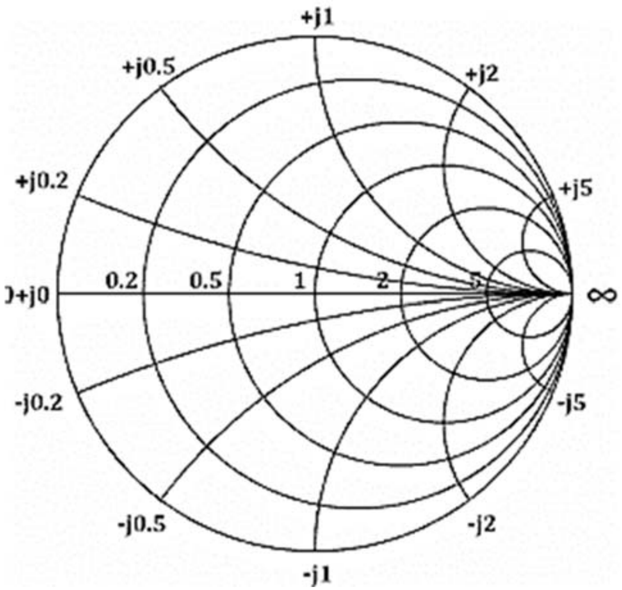
Lecture 4 Smath Chart

1. What is the Smith Chart?
2. Derivation of the Smith Chart Equations
3. The Real Smith Chart
4. Getting Familiar with the Smith Chart
5. Admittance Smith Chart − Y Chart
6. ZY Chart
7. Smith Chart Examples

1. What is the Smith Chart?

- Zin vs Γ calculation diagram



Smith chart

ARINST VR 23-6200 ($510)

- Invention

- Philip Smith, 1939, Electronics magazine

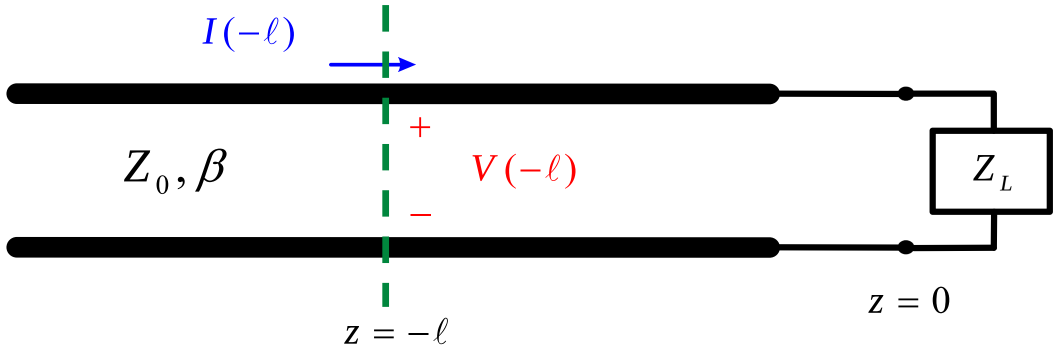
- Mizuhashi Tosaku, 1936, Journal of Electrical Communications Engineers of Japan

Applications

- Impedance matching: $5 - j\,210\ \ \Omega \rightarrow\ 50 + j\,0\ \ \Omega$

- Impedance calculation

Zin vs Γ



$$V(-\ell) = V_0^+ e^{-j\beta\ell}[1+\Gamma(-\ell)]$$

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{-j\beta\ell}[1+\Gamma(-\ell)]$$

$$\boxed{Z_{\text{in}} = Z = Z_0 \frac{1+\Gamma(z)}{1-\Gamma(z)}} \text{ (input impedance)}$$

$$\boxed{\Gamma = \frac{Z - Z_0}{Z + Z_0}} \text{ (reflection coefficient)}$$

$$\Gamma(-\ell) = \Gamma_L e^{-j2\beta\ell}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

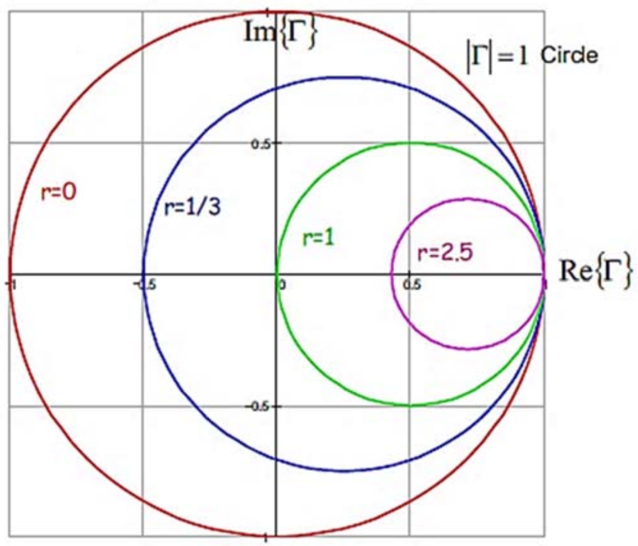
2. Derivation of the Smith Chart Equations

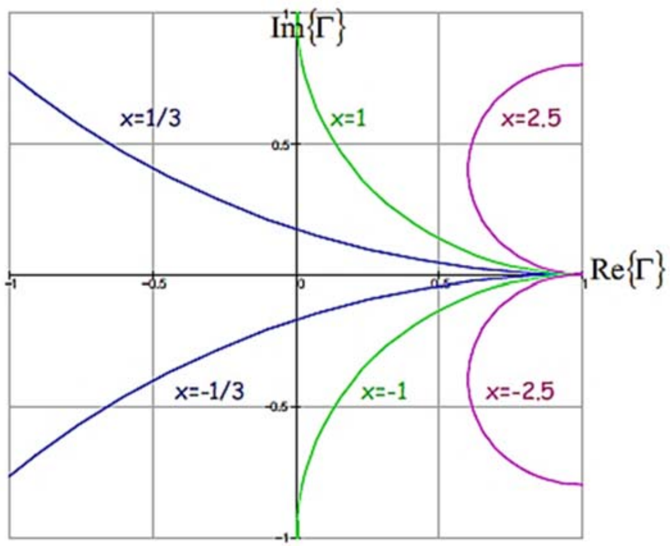
$$z = \frac{Z}{Z_0} = r + jx = \frac{1+\Gamma}{1-\Gamma} = \frac{1+u+jv}{1-(u+jv)} \quad \text{(normalized impedance)}$$

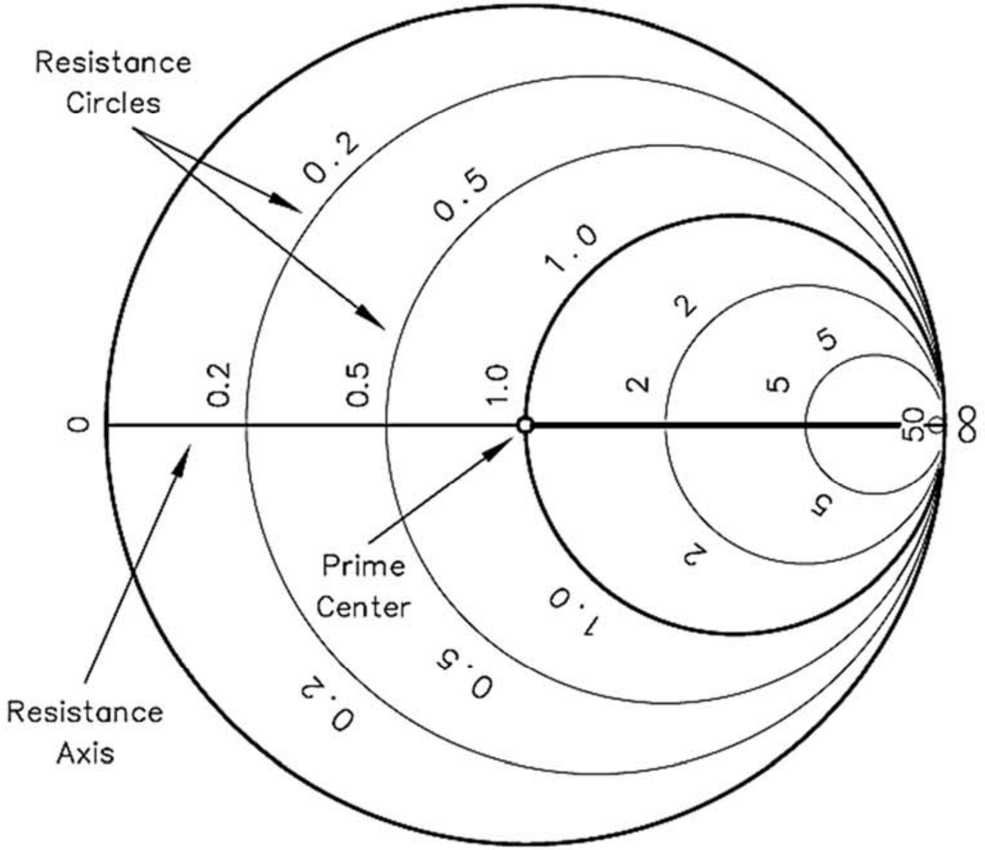
$$\Gamma = u + jv = \rho e^{j\phi} \quad \text{(reflection coefficient)}$$

$$r = \frac{1-u^2-v^2}{(1-u)^2+v^2} \rightarrow \left(u - \frac{r}{1+r}\right)^2 + v^2 = \left(\frac{1}{1+r}\right)^2 \quad \text{(circles of constant } r\text{)}$$

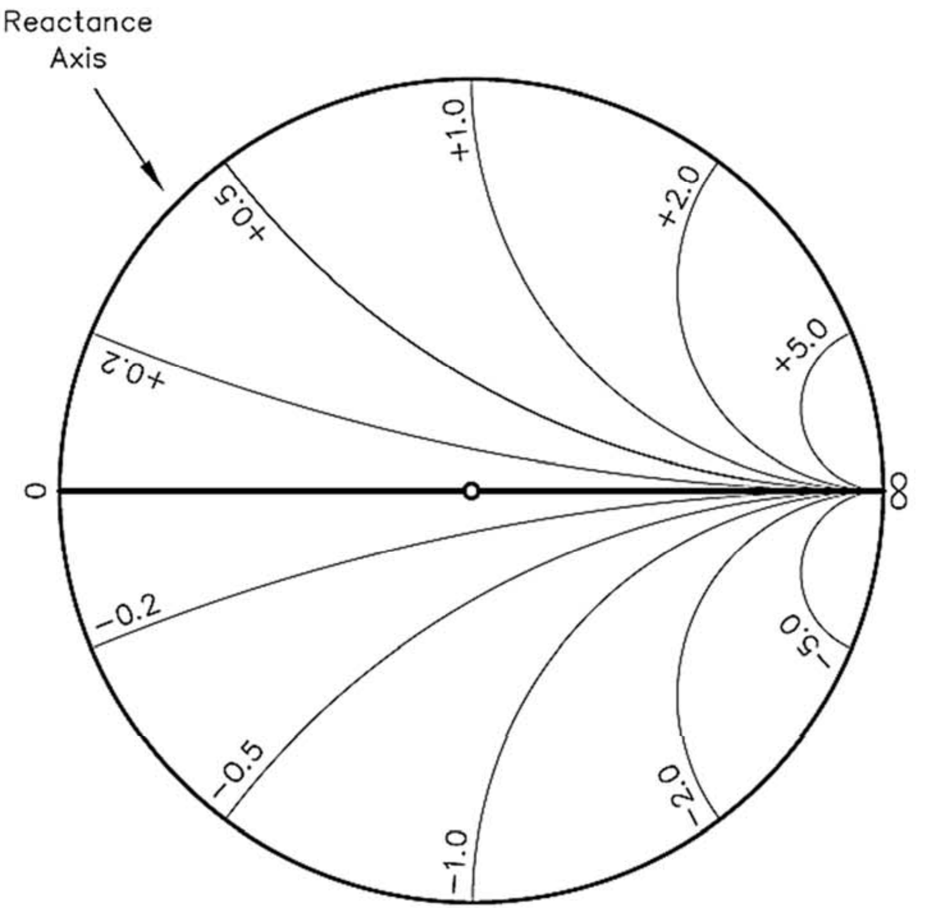
$$x = \frac{2v}{(1-u)^2+v^2} \rightarrow (u-1)^2 + \left(v - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2 \quad \text{(circles of constant } r\text{)}$$



