Lecture 6 Impedance Matching (2)

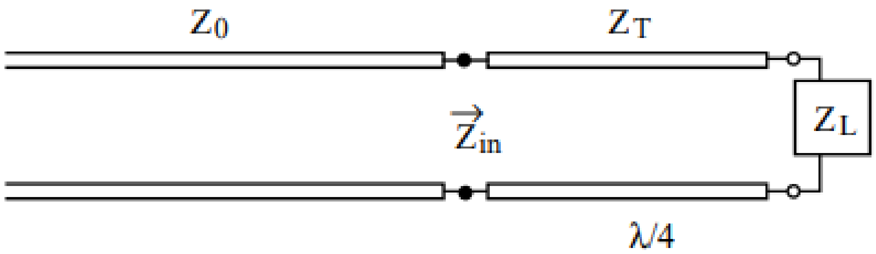
1. Quarter-wave Transformer
2. Single-stub Matching
3. Double-stub Matching
4. *LC* Matching
5. Coding Example

1. Quarter-wave Transformer

- Theory

- Quarter-wave = 1/4 wavelength

- Z_L real, if not make it real by adding L, C, or a length of transmission line.



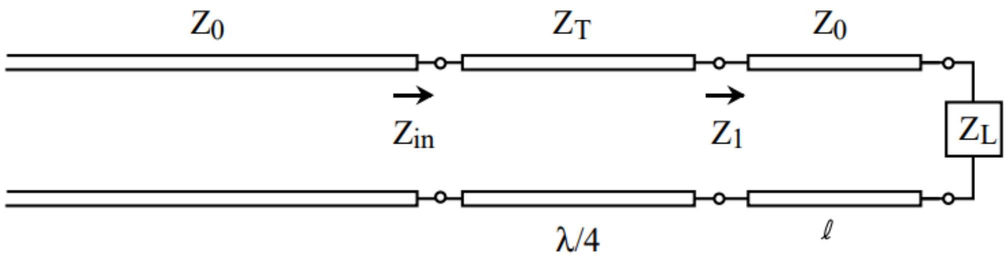
$$Z_{in} = Z_T \frac{Z_L + jZ_T \tan\frac{\pi}{2}}{Z_T + jZ_L \tan\frac{\pi}{2}} = \frac{Z_T^2}{Z_L},$$

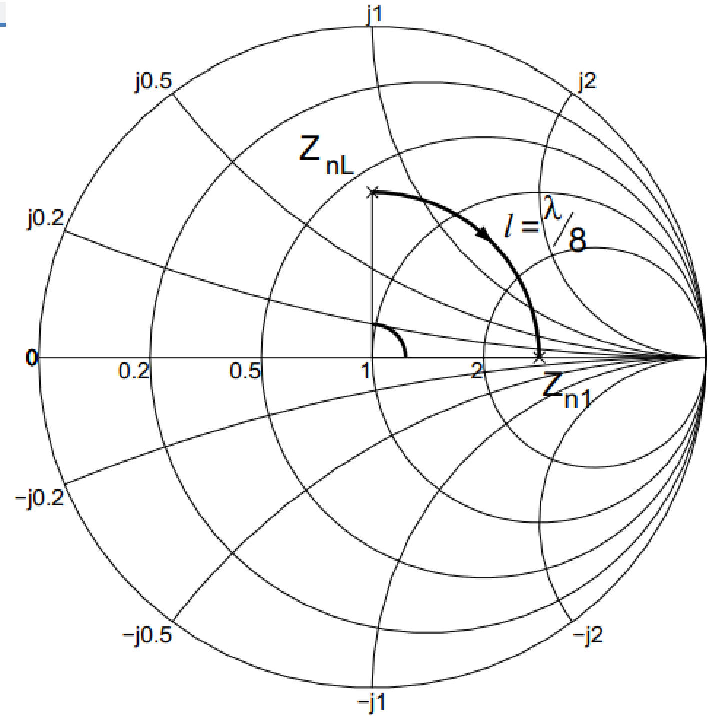
$$\tan \beta l = \tan \frac{2\pi}{\lambda}\frac{\lambda}{4} = \tan\frac{\pi}{2} = \infty$$

$$Z_T^2 = Z_0 Z_L \Rightarrow Z_T = \sqrt{Z_0 Z_l}$$

Example

- Quarter-wave transformer with a transmission line section added





Given that $Z_L = (30 + j40)\Omega$, $Z_0 = 50\Omega$, find the shortest l and Z_T so that the above circuit is matched. Assume that Z_T is real and lossless.

We want Z_1 to be real and Z_{in} to be $Z_0 = 50\Omega$ in order for Z_T to be real and the matching condition satisfied. We find that $Z_{nL} = 0.6 + j0.8$. In order to make Z_{n1} real, the shortest l from the Smith Chart is $\frac{\lambda}{8}$. Then $Z_{n1} = 3.0$, and $Z_1 = 150\Omega$. Since $Z_{in} = 50\Omega$, we need

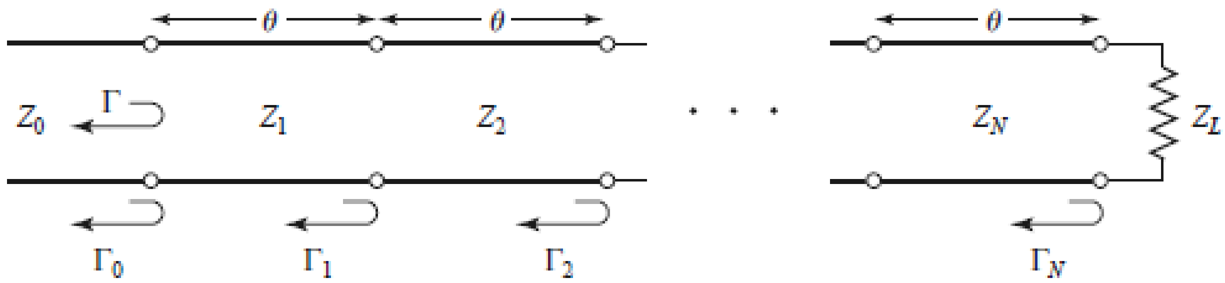
$$Z_T = \sqrt{Z_{in} Z_1} = \sqrt{50 \times 150} = 86.6\Omega$$

in order for matching condition to be satisfied.

