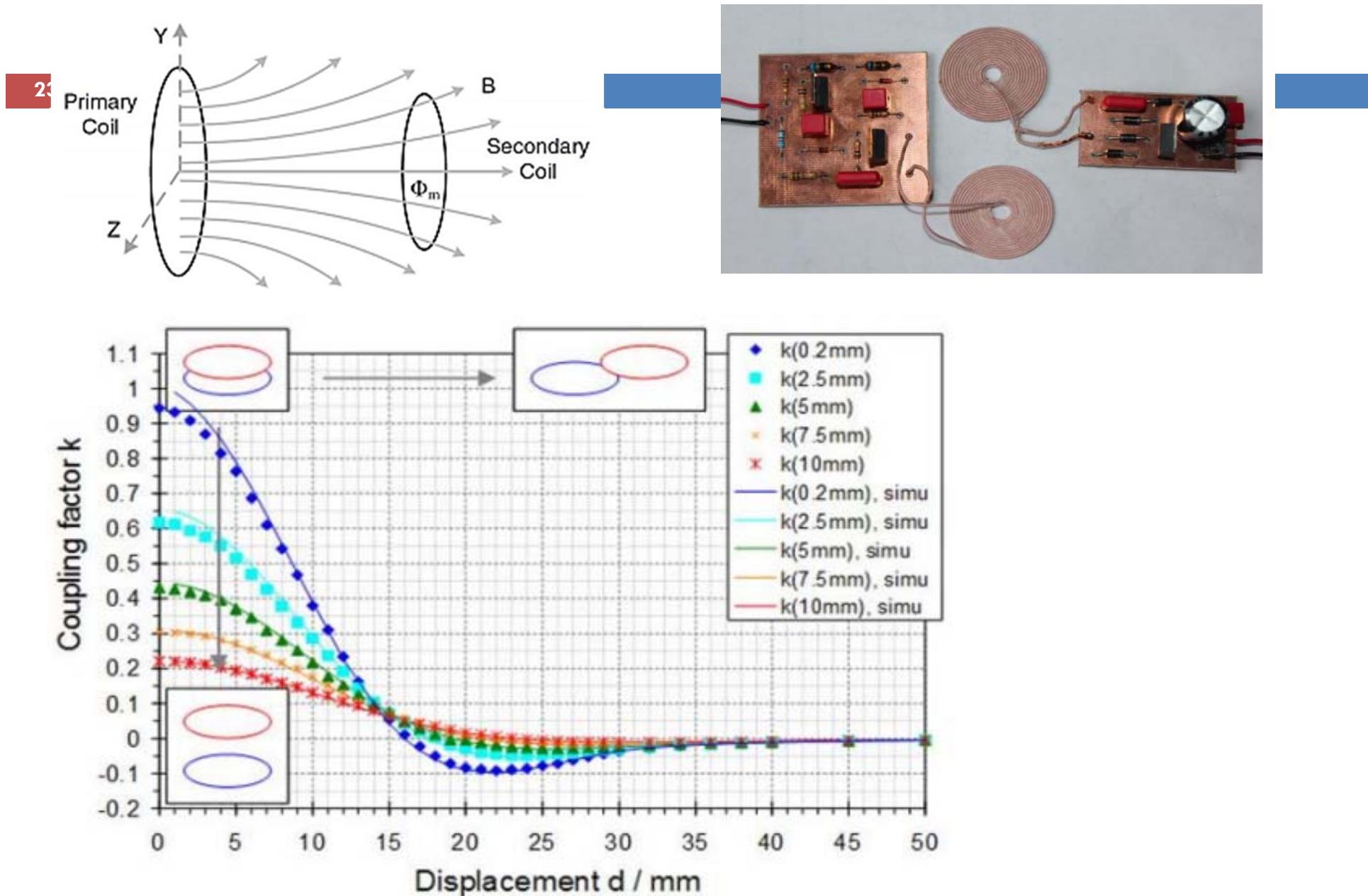
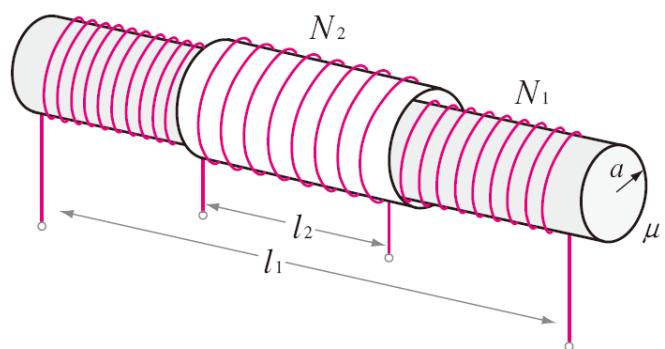