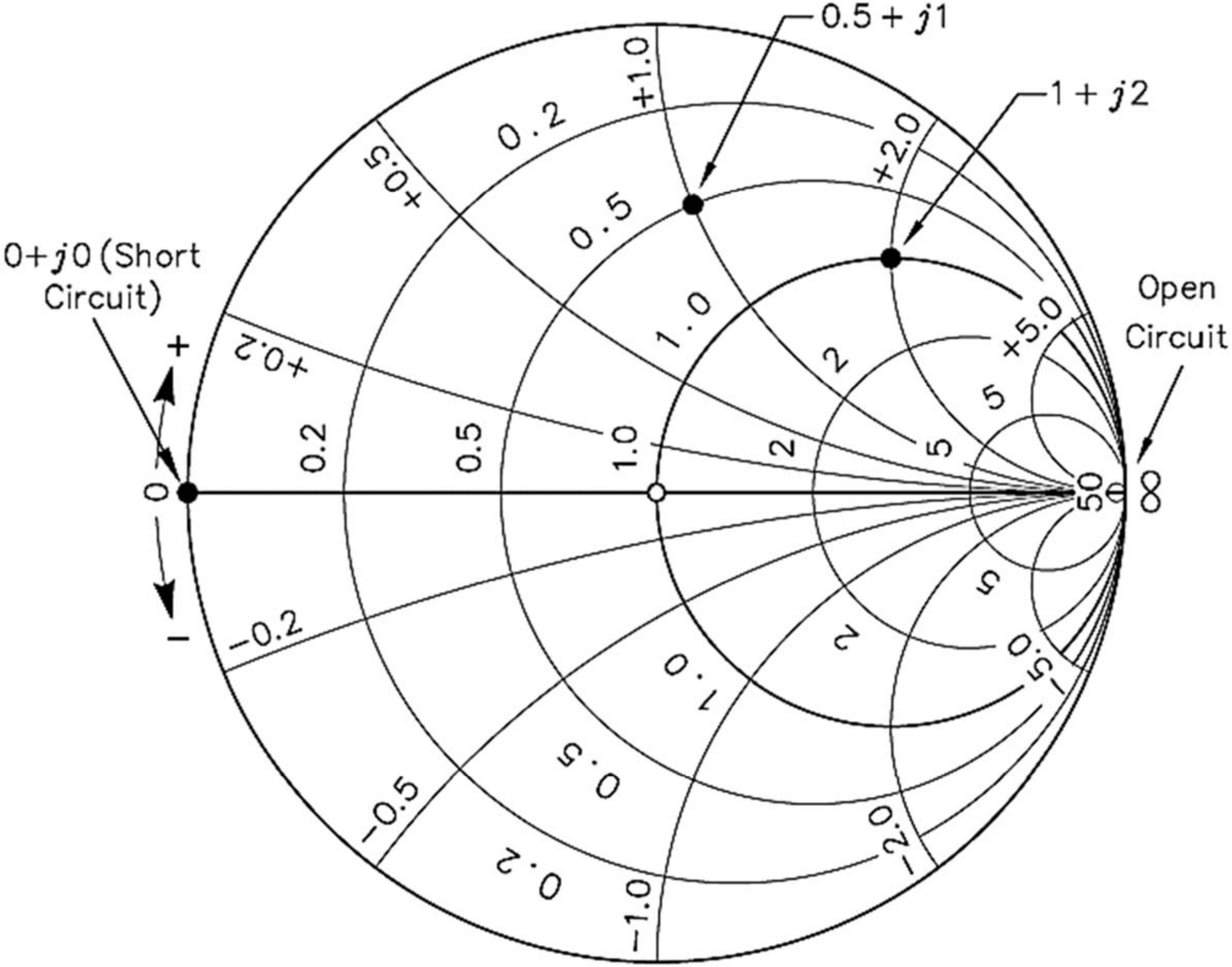




Resistance Circles
0.2
0.5
1.0
2
5
0
50
∞
Prime Center
Resistance Axis

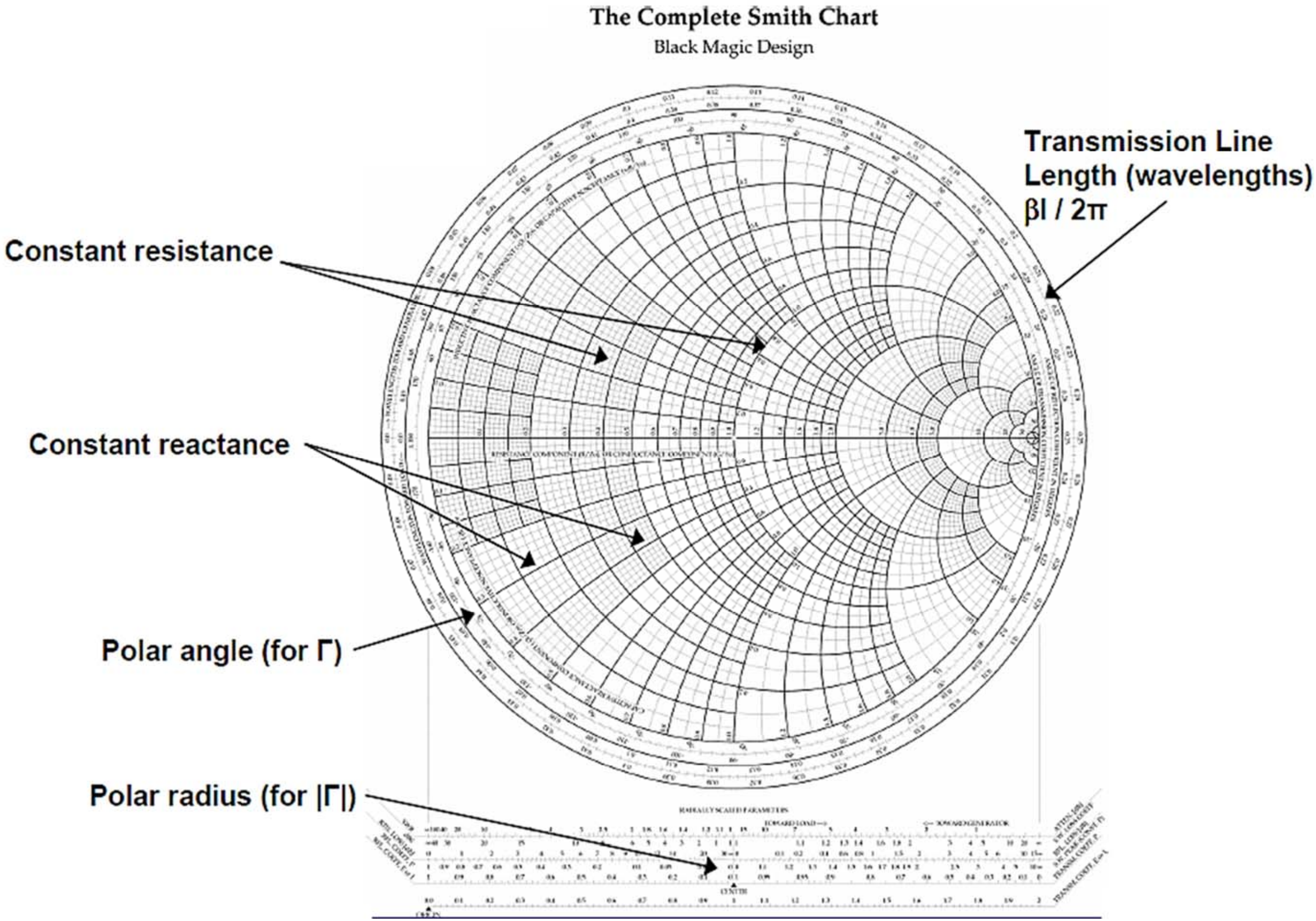


Reactance Axis
+0.2
+0.5
+1.0
+2.0
+5.0
0
∞
−0.2
−0.5
−1.0
−2.0
−5.0

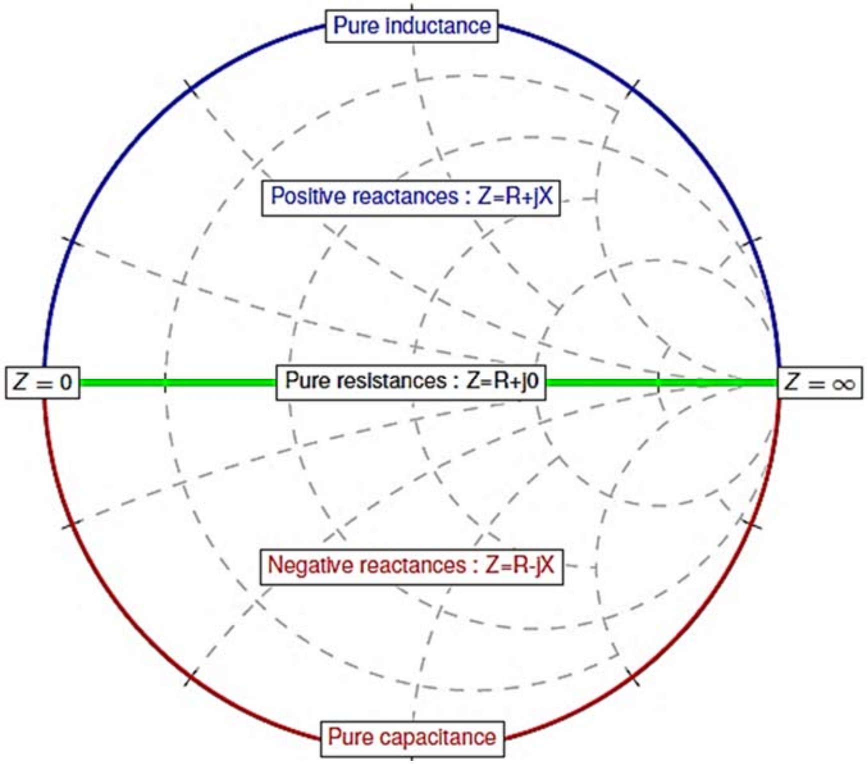


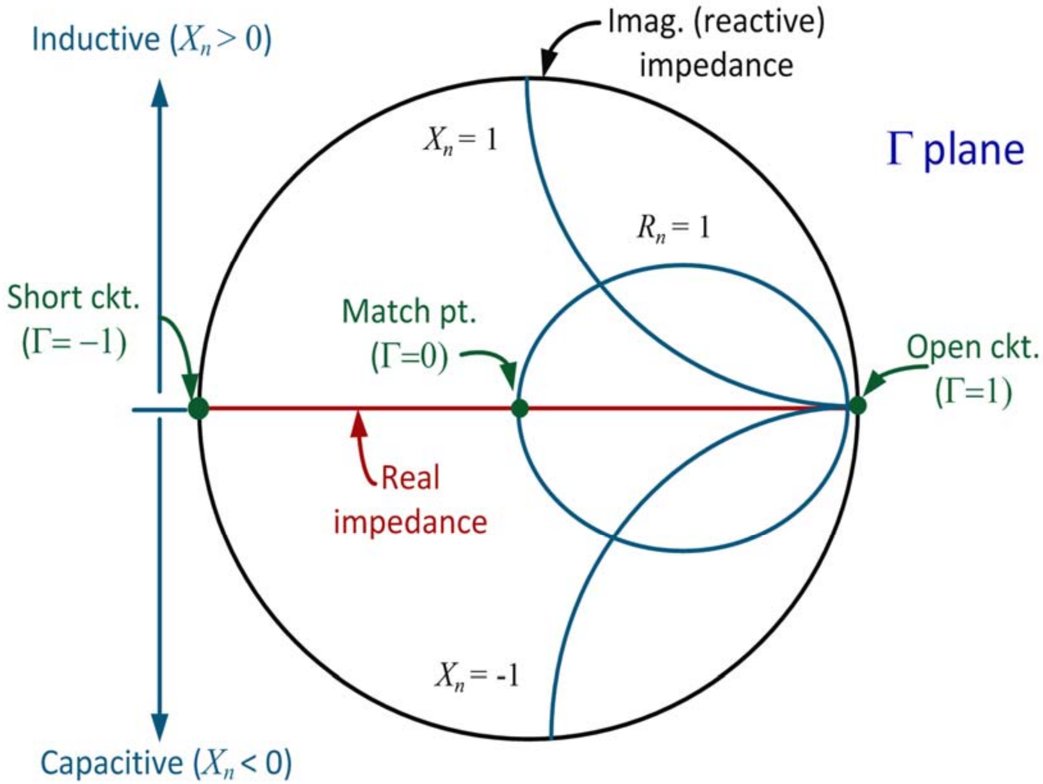
0.5 + j1
1 + j2
0+j0 (Short
Circuit)
Open
Circuit
+1.0
+2.0
+0.5
+0.2
+5.0
0.2
0.5
1.0
2
5
50
∞
−0.2
−0.5
−1.0
−2.0