Multisection quarter-wave transformer

- Single section transformer: narrow band, compact

- Multi-section transformer: wideband, not compact



$$\Gamma_0 = \frac{Z_1 - Z_0}{Z_1 + Z_0},$$

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n},$$

$$\Gamma_N = \frac{Z_L - Z_N}{Z_L + Z_N}.$$

$$\Gamma(\theta) = e^{-jN\theta}\left\{\Gamma_0[e^{jN\theta} + e^{-jN\theta}] + \Gamma_1[e^{j(N-2)\theta} + e^{-j(N-2)\theta}] + \cdots\right\}.$$

$$\beta l = \theta$$

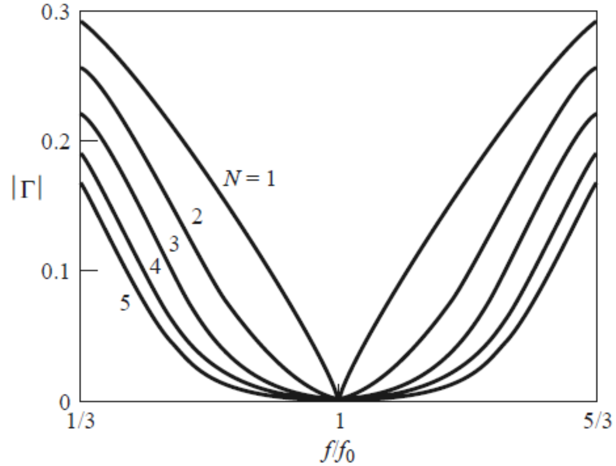
$$\theta = \pi/2 \text{ at } f_0$$

1) Binomial transformer

$$\Gamma(\theta) = A(1 + e^{-2j\theta})^N.$$ (binomial function)

$$\Gamma_n = 2^{-N} \frac{Z_L - Z_0}{Z_L + Z_0} \frac{N!}{(N-n)!n!}$$

$$A = 2^{-N} \frac{Z_L - Z_0}{Z_L + Z_0}.$$



$$\Gamma_m = 2^N |A| \cos^N \theta_m,$$ (maximum reflection coefficient tolerable)

$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4\theta_m}{\pi}$$

	$N=2$		$N=3$			$N=4$			
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1067	1.3554	1.0520	1.2247	1.4259	1.0257	1.1351	1.3215	1.4624
2.0	1.1892	1.6818	1.0907	1.4142	1.8337	1.0444	1.2421	1.6102	1.9150
3.0	1.3161	2.2795	1.1479	1.7321	2.6135	1.0718	1.4105	2.1269	2.7990
4.0	1.4142	2.8285	1.1907	2.0000	3.3594	1.0919	1.5442	2.5903	3.6633
6.0	1.5651	3.8336	1.2544	2.4495	4.7832	1.1215	1.7553	3.4182	5.3500
8.0	1.6818	4.7568	1.3022	2.8284	6.1434	1.1436	1.9232	4.1597	6.9955
10.0	1.7783	5.6233	1.3409	3.1623	7.4577	1.1613	2.0651	4.8424	8.6110

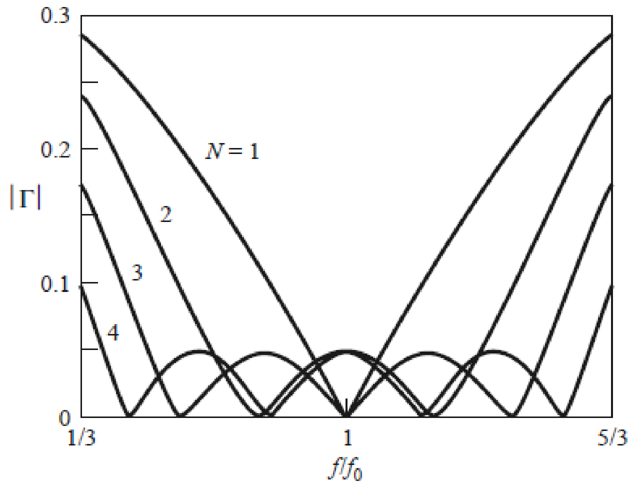
2) Chebyshev transformer

$$\Gamma(\theta) = 2e^{-jN\theta}[\Gamma_0 \cos N\theta + \Gamma_1 \cos(N-2)\theta + \cdots + \Gamma_n \cos(N-2n)\theta + \cdots]$$
$$= Ae^{-jN\theta} T_N(\sec\theta_m \cos\theta),$$

$\Gamma_m = |A|$ (maximum reflection coefficient tolerable)

$$\sec\theta_m = \cosh\left[\frac{1}{N}\cosh^{-1}\left(\frac{1}{\Gamma_m}\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|\right)\right]$$

$$\frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi}. \qquad \Gamma_n \simeq \frac{1}{2}\ln\frac{Z_{n+1}}{Z_n}$$



	$N = 2$				$N = 3$					
	$\Gamma_m = 0.05$		$\Gamma_m = 0.20$		$\Gamma_m = 0.05$			$\Gamma_m = 0.20$		
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1347	1.3219	1.2247	1.2247	1.1029	1.2247	1.3601	1.2247	1.2247	1.2247
2.0	1.2193	1.6402	1.3161	1.5197	1.1475	1.4142	1.7429	1.2855	1.4142	1.5558
3.0	1.3494	2.2232	1.4565	2.0598	1.2171	1.7321	2.4649	1.3743	1.7321	2.1829
4.0	1.4500	2.7585	1.5651	2.5558	1.2662	2.0000	3.1591	1.4333	2.0000	2.7908
6.0	1.6047	3.7389	1.7321	3.4641	1.3383	2.4495	4.4833	1.5193	2.4495	3.9492
8.0	1.7244	4.6393	1.8612	4.2983	1.3944	2.8284	5.7372	1.5766	2.8284	5.0742
10.0	1.8233	5.4845	1.9680	5.0813	1.4385	3.1623	6.9517	1.6415	3.1623	6.0920

- Chebyshev transformer: example, N = 4, A = 0.1

$$T_0(\sec\theta_m \cos\theta) = 1$$
$$T_1(\sec\theta_m \cos\theta) = \sec\theta_m \cos\theta$$
$$T_2(\sec\theta_m \cos\theta) = \sec^2\theta_m (1+\cos 2\theta) - 1 \qquad (2.5)$$
$$T_3(\sec\theta_m \cos\theta) = \sec^3\theta_m (\cos 3\theta + 3\cos\theta) - 3\sec\theta_m \cos\theta$$
$$T_4(\sec\theta_m \cos\theta) = \sec^4\theta_m (\cos 4\theta + 4\cos 2\theta + 3) - 4\sec^2\theta_m (\cos 2\theta + 1) + 1$$

It is notable that it is sufficient to calculate Γ_0, Γ_1 and Γ_2. Due to the symmetry we have $\Gamma_0 = \Gamma_4, \Gamma_1 = \Gamma_3$.