그림 E7-18과 같이 텐수가 N_1, N_2 인 코일이 반지름이 a 이고 투자율이 μ 인 원통형 코어에 감겨있다. 코일간의 상호인덕턴스를 구하라.



$$B = \mu nI = \mu \left(\frac{N_1}{l_1} \right) I_1 \text{ (T)}$$

$$\Phi_1 = BS = \mu \left(\frac{N_1}{l_1} \right) (\pi a^2) I_1 \text{ (Wb)}$$

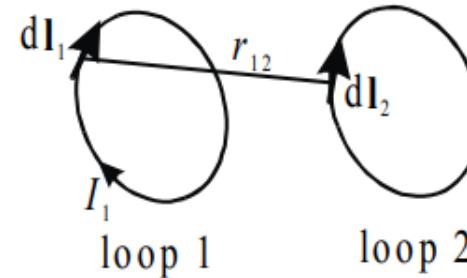
$$\Lambda_{12} = N_2 \Phi_1 = \mu \frac{N_1 N_2}{l_1} (\pi a^2) I_1$$

$$M_{12} = \frac{\Lambda_{12}}{I_1} = \mu \frac{N_1 N_2}{l_1} (\pi a^2)$$

$$M_{12} = M_{21}$$

$$M = \mu n_1 n_2 l_2 (\pi a^2) \text{ (H)}$$

Inductance Calculation; Neumann Formula



$$d\mathbf{A}_2 = \frac{\mu_0 I_1}{4\pi} \frac{d\mathbf{l}_1}{r_{12}}.$$

Then at this point the total vector potential is given by integrating the expression around the whole of loop 1:

$$\mathbf{A}_2 = \frac{\mu_0 I_1}{4\pi} \oint_{\text{loop } 1} \frac{d\mathbf{l}_1}{r_{12}}.$$

The magnetic flux threading loop 2 is then found by taking the line integral of \mathbf{A}_2 around loop 2

$$\Phi_2 = \int_{\text{loop } 2} \mathbf{A}_2 \cdot d\mathbf{l}_2$$

so in this case

$$\Phi_2 = \frac{\mu_0 I_1}{4\pi} \oint_{\text{loop } 2} \oint_{\text{loop } 1} \frac{d\mathbf{l}_1 \cdot d\mathbf{l}_2}{r_{12}}.$$

This gives a general expression for calculating the mutual inductance

$$M_{21} = \frac{\Phi_2}{I_1} = \frac{\mu_0}{4\pi} \oint_{\text{loop } 2} \oint_{\text{loop } 1} \frac{d\mathbf{l}_1 \cdot d\mathbf{l}_2}{r_{12}} \quad (9.14)$$

but this is seen to be symmetric in the indices 1 and 2; the flux in loop 2 due to a current in loop 1 is equal to the flux in loop 1 due to the same current in loop 2. And thus we conclude that

$$M_{12} = M_{21}. \quad (9.15)$$

Tech Brief 11: Inductive Sensors

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LVDT can measure displacement with submillimeter precision