3. The Real Smith Chart



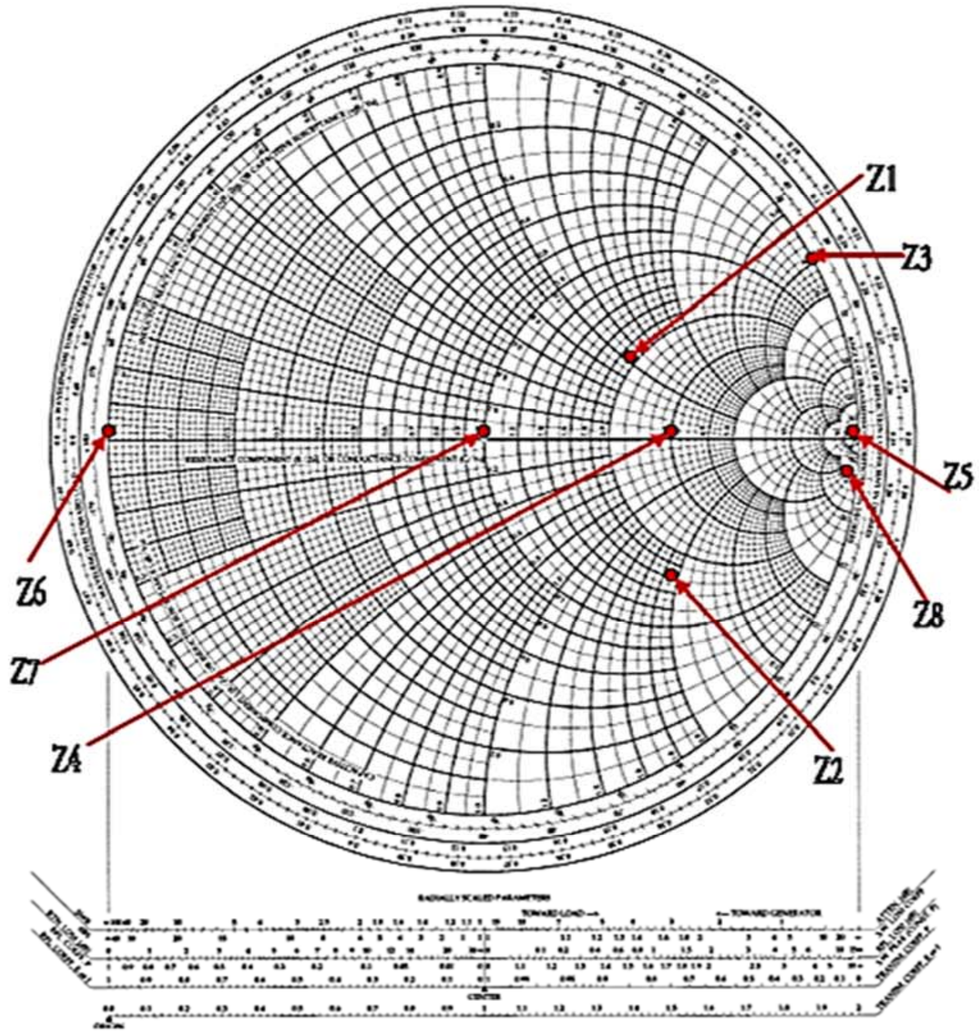
4. Getting Familiar with the Smith Chart

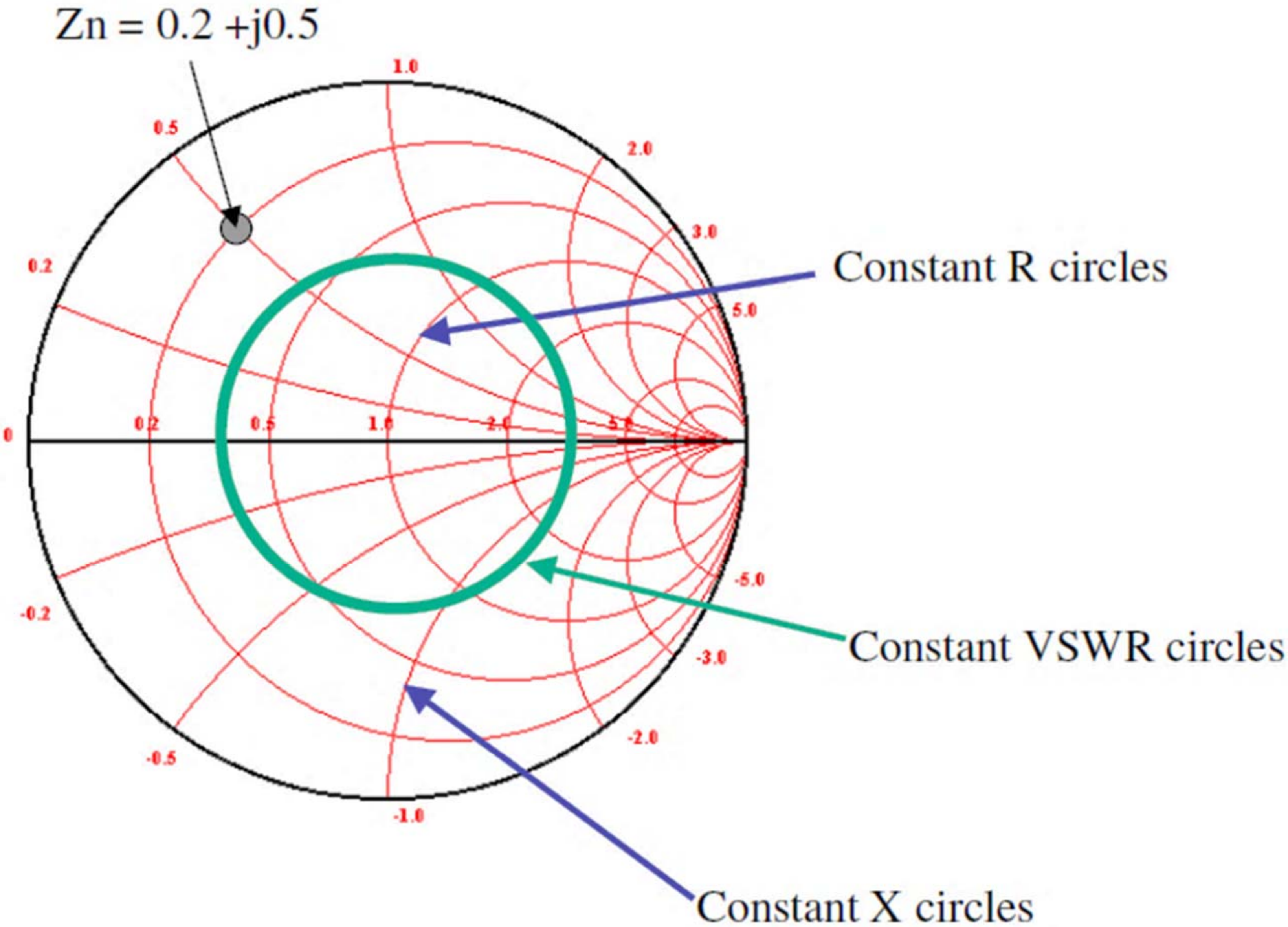




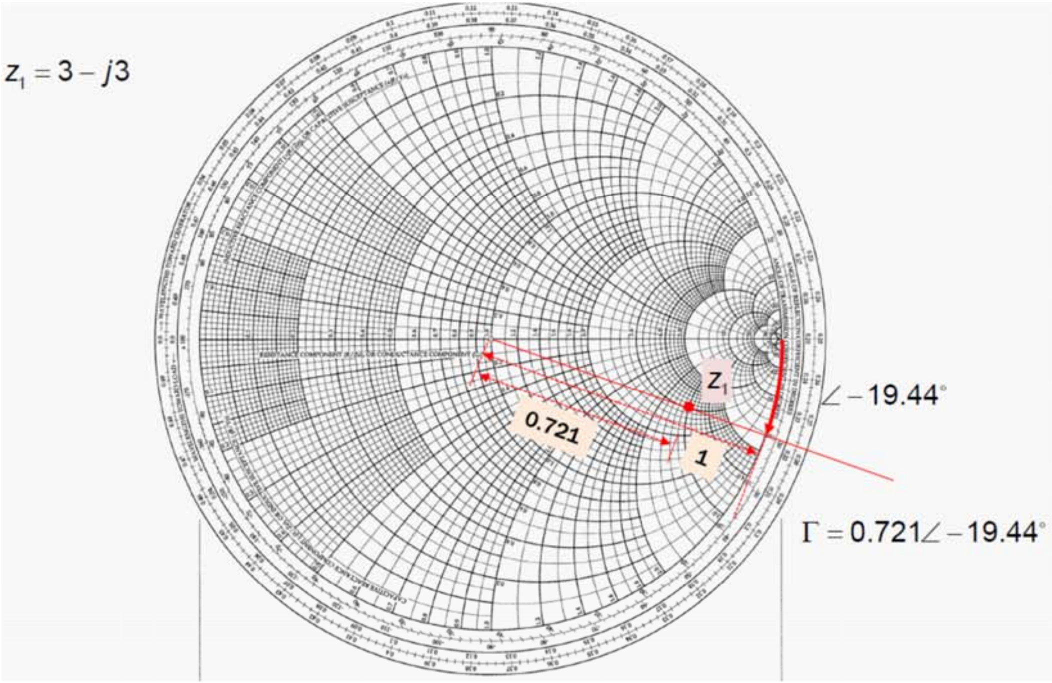
- Impedance divided by line impedance (50 Ohms)
  - Z1 = 100 + j50
  - Z2 = 75 -j100
  - Z3 = j200
  - Z4 = 150
  - Z5 = infinity (an open circuit)
  - Z6 = 0 (a short circuit)
  - Z7 = 50
  - Z8 = 184 -j900

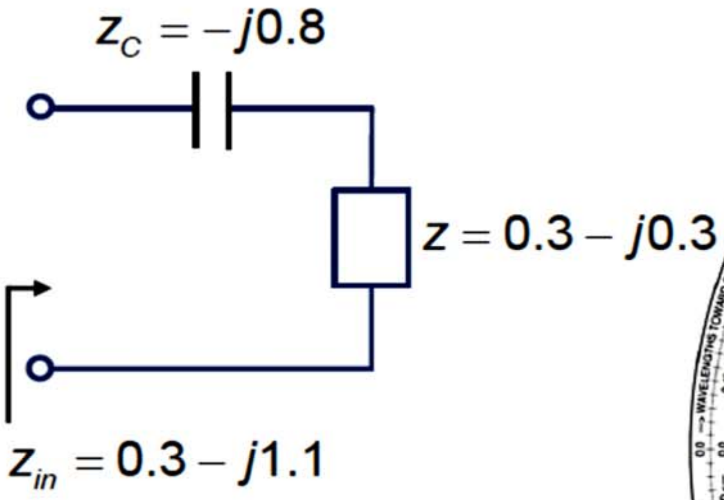
- Then, normalize and plot. The points are plotted as follows:
  - z1 = 2 + j
  - z2 = 1.5 -j2
  - z3 = j4
  - z4 = 3
  - z5 = infinity
  - z6 = 0
  - z7 = 1
  - z8 = 3.68 -j18S