$$\Gamma(\theta) = 2e^{-j4\theta}\left[\Gamma_0(e^{j4\theta} + e^{-j4\theta}) + \Gamma_1(e^{j2\theta} + e^{-j2\theta}) + \Gamma_2/2\right]$$
$$= Ae^{-j4\theta}\left[\sec^4\theta_m (\cos 4\theta + 4\cos 2\theta + 3) - 4\sec^2\theta_m (\cos 2\theta + 1) + 1\right] =$$
$$= 2e^{-j4\theta}\left[\Gamma_0 \cos 4\theta + \Gamma_1 \cos 2\theta + \frac{\Gamma_2}{2}\right]$$

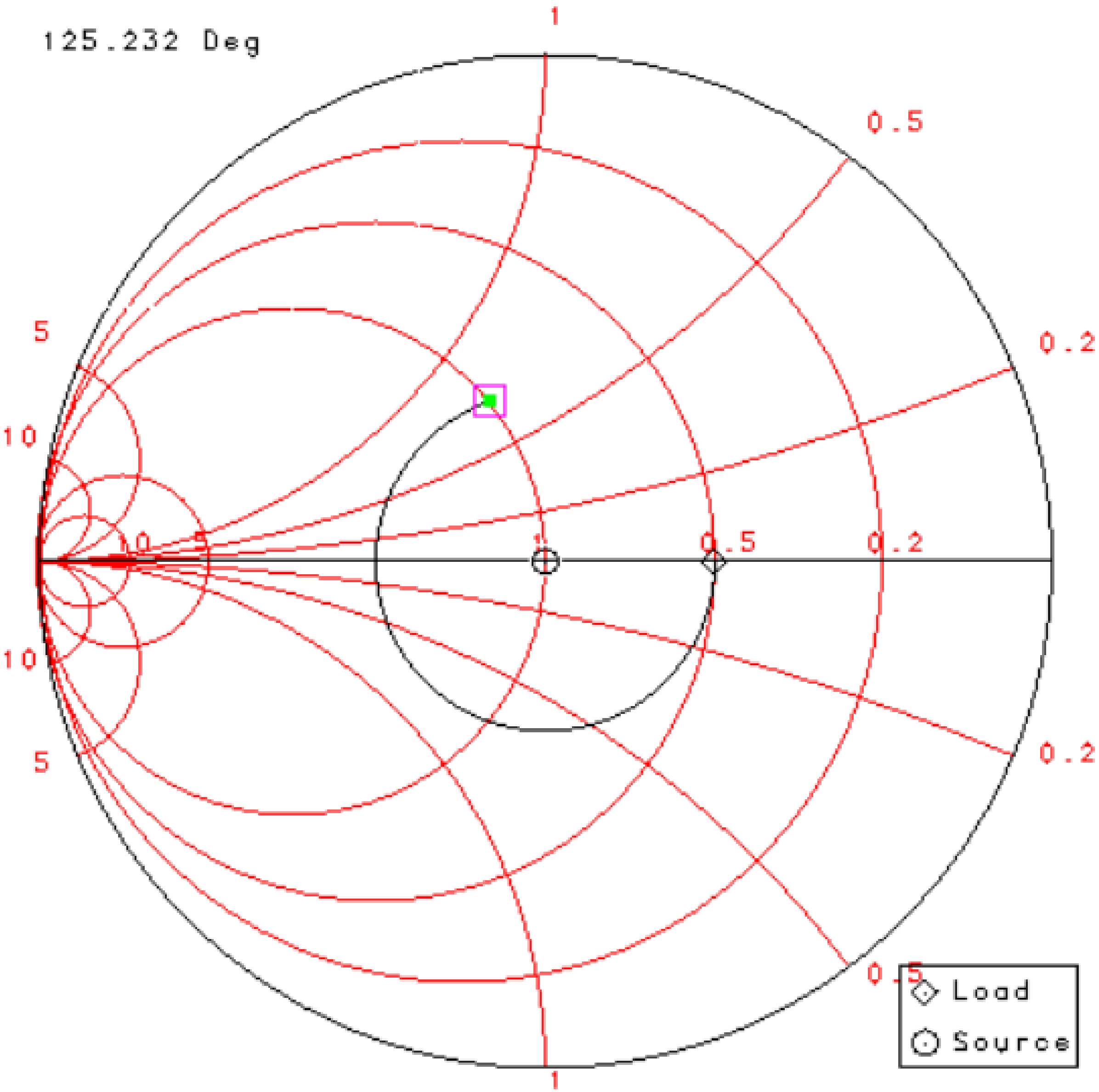
$$2\Gamma_0 = A\sec^4\theta_m \Rightarrow \Gamma_0 = \frac{1}{2}0.1\sec^4(38°) = 0.131$$
$$2\Gamma_1 = 4A\left(\sec^4\theta_m - \sec^2\theta_m\right) \Rightarrow$$
$$\Rightarrow \Gamma_1 = \frac{1}{2}0.1\left(\sec^4(38°) - \sec^2(38°)\right) = 0.2$$
$$\Gamma_2 = A\left(3\sec^4\theta_m - 4\sec^2\theta_m + 1\right) \Rightarrow$$
$$\Rightarrow \Gamma_2 = 0.1\left(3\sec^4(38°) - 4\sec^2(38°) + 1\right) = 0.239$$

$$Z_1 = Z_0 e^{2\Gamma_0} = 65\,\Omega$$
$$Z_2 = Z_1 e^{2\Gamma_1} = 97.1\,\Omega$$
$$Z_3 = Z_2 e^{2\Gamma_2} = 156.7\,\Omega$$
$$Z_4 = Z_3 e^{2\Gamma_3} = 234.2\,\Omega$$