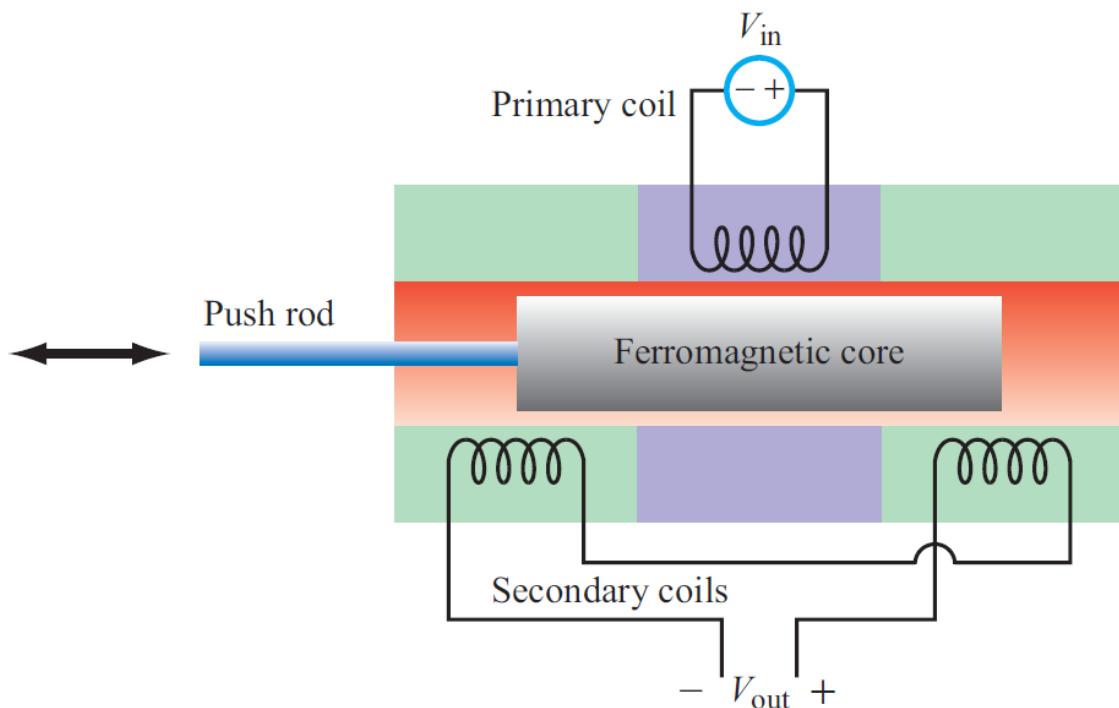


Figure TF11-1: Linear variable differential transformer (LVDT) circuit.

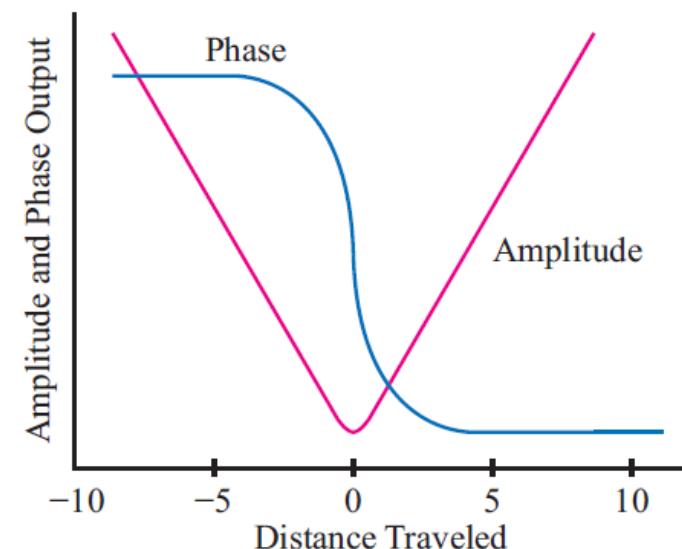


Figure TF11-2: Amplitude and phase responses as a function of the distance by which the magnetic core is moved away from the center position.