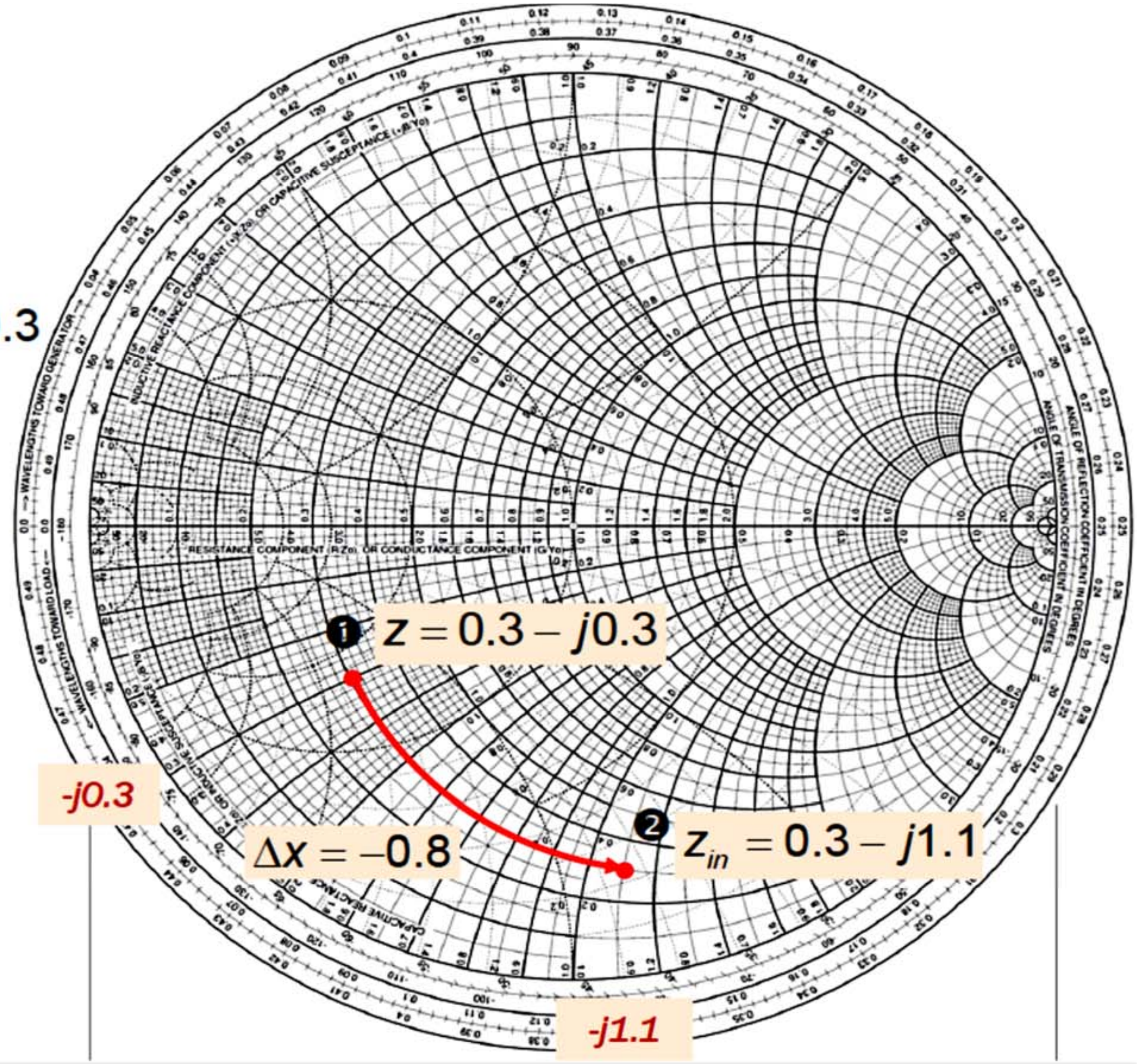


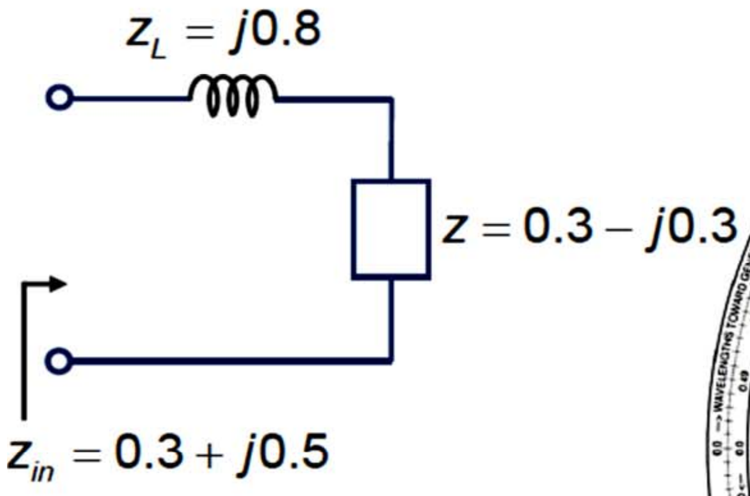


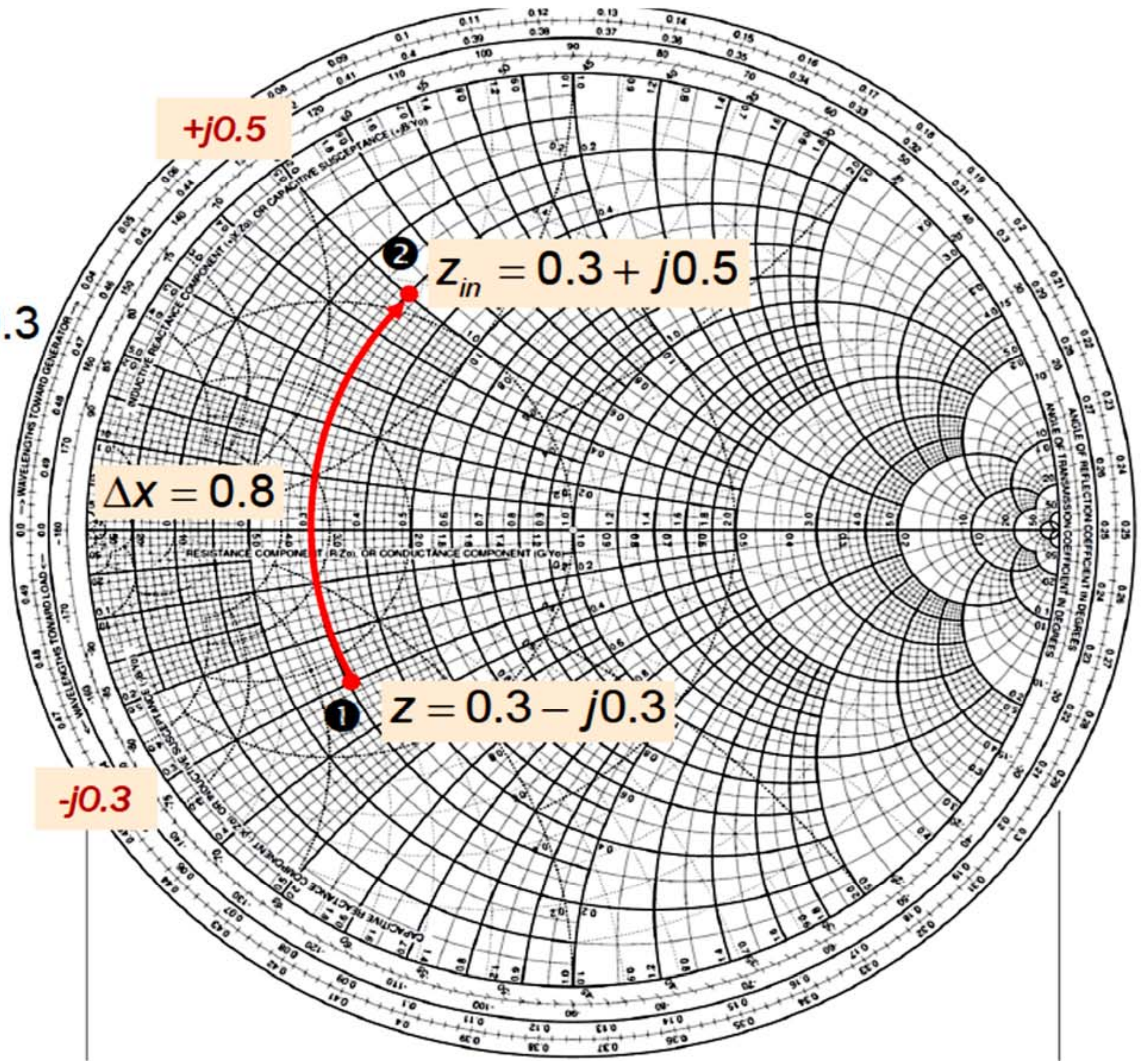
Zn = 0.2 +j0.5
1.0
0.5
2.0
3.0
0.2
5.0
0
0.2
0.5
1.0
2.0
5.0
-5.0
-0.2
-3.0
-2.0
-0.5
-1.0
Constant R circles
Constant VSWR circles
Constant X circles


$z_1 = 3 - j3$
z_1
$\angle -19.44^\circ$
0.721
1
$\Gamma = 0.721\angle -19.44^\circ$