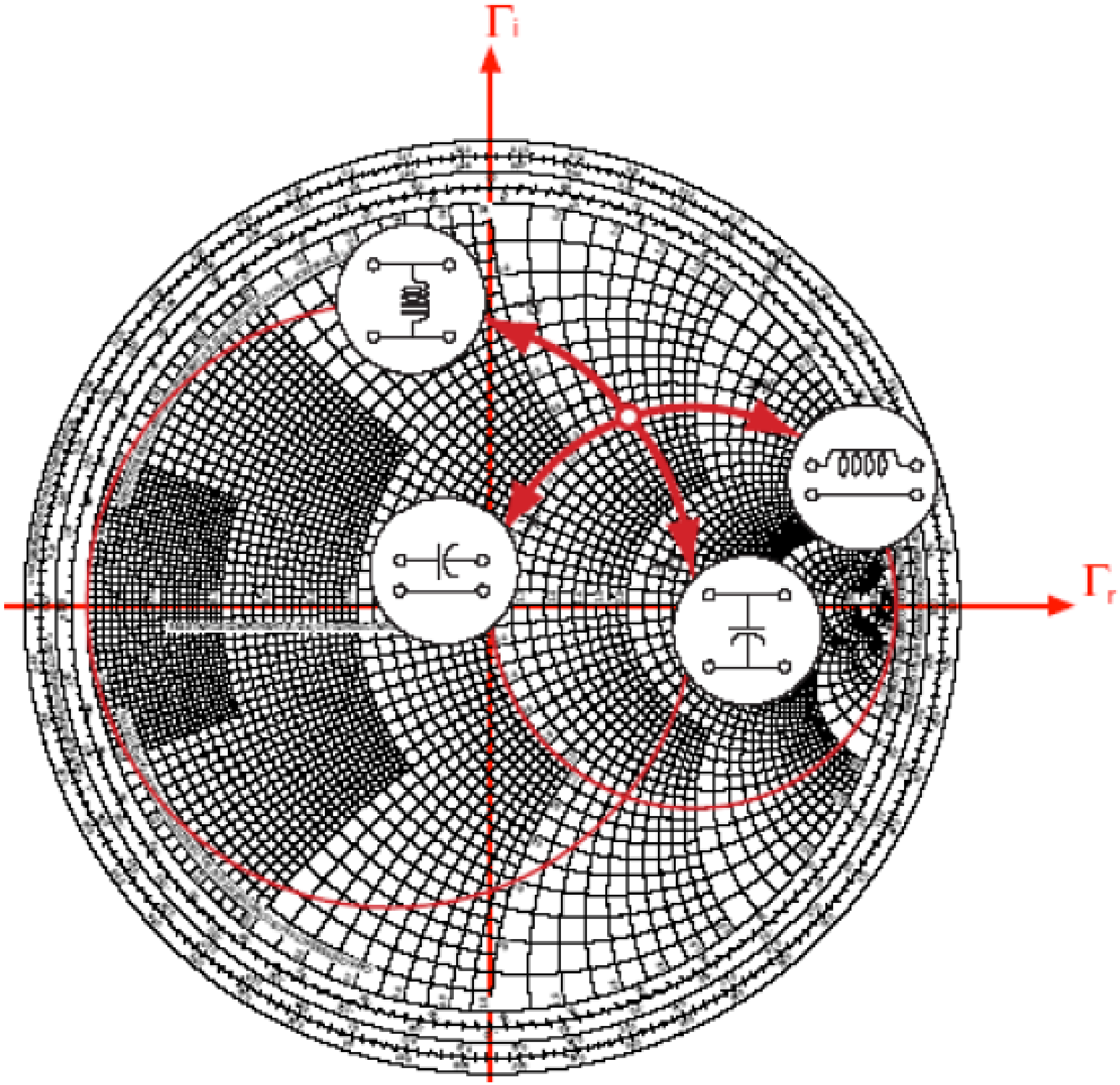
2. Single-stub Matching

- Load + transmission line section: non-adjustable

$$\Gamma(-\ell) = \Gamma(0)e^{-j2\beta\ell},\ \theta = 2\beta\ell \quad (\ell = \lambda/2 \rightarrow \theta = 2\pi)$$