Proximity Sensor

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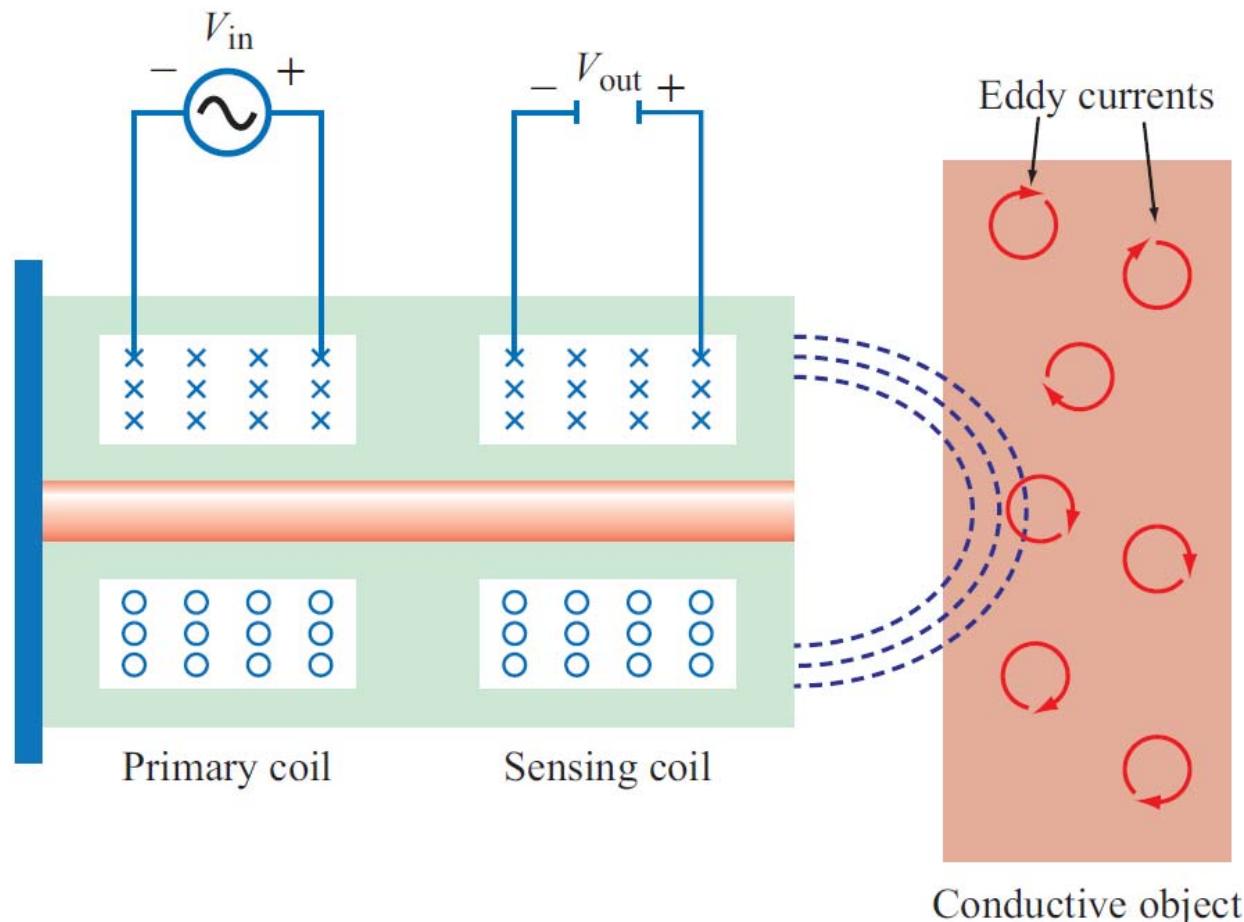


Figure TF11-5: Eddy-current proximity sensor.

5.8 Magnetic Energy Density

인덕터에 저장되는 에너지: 코일에 / 의 전류를 흘리는데 소요되는 에너지

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$$W = \int pdt = \int vidt = L \int_0^I idi = \frac{1}{2} LI^2 \text{ (J)}$$

$v = Ldi/dt$

: 코일내부에 자기장의 형태로 저장

솔레노이드 인덕터를 사용하여 에너지를 자기장을 사용하여 표현:

$$\begin{aligned} W_m &= \frac{1}{2} LI^2 \\ &= \frac{1}{2} \left(\mu \frac{N^2}{l} S \right) \left(\frac{Bl}{\mu N} \right)^2 \\ &= \frac{1}{2} \frac{B^2}{\mu} (lS) = \frac{1}{2} \mu H^2 v, \quad \rightarrow \end{aligned}$$

$$w_m = \frac{W_m}{v} = \frac{1}{2} \mu H^2 \quad (\text{J/m}^3).$$

(자기장 에너지 밀도)

$$\begin{aligned} W_m &= \frac{1}{2} \int_v \vec{H} \cdot \vec{B} dv \\ &= \frac{1}{2} \int_v \frac{B^2}{\mu} dv \\ &= \frac{1}{2} \int_v \mu H^2 dv \text{ (J)} \end{aligned}$$