$z_C = -j0.8$
$z = 0.3 - j0.3$
$z_{in} = 0.3 - j1.1$


$z = 0.3 - j0.3$
-j0.3
$\Delta x = -0.8$
$z_{in} = 0.3 - j1.1$
-j1.1


$z_L = j0.8$
$z = 0.3 - j0.3$
$z_{in} = 0.3 + j0.5$


+j0.5
❷ $z_{in} = 0.3 + j0.5$
$\Delta x = 0.8$
❶ $z = 0.3 - j0.3$
-j0.3

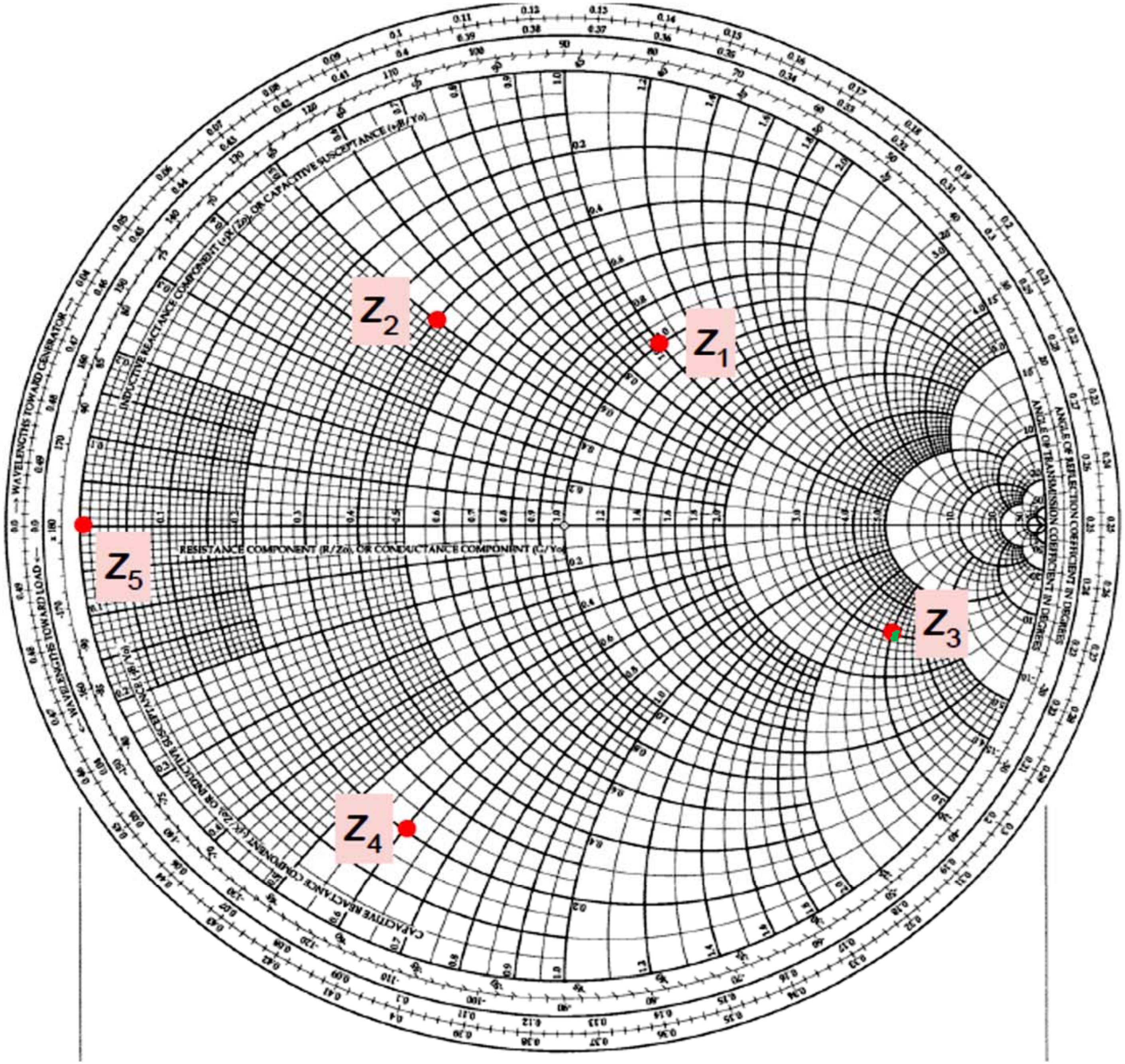
$$z_1 = 1 + j1$$

$$z_2 = 0.4 + j0.5$$

$$z_3 = 3 - j3$$

$$z_4 = 0.2 - j0.6$$

$$z_5 = 0$$



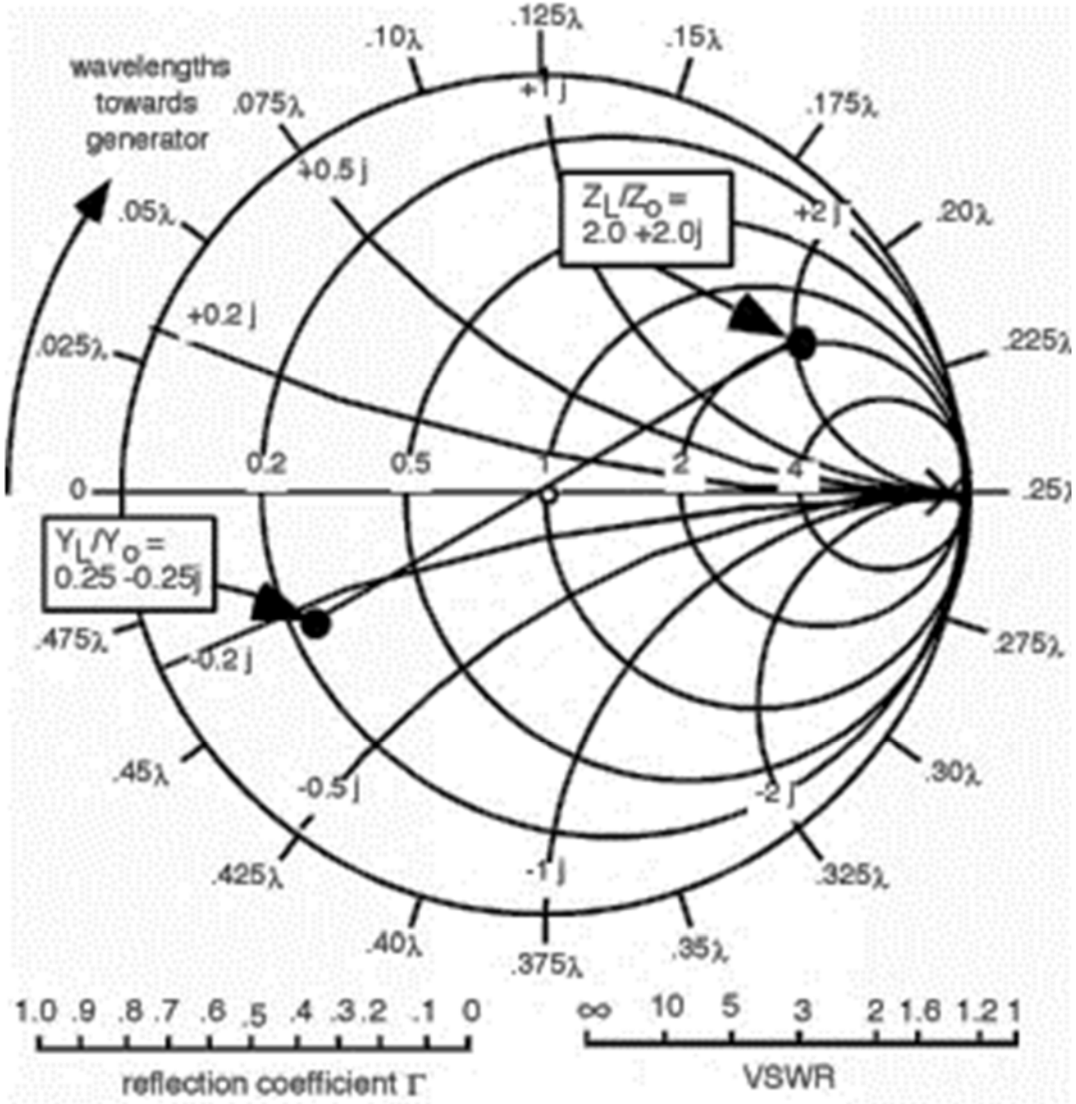
5. Admittance Smith Chart − Y Chart

- Usign Z chart for admittance

$$Y = \frac{1}{Z} = \frac{1}{Z_0}\frac{1-\Gamma}{1+\Gamma} = Y_0\frac{1+(-\Gamma)}{1-(-\Gamma)} \leftrightarrow Z = Z_0\frac{1+\Gamma}{1-\Gamma}$$

$$\boxed{\begin{array}{l}\Gamma \rightarrow -\Gamma \text{ (rotate } 180^\circ) \\ Z_0 \rightarrow Y_0 = 1/Z_0\end{array}}$$

- Rotate *z* by 180º and read a *z* value on the Smith chart. It is then *y* value.