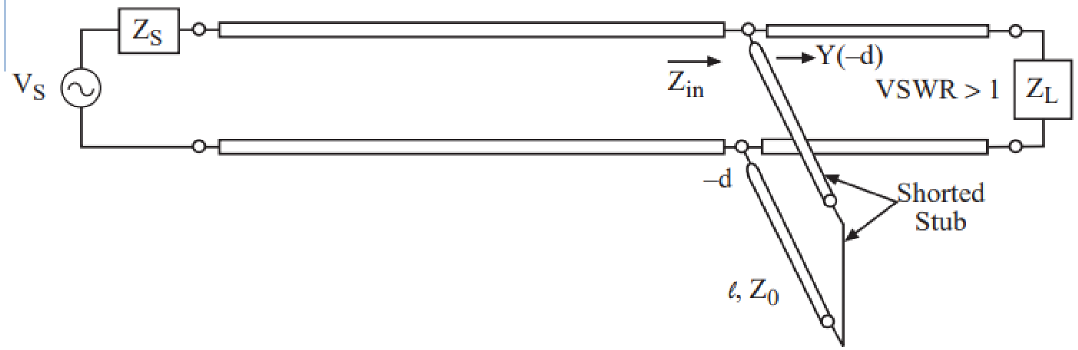


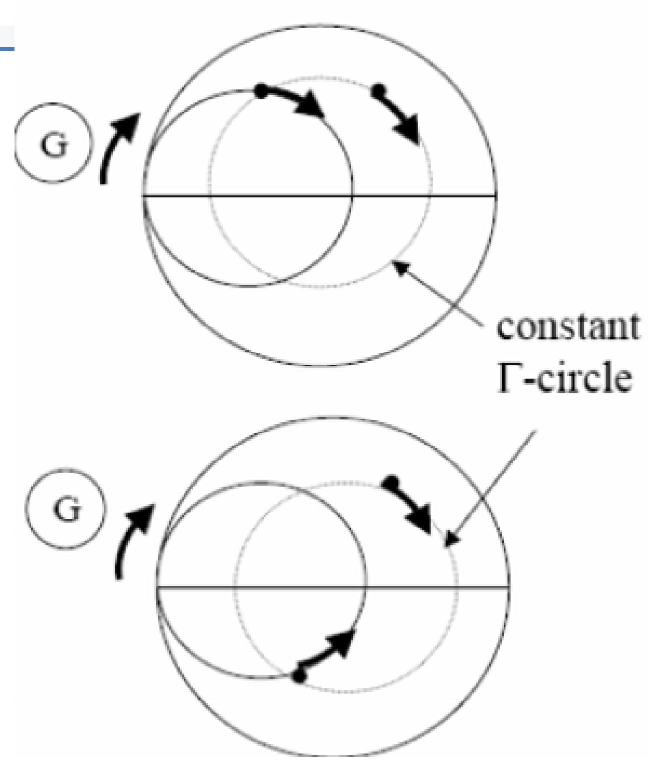
- Adding series and shunt elements



▪ Theory

- Design parameters: *d*, *l*



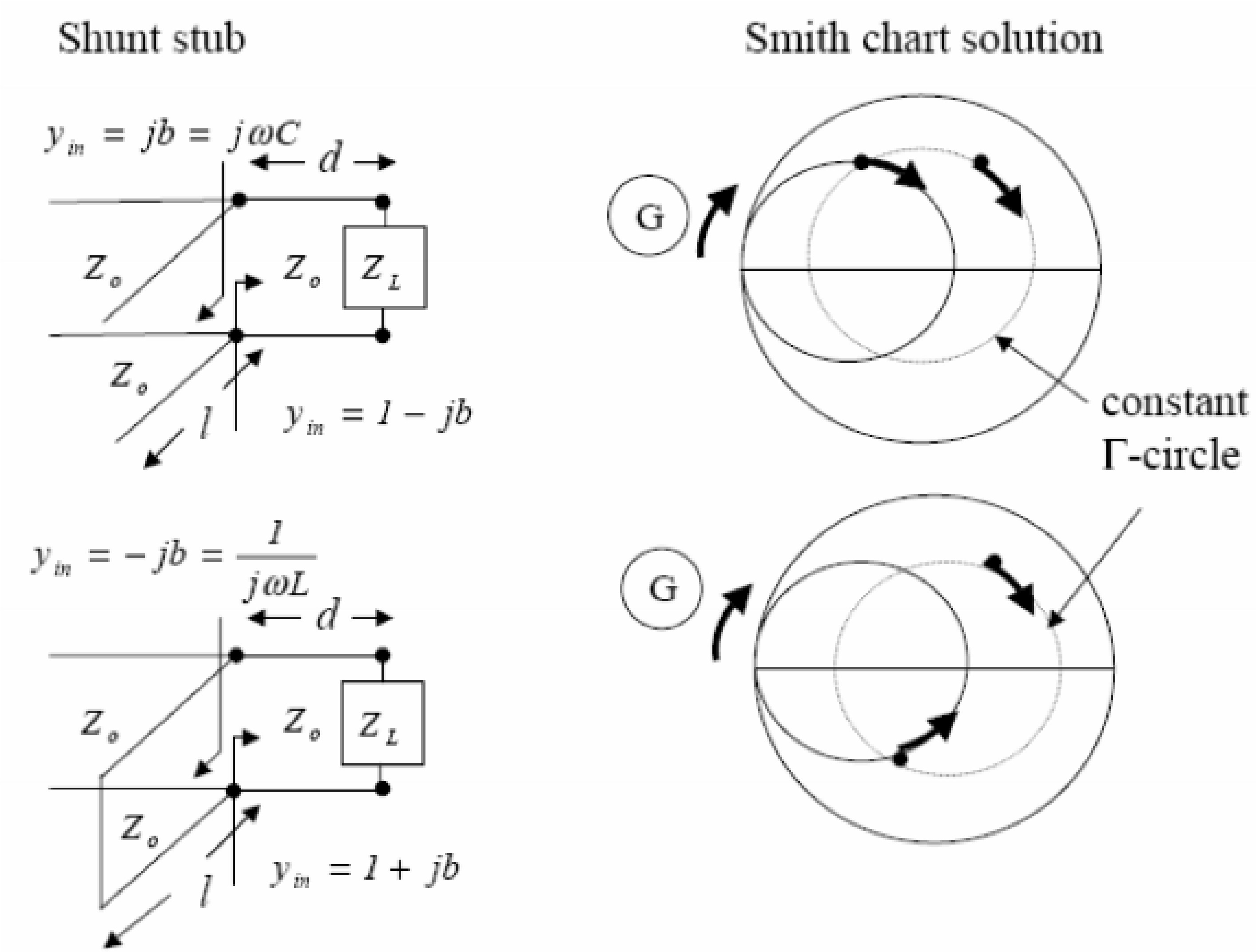


The location d is chosen so that the admittance $Y(-d)$ looking toward the load is $Y_0 + jB$ ($Y_0 = \frac{1}{Z_0}$). The length l of the shorted stub is chosen so that its admittance is $-jB$. Hence, when the stub is connected in parallel to the transmission line at $z = -d$, the impedance $Z_{in} = Z_0$, so that matching condition is achieved.

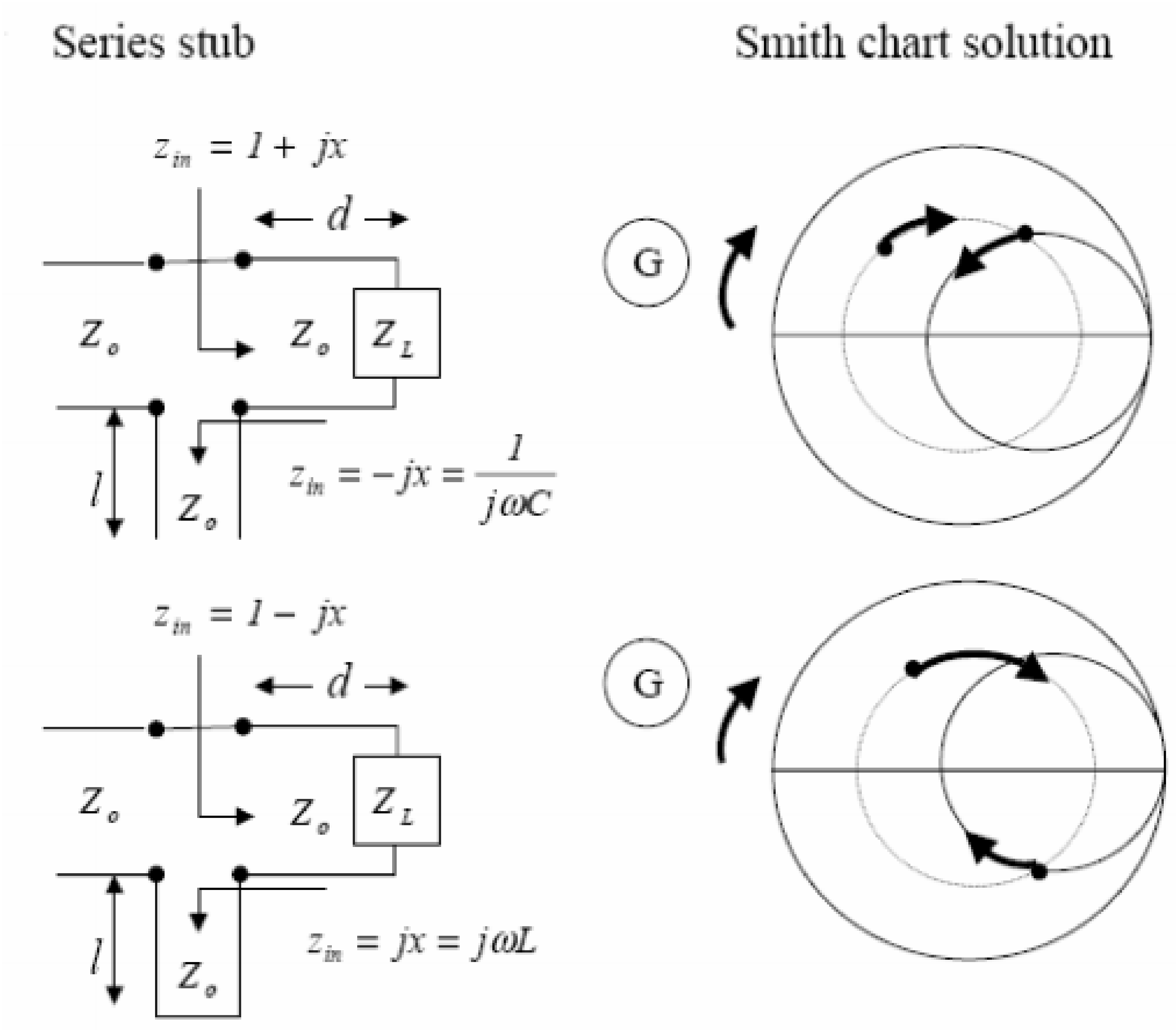
$$Z_s = jZ_0 \tan \beta l, \qquad Z_{op} = -jZ_0 \cot \beta l,$$

$$Y_s = -jY_0 \cot \beta l. \qquad Y_{op} = jY_0 \tan \beta l.$$

- For some transmission lines, shunt stubs are easier to implement.