Example 5-8: Magnetic Energy in a Coaxial Cable

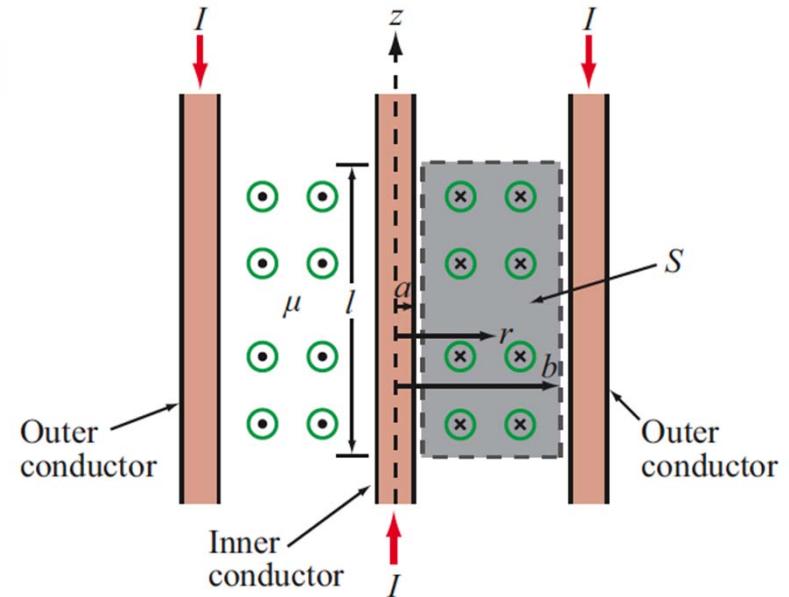
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Magnetic field in the insulating material is

$$H = \frac{B}{\mu} = \frac{I}{2\pi r}$$

The magnetic energy stored in the coaxial cable is

$$W_m = \frac{1}{2} \int_V \mu H^2 dV = \frac{\mu I^2}{8\pi^2} \int_V \frac{1}{r^2} dV$$



$$\begin{aligned} W_m &= \frac{\mu I^2}{8\pi^2} \int_a^b \frac{1}{r^2} \cdot 2\pi r l dr \\ &= \frac{\mu I^2 l}{4\pi} \ln\left(\frac{b}{a}\right) \\ &= \frac{1}{2} L I^2 \quad (\text{J}), \end{aligned}$$

Summary

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Chapter 5 Relationships

Maxwell's Magnetostatics Equations

Gauss's Law for Magnetism

$$\nabla \cdot \mathbf{B} = 0 \quad \leftrightarrow \quad \oint_S \mathbf{B} \cdot d\mathbf{s} = 0$$

Ampère's Law

$$\nabla \times \mathbf{H} = \mathbf{J} \quad \leftrightarrow \quad \oint_C \mathbf{H} \cdot d\ell = I$$

Lorentz Force on Charge q

$$\mathbf{F} = q(\mathbf{E} + \mathbf{u} \times \mathbf{B})$$

Magnetic Force on Wire

$$\mathbf{F}_m = I \oint_C d\mathbf{l} \times \mathbf{B} \quad (\text{N})$$

Magnetic Torque on Loop

$$\mathbf{T} = \mathbf{m} \times \mathbf{B} \quad (\text{N}\cdot\text{m})$$

$$\mathbf{m} = \hat{\mathbf{n}} NIA \quad (\text{A}\cdot\text{m}^2)$$

Biot-Savart Law

$$\mathbf{H} = \frac{I}{4\pi} \int_l \frac{d\mathbf{l} \times \hat{\mathbf{R}}}{R^2} \quad (\text{A/m})$$

Magnetic Field

Infinitely Long Wire $\mathbf{B} = \hat{\phi} \frac{\mu_0 I}{2\pi r} \quad (\text{Wb/m}^2)$

Circular Loop $\mathbf{H} = \hat{\mathbf{z}} \frac{Ia^2}{2(a^2 + z^2)^{3/2}} \quad (\text{A/m})$

Solenoid $\mathbf{B} \simeq \hat{\mathbf{z}} \mu n I = \frac{\hat{\mathbf{z}} \mu NI}{l} \quad (\text{Wb/m}^2)$

Vector Magnetic Potential

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (\text{Wb/m}^2)$$

Vector Poisson's Equation

$$\nabla^2 \mathbf{A} = -\mu \mathbf{J}$$

Inductance

$$L = \frac{\Lambda}{I} = \frac{\Phi}{I} = \frac{1}{I} \int_S \mathbf{B} \cdot d\mathbf{s} \quad (\text{H})$$

Magnetic Energy Density

$$w_m = \frac{1}{2} \mu H^2 \quad (\text{J/m}^3)$$

Coding Example

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Torrodal inductor's inductance calculation

$$L = \frac{N\Phi}{I} = \mu N^2 \frac{h(b-a)}{\pi(b+a)}$$

Input: ur(relative permeability), N, h, b, a
Output: L of a toroidal inductor