$y_1 = 1 + j1$

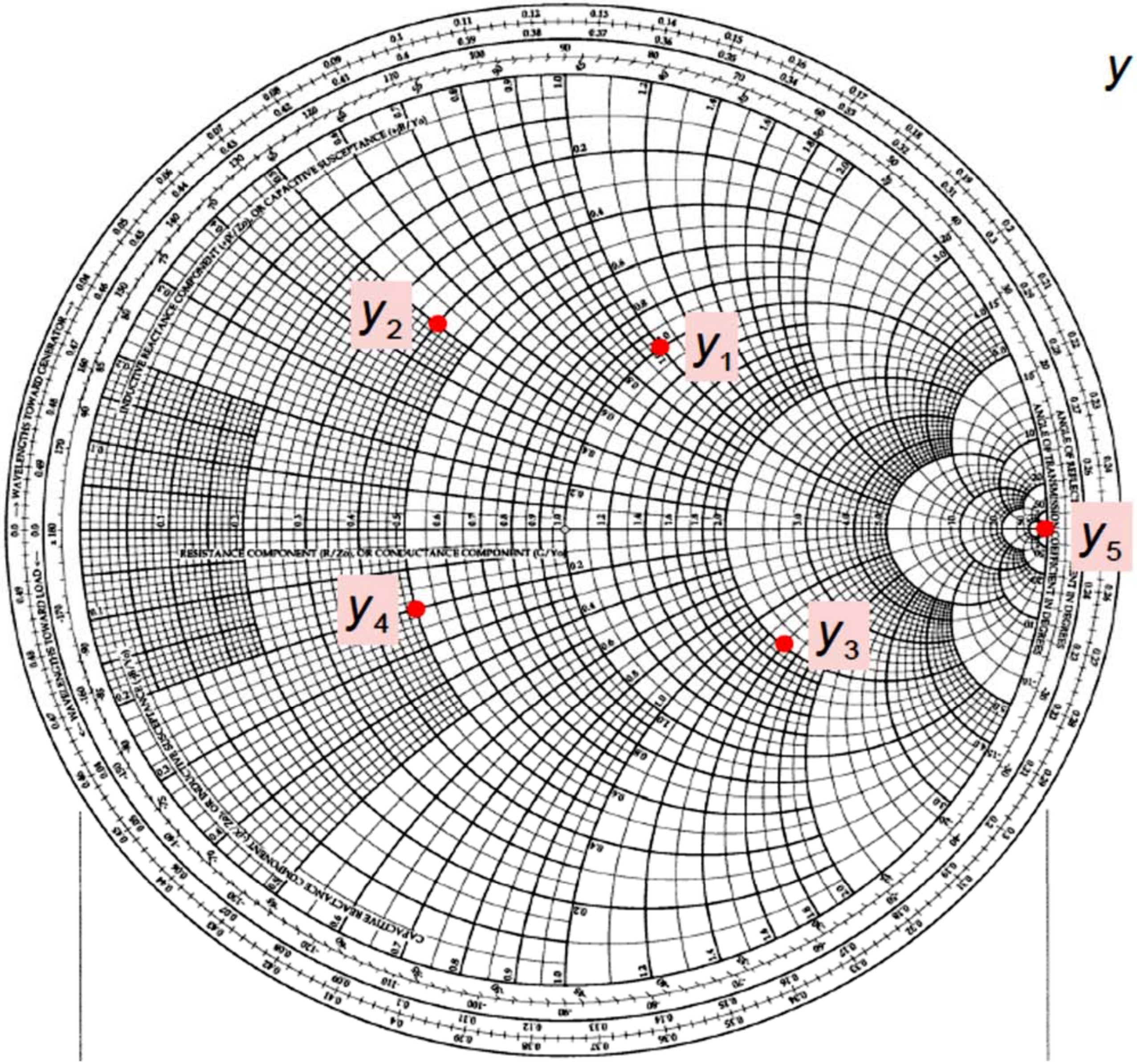
$y_2 = 0.4 + j0.5$

$y_3 = 2 - j1.4$

$y_4 = 0.5 - j0.2$

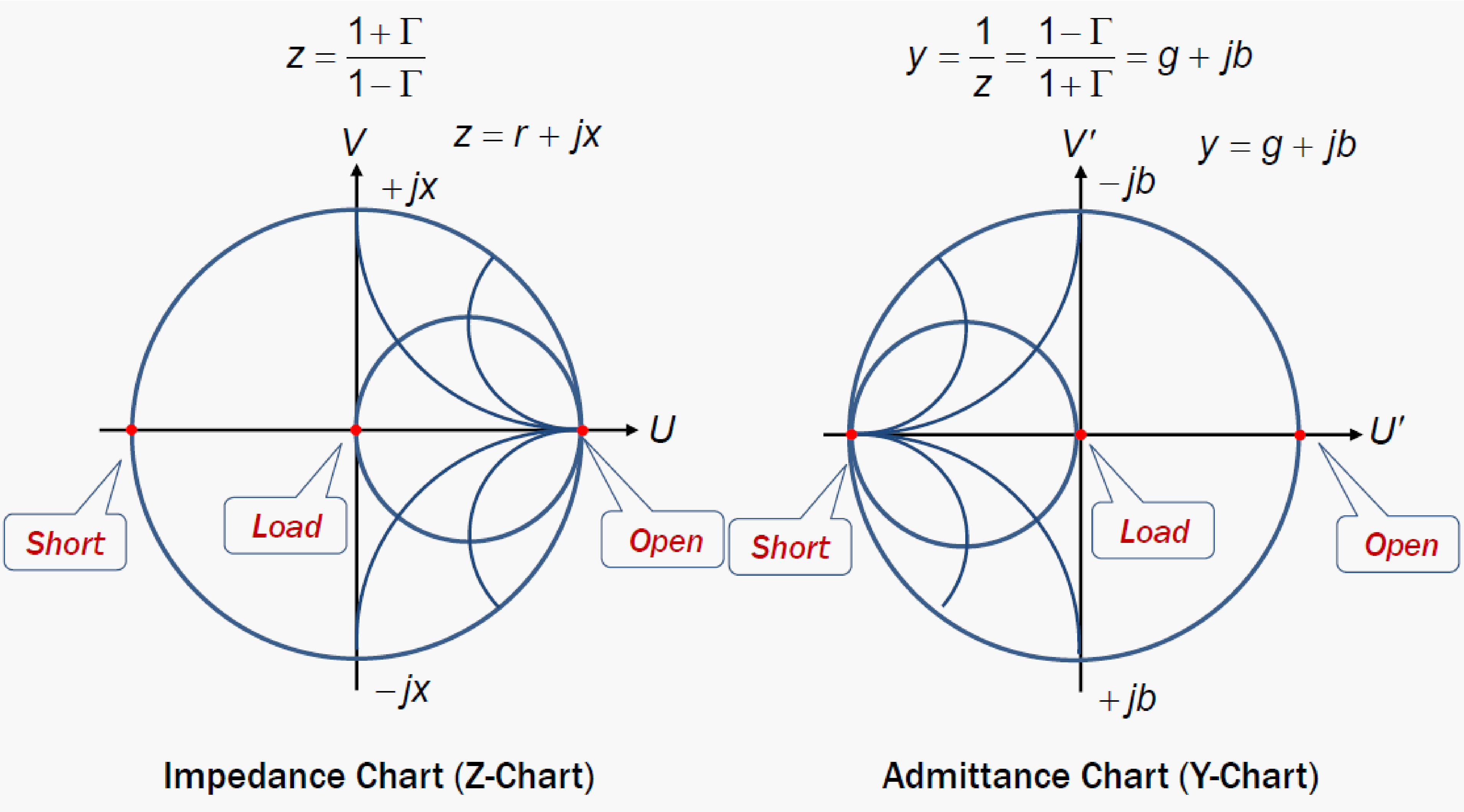
$y_5 = \infty$

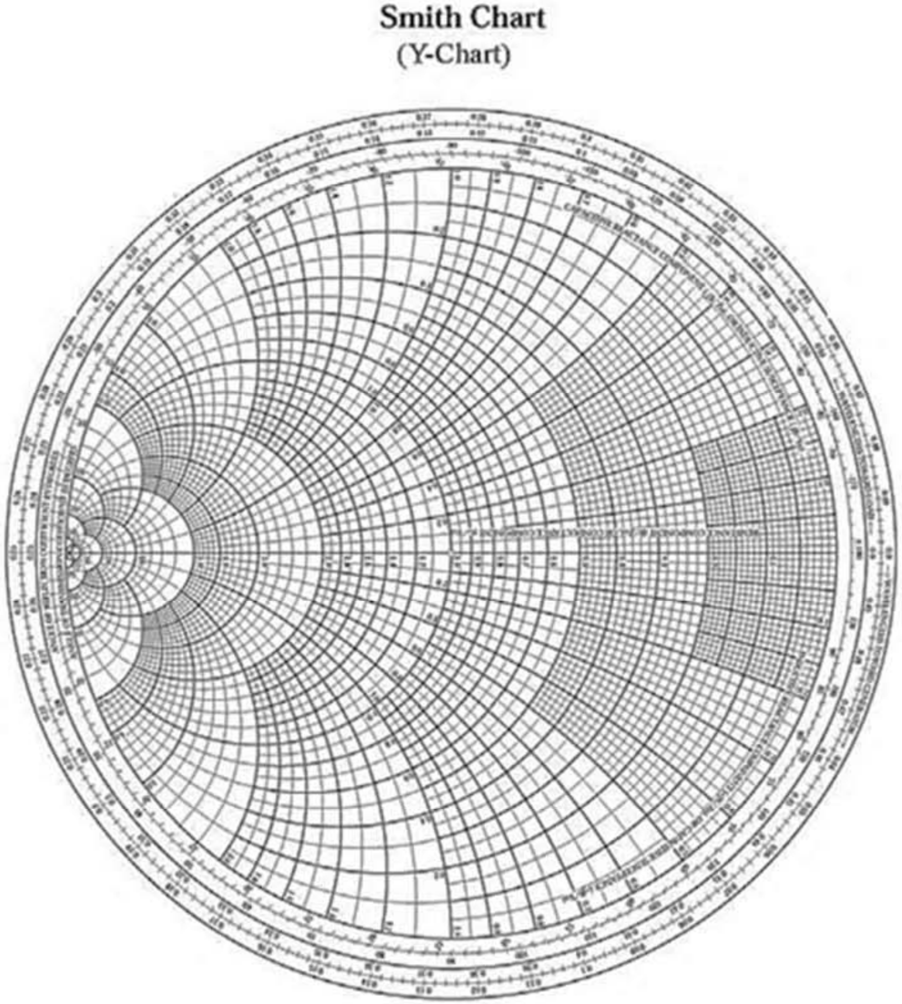
$y = g + jb$

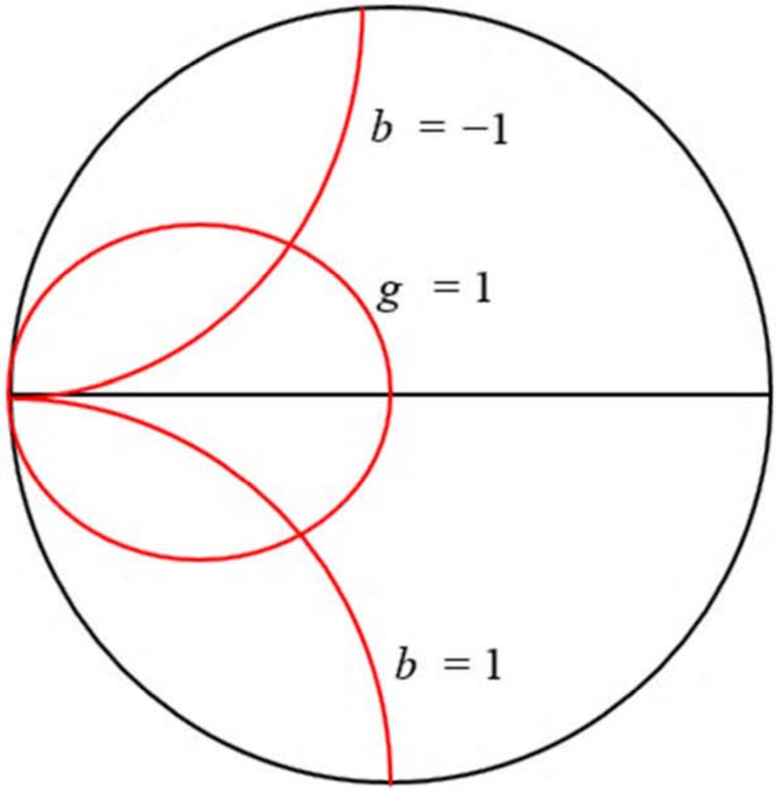


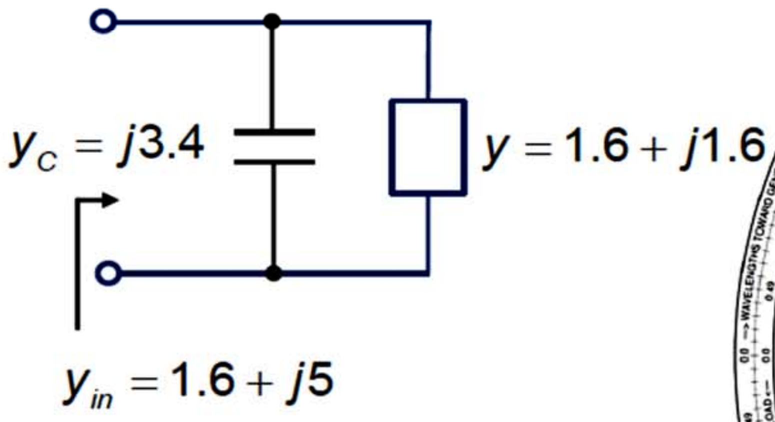
- Using the Y Smith chart

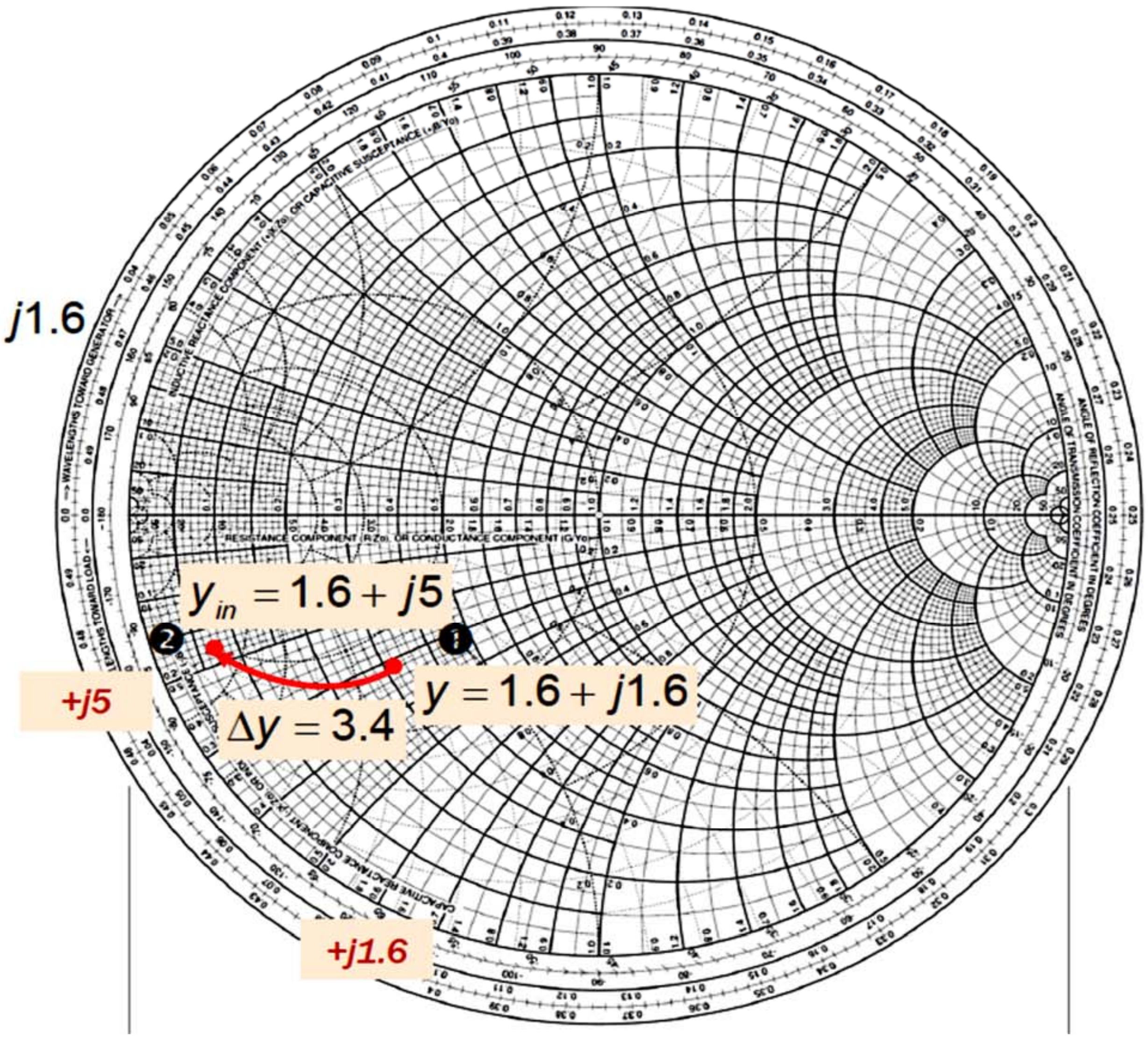
- Rotate the z Smith chart by 180°

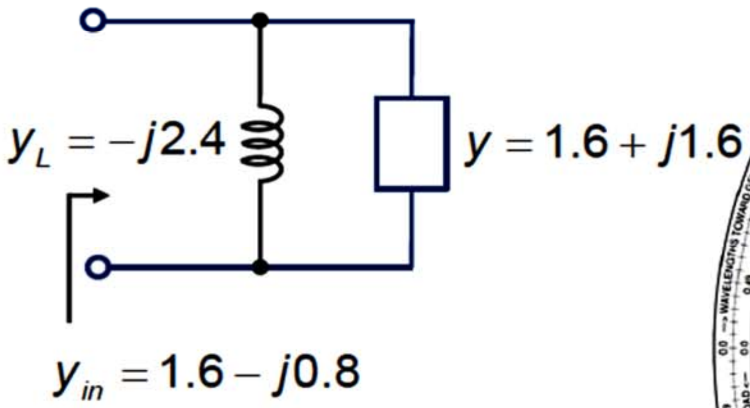


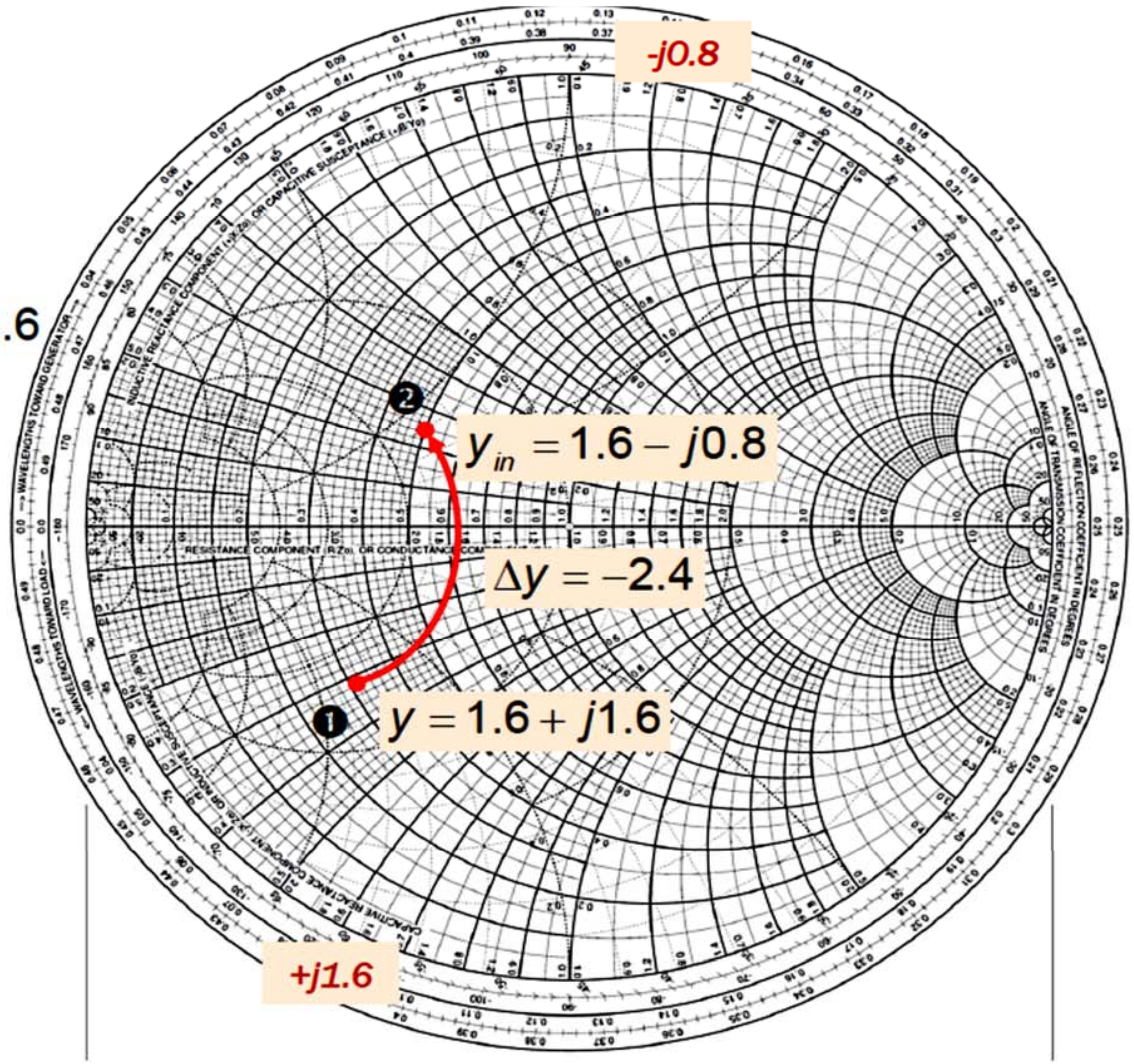

Smith Chart
(Y-Chart)