- For some transmission lines, series stubs are easier to implement.



- Example

$$Z_L = 100 + j80\Omega$$

$$Z_i = 50\Omega$$

$$f = 2 \text{ GHz}$$

$$z_L = \frac{Z_L}{Z_0} = 2 + j1.6$$

$z = 1 + jx$ circle

$$d_1 = 0.328 - 0.208 = 0.120\lambda$$

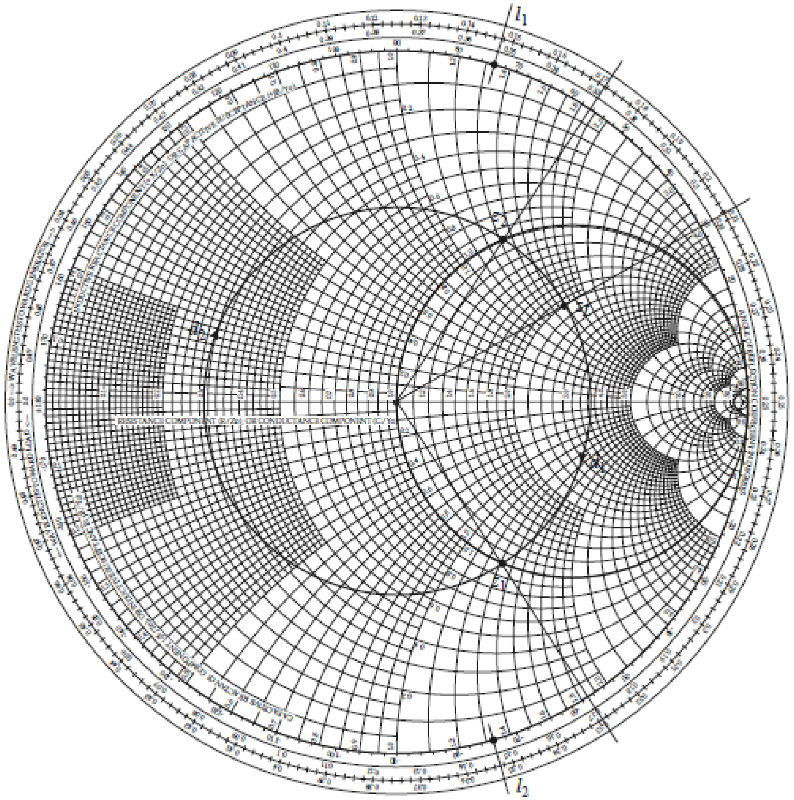
$$d_2 = (0.5 - 0.208) + 0.172 = 0.463\lambda$$

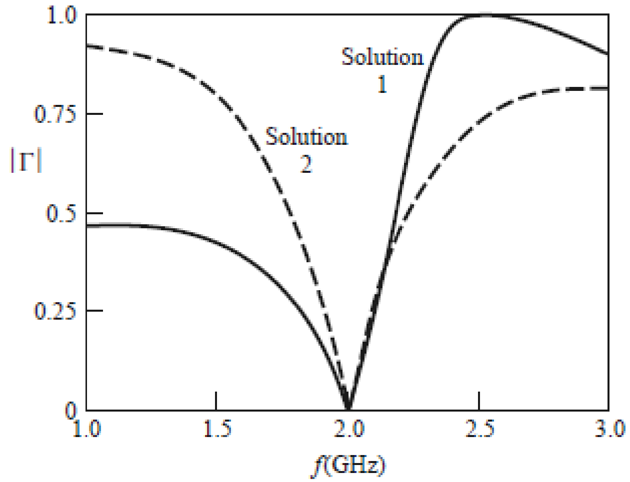
$$z_1 = 1 - j1.33$$

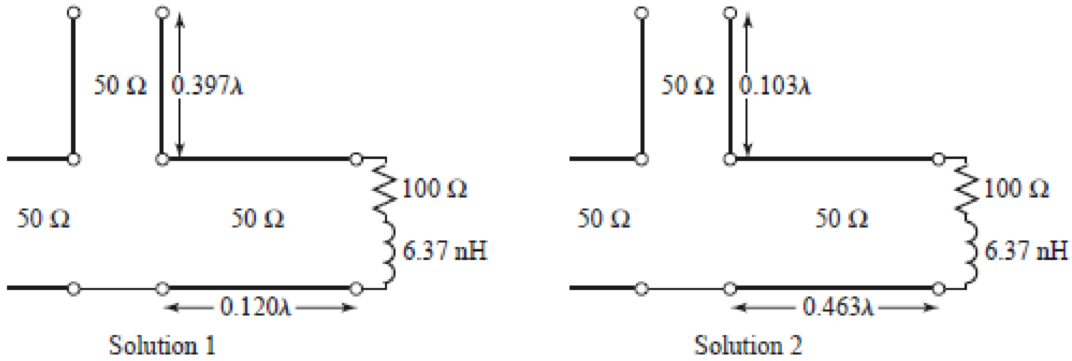
$$z_2 = 1 + j1.33$$

$\ell_1 = 0.397\lambda$ (open stub with $Z_0 = 50\Omega$)

$\ell_2 = 0.103\lambda$ (open stub with $Z_0 = 50\Omega$)