$b = -1$
$g = 1$
$b = 1$

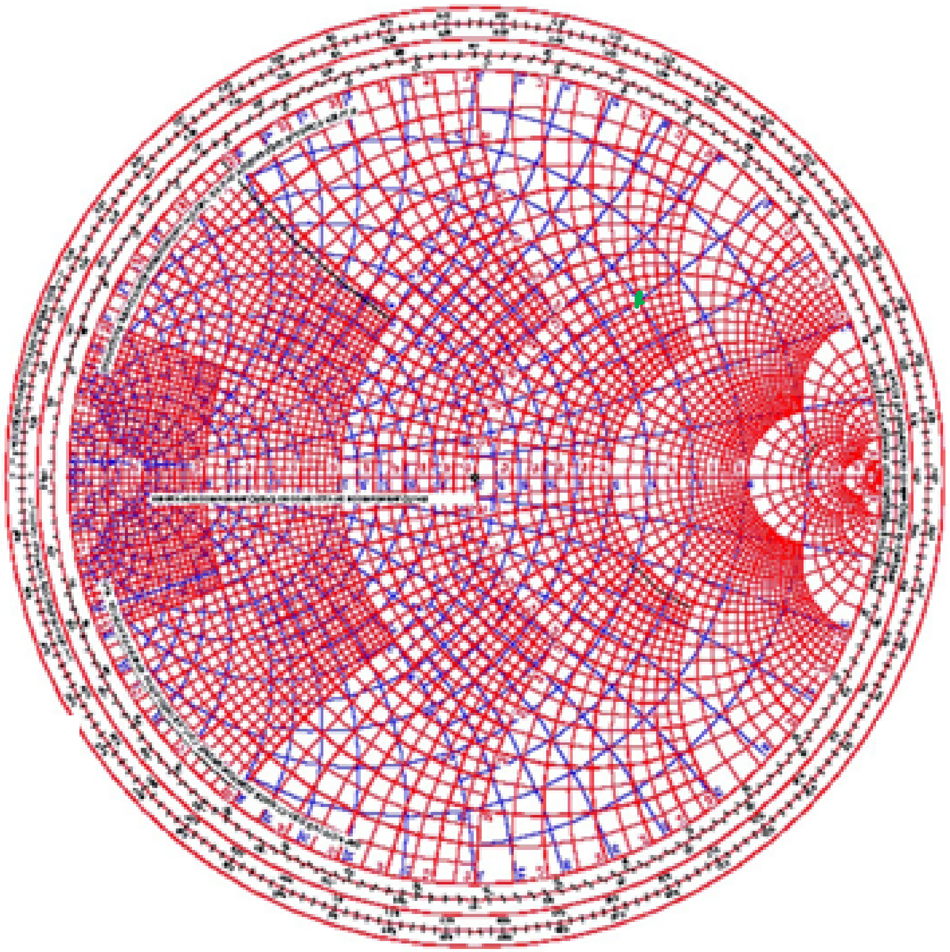

$y_C = j3.4$
$y = 1.6 + j1.6$
$y_{in} = 1.6 + j5$

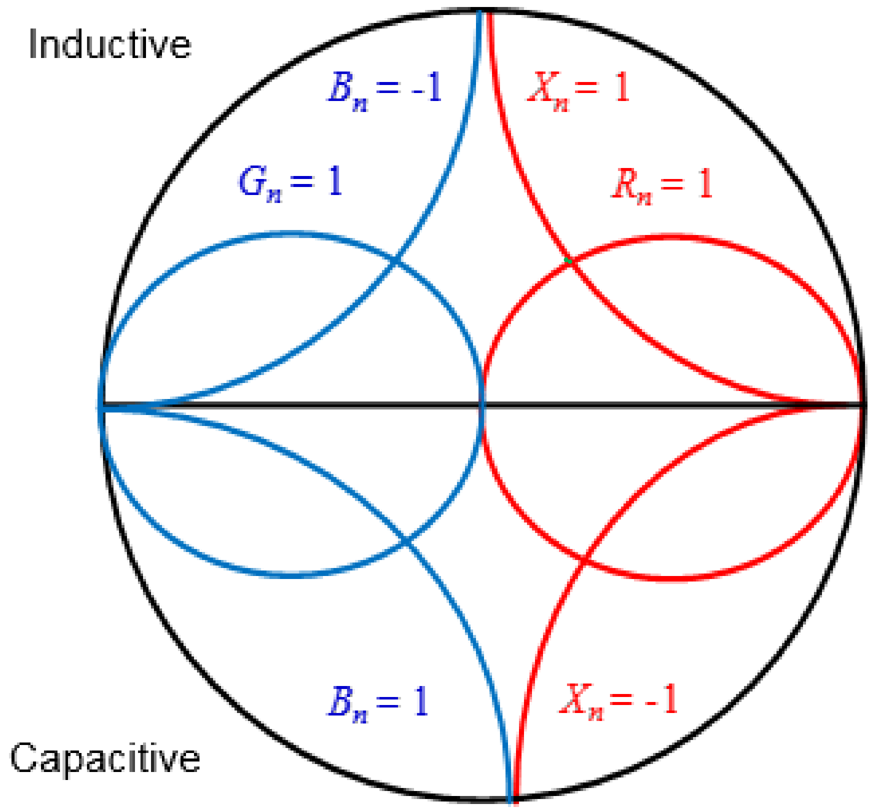

$y_{in} = 1.6 + j5$
$y = 1.6 + j1.6$
$\Delta y = 3.4$
+j5
+j1.6

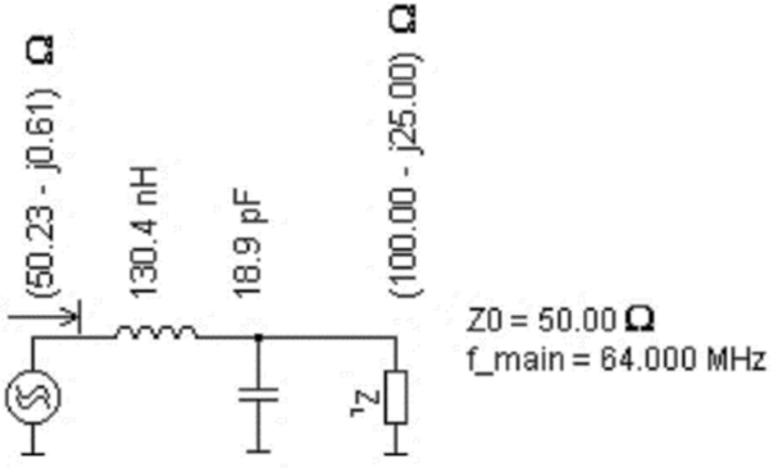

$y_L = -j2.4$
$y = 1.6 + j1.6$
$y_{in} = 1.6 - j0.8$

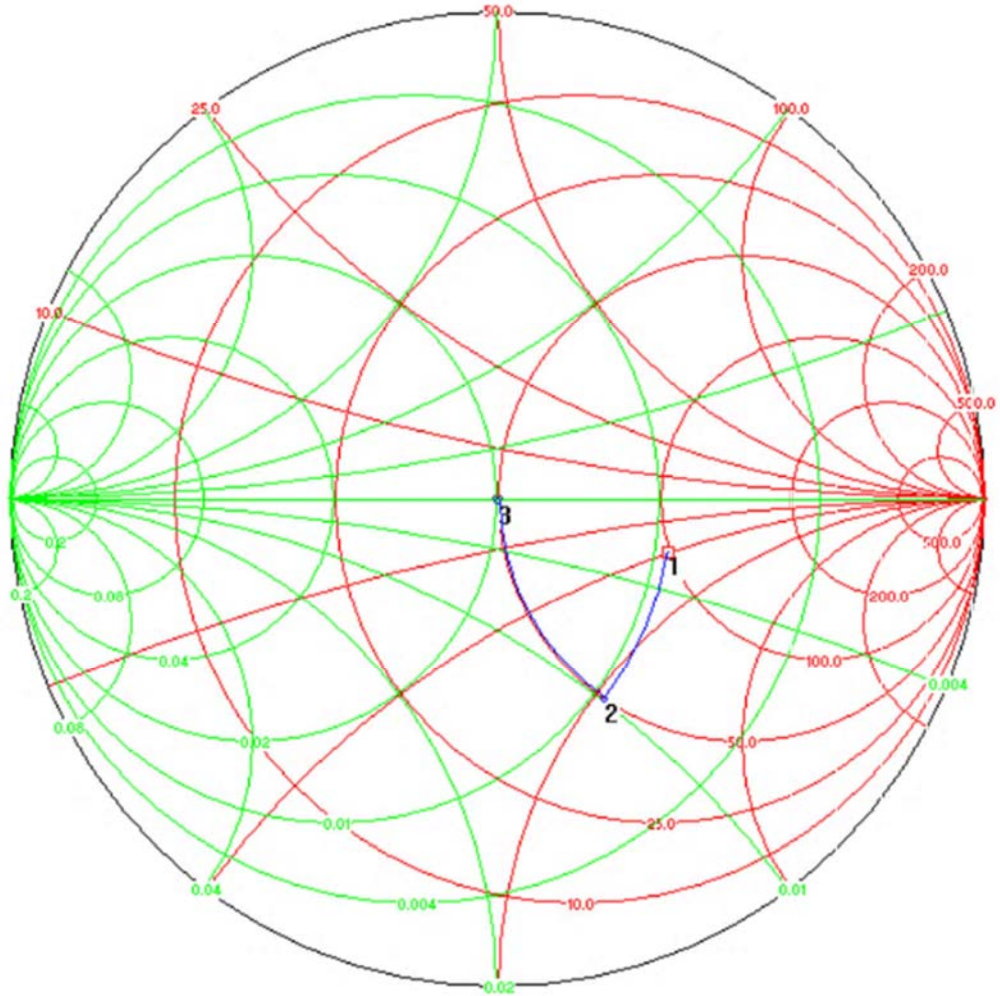

-j0.8
$y_{in} = 1.6 - j0.8$
$\Delta y = -2.4$
$y = 1.6 + j1.6$
+j1.6

6. ZY Chart

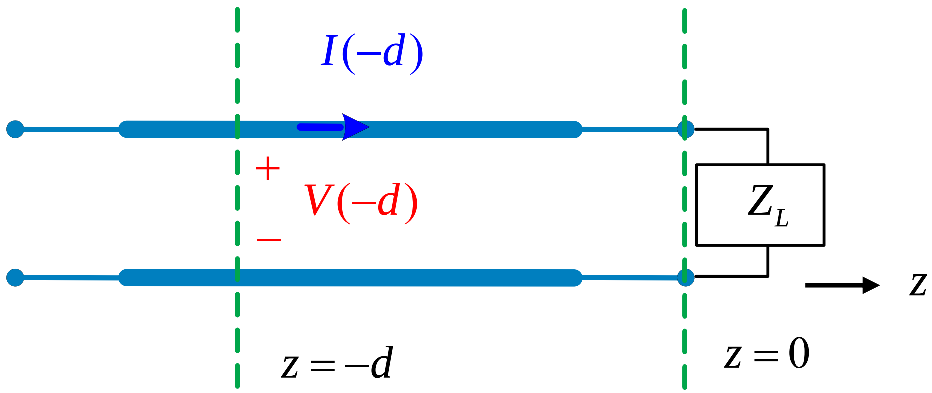


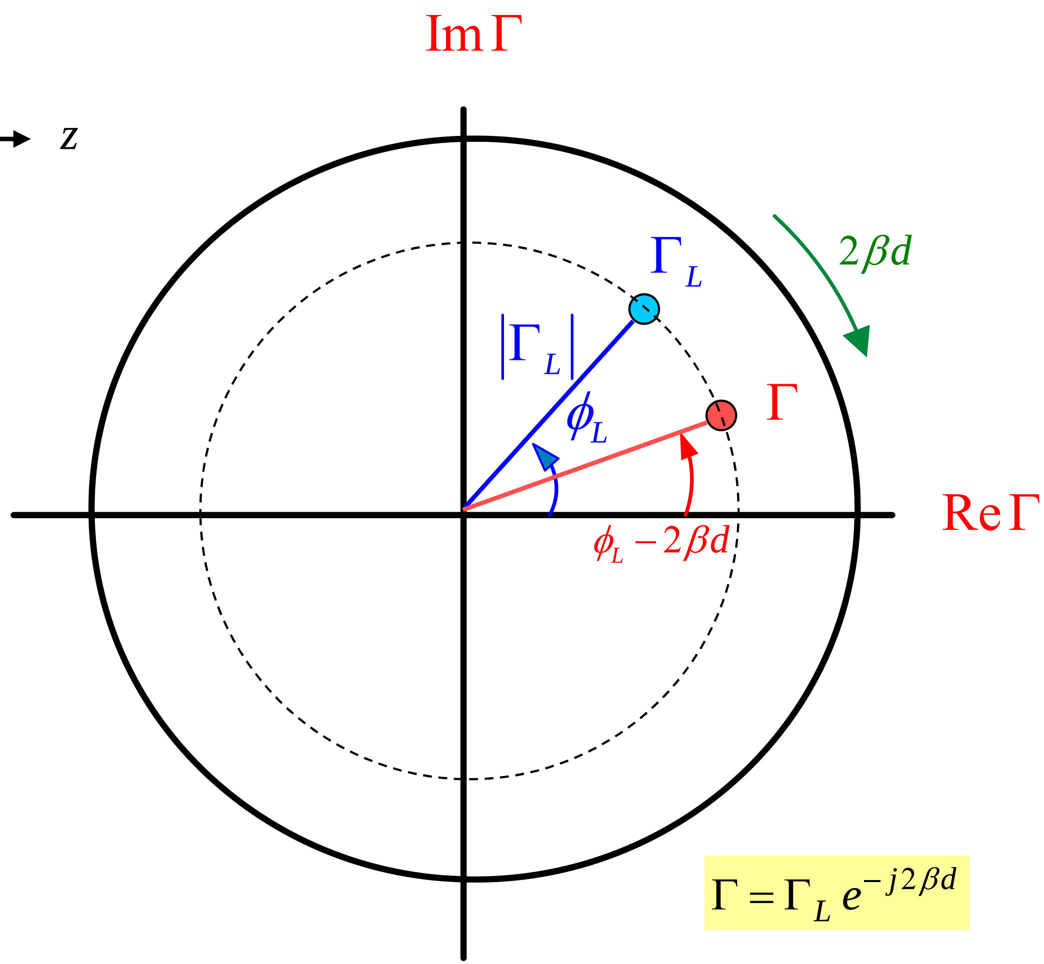



(50.23 - j0.61) Ω
130.4 nH
18.9 pF
(100.00 - j25.00) Ω
Z_L
Z0 = 50.00 Ω
f_main = 64.000 MHz



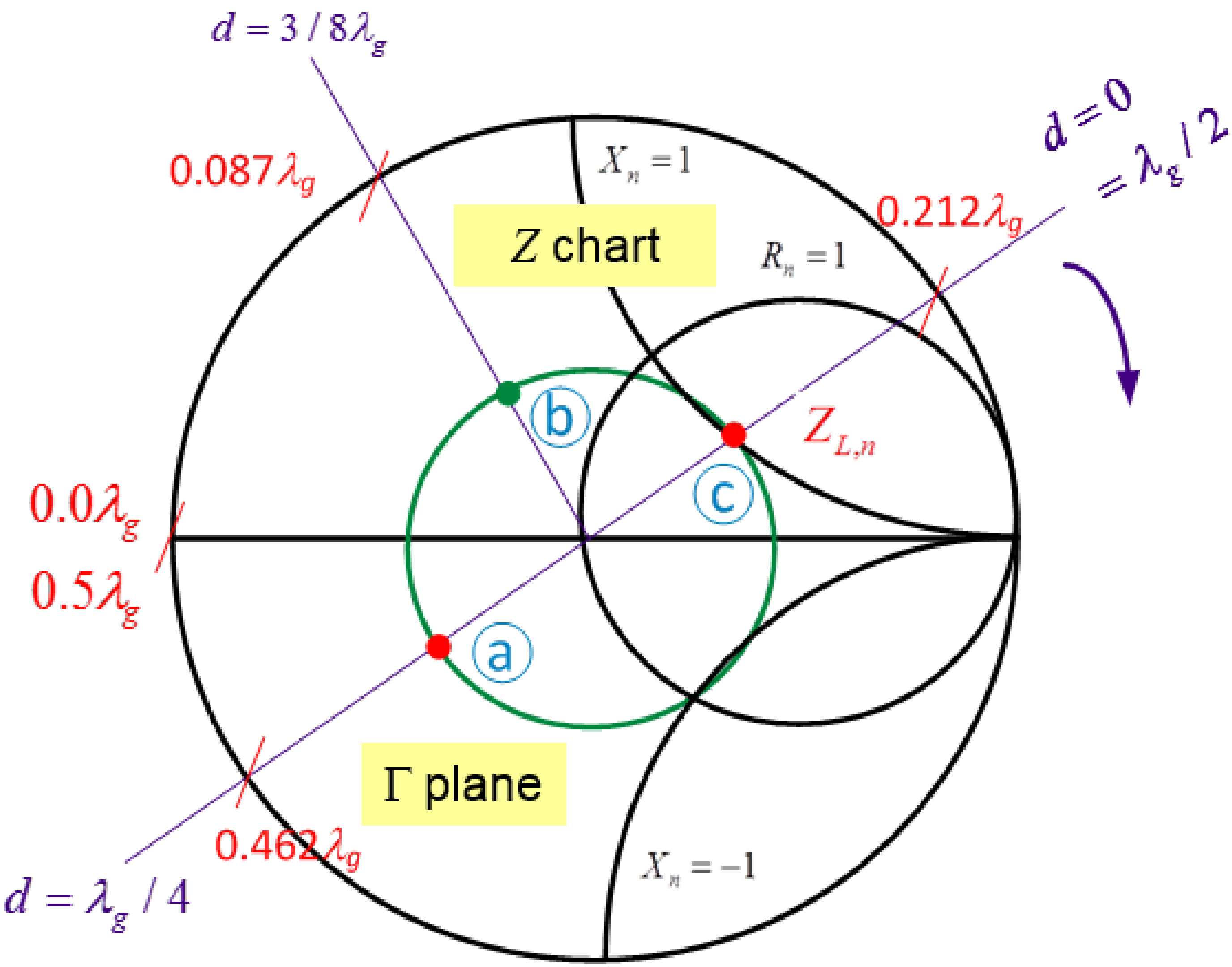
- Calculating Γ on different points in a transmission line





$$\Gamma = \Gamma_L e^{-j2\beta d}$$

- Γ with a quarter-wave TL and a half-wave TL

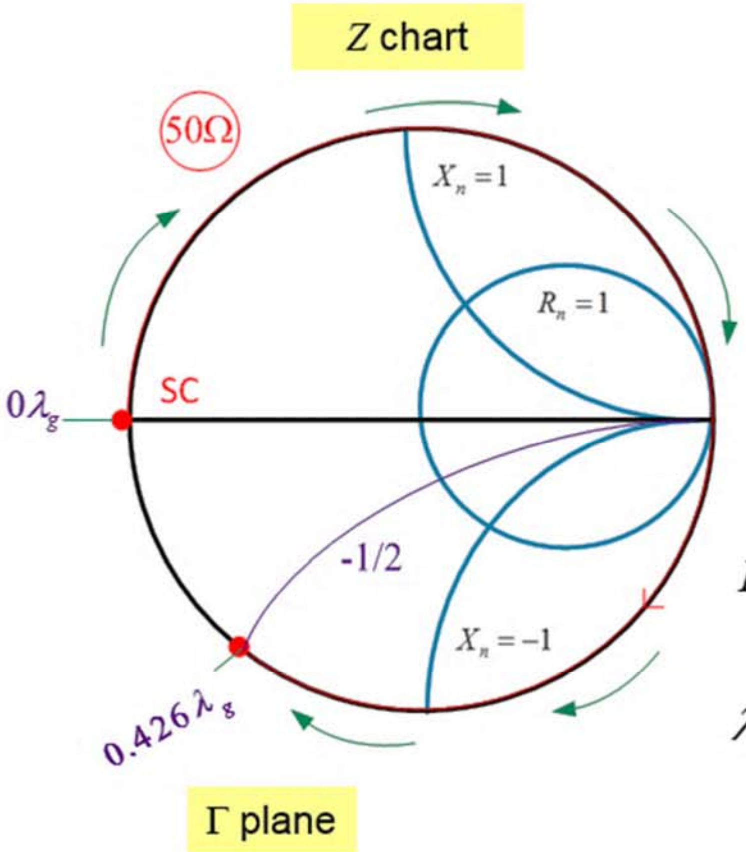


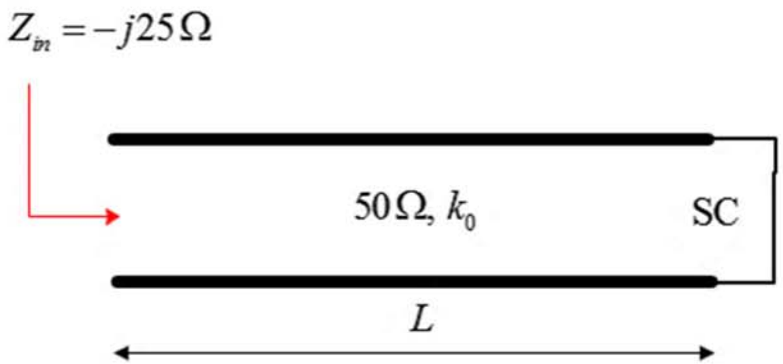
Short-circuited transmission line