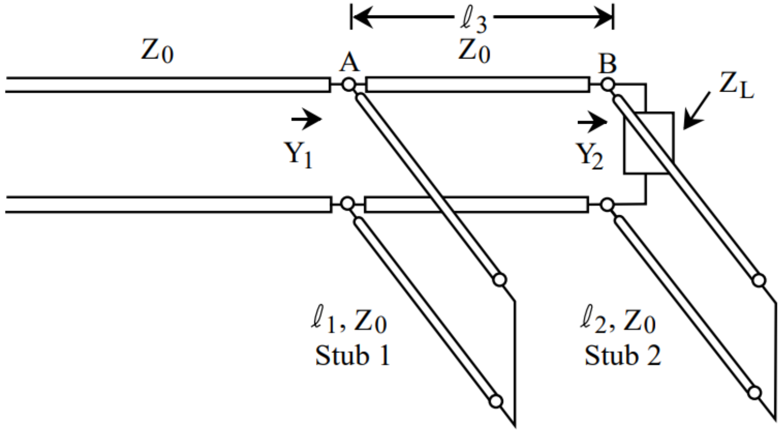


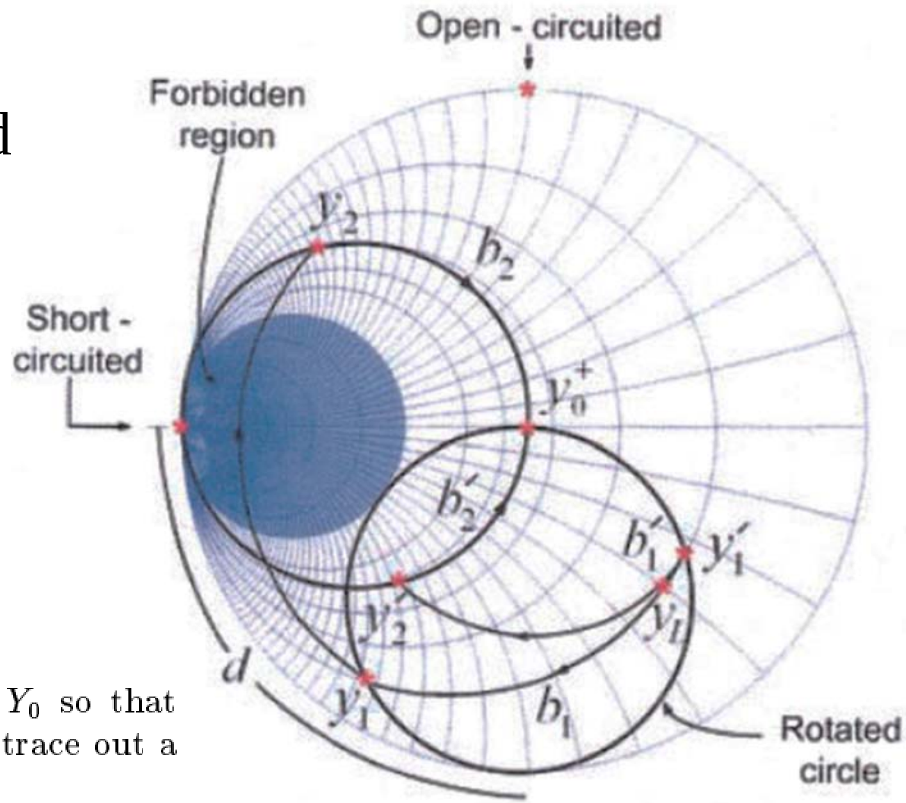




3. Double-stub Matching: adjustable

- Theory $\ell_3 = \lambda/8\ (90°):$ fixed





(1) In order to have a matched circuit, we should have $Y_1 = Y_0$ so that $Y_{n1} = 1$. However, if we change l_1, the possible values of Y_{n1} trace out a circle C_1 as shown.

(2) If Y_{nL} is as shown, by changing l_2, the possible values of Y_{n2} trace out a circle C_2 as shown.

(3) When l_3 is added, all the possible values of Y_{n1} at A is transformed to B by a rotation according to the length of l_3. This constitute a circle C_3 which is all the possible values of Y_{n2} obtained from Y_{n1}. There are only two points, P and Q that the two circles C_2 and C_3 intersect. If we pick P, then this point should correspond to the value of Y_{n2}.

$$Y_{n2} = Y_{nl} + Y_{nstub2} \tag{25.1}$$

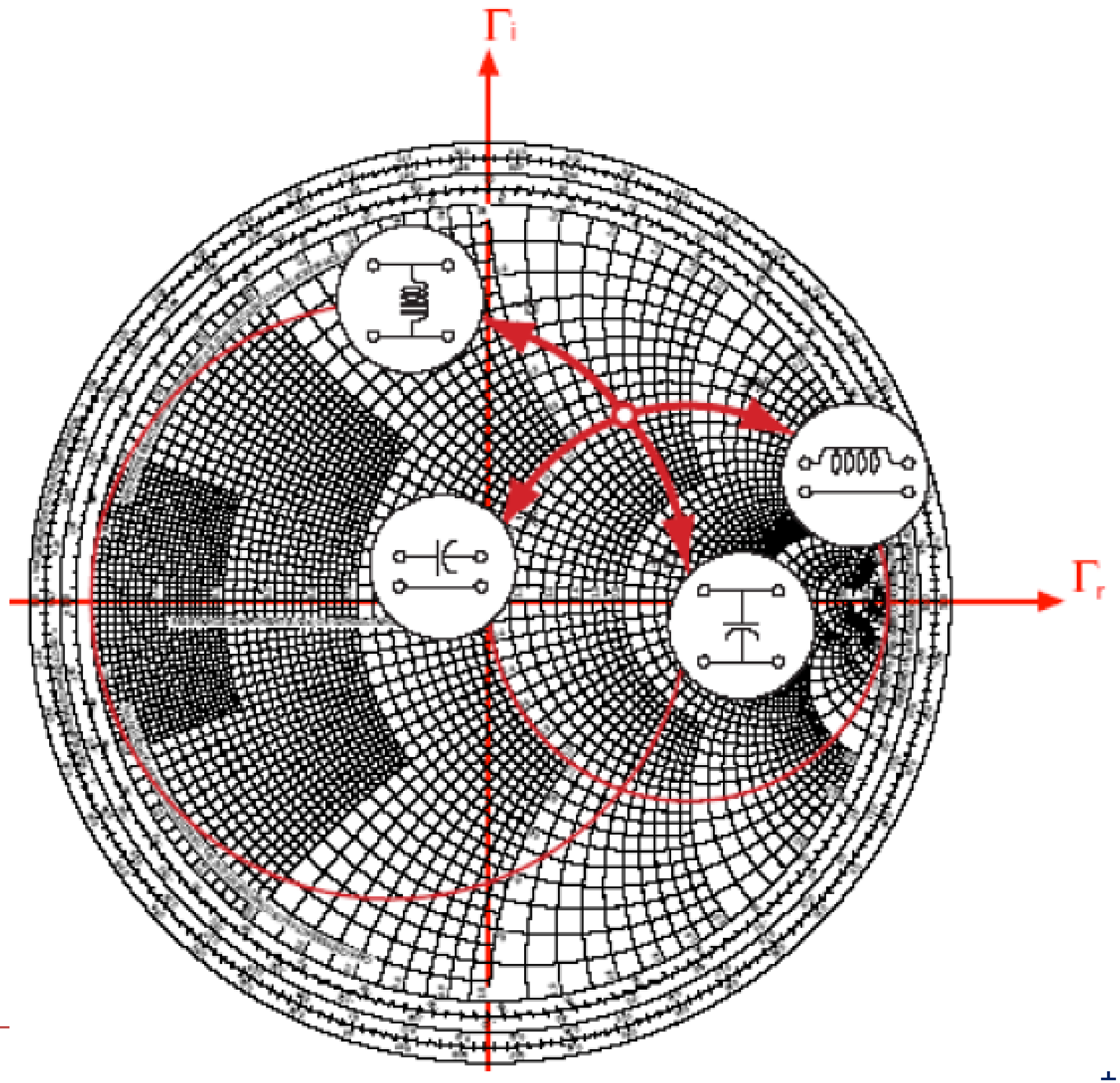
We can figure out Y_{nstub2} and hence the length l_2.