Use a short-circuited section of air-filled TEM, 50 Ω transmission line ($\beta = k_0$, $\lambda_g = \lambda_d = \lambda_0$) to create an impedance of $Z_{in} = -j25\ \Omega$ at $f = 10$ GHz.

Use the *Z* chart.

$$Z_{in,n} = -j\frac{25}{50} = -j(0.5)$$





$$L = 0.426\lambda_g - 0\lambda_g = 0.426\lambda_g = 0.426\lambda_0$$

$$\lambda_0 = \frac{c}{f} = \frac{2\pi}{k_0} = \frac{2\pi}{\omega\sqrt{\mu_0\varepsilon_0}}$$

$$\lambda_0 = 3.0\ \text{cm}$$

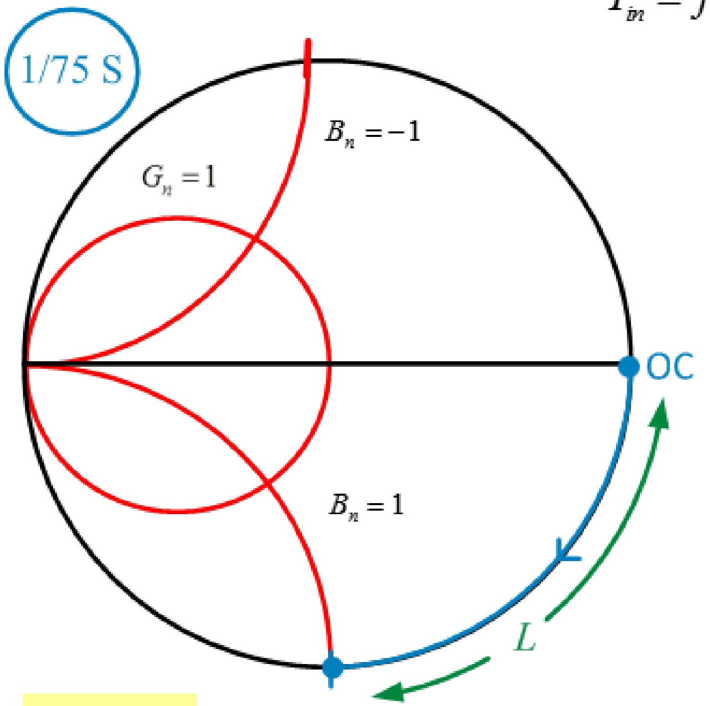
$$\Rightarrow L = 1.28\,\text{cm}$$

- Open-circuited transmission line

Use an open-circuited section of 75 Ω ($Y_0 = 1/75$ S) air-filled transmission line at $f = 10$ GHz to create an admittance of $Y_{in} = j1/75$ S:

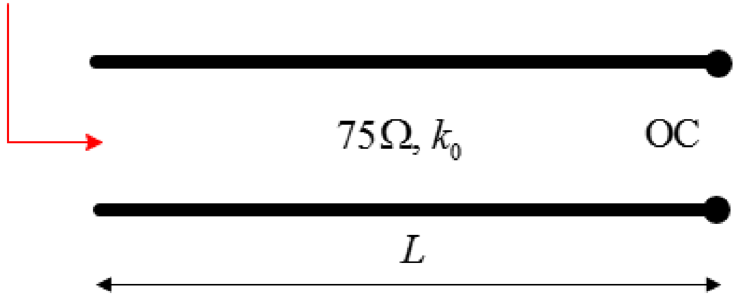
Use the Y chart.

Y chart



Γ plane

$$Y_{in} = j(1/75)\text{S} \Rightarrow Y_{in,n} = j(1)$$



$$L = 0.125\lambda_g = 0.125\lambda_0 \qquad \lambda_0 = 3.0 \text{ cm}$$

$\Rightarrow$ $L = 0.375\,\text{cm}$

Fin

(End)