(4) The length l_3 rotates the point P to the point R. Then R has the impedance $Y_{n1} - Y_{nstub1} = 1 - Y_{nstub1}$. We can figure out Y_{nstub1} from the Smith Chart and hence the length l_1.

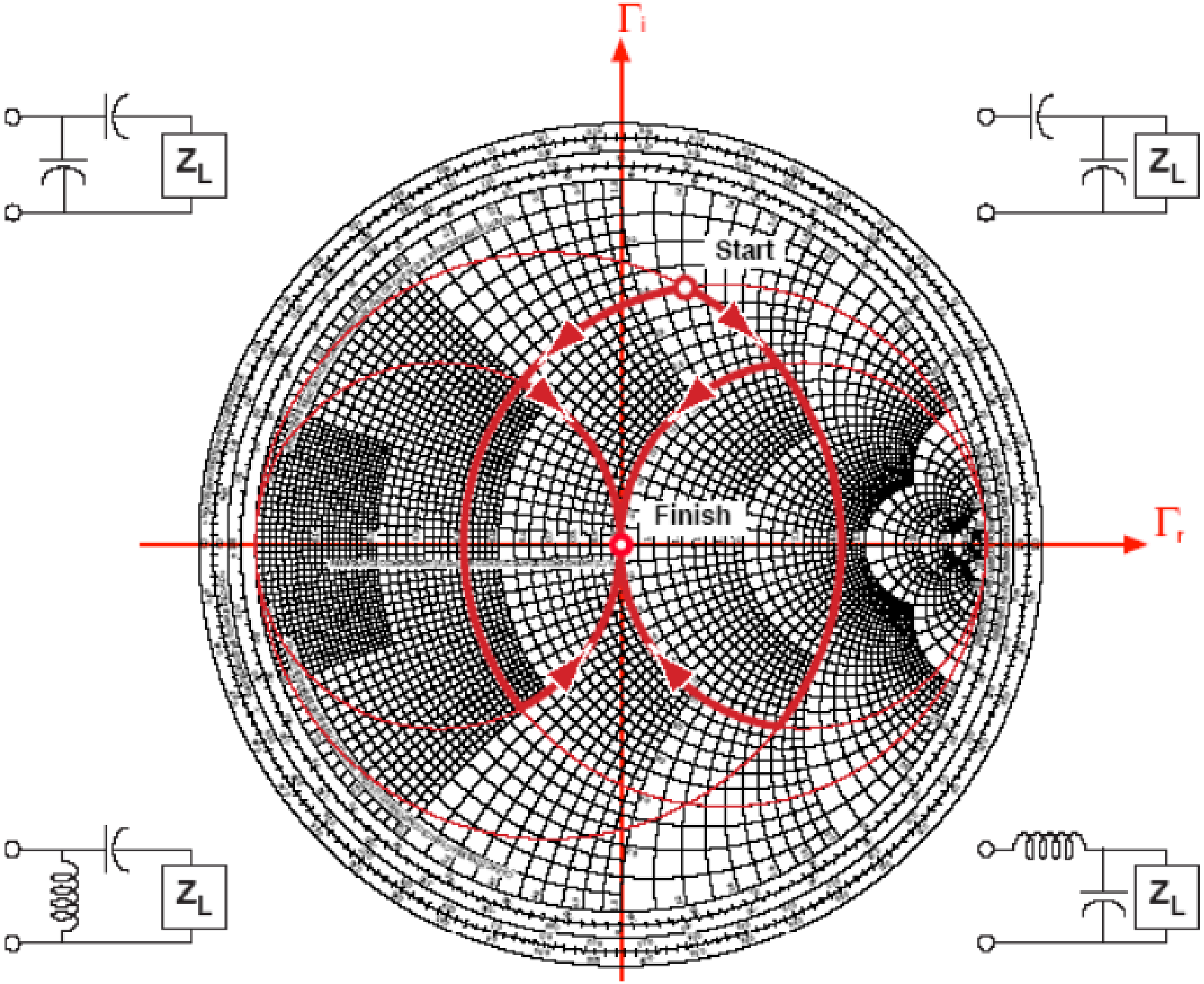
4. *LC* Matching

- Theory

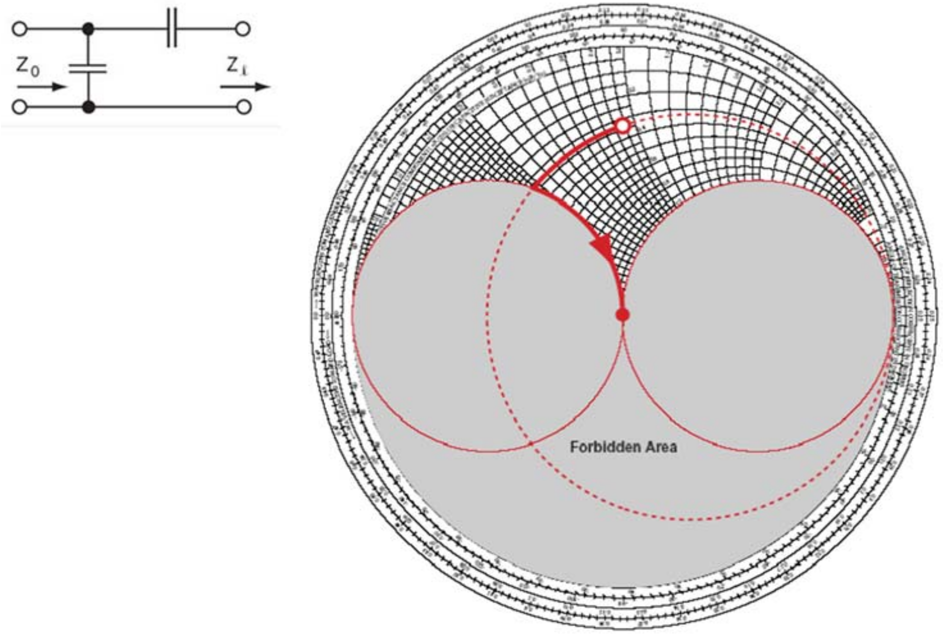
- Adding shunt or series *L* and *C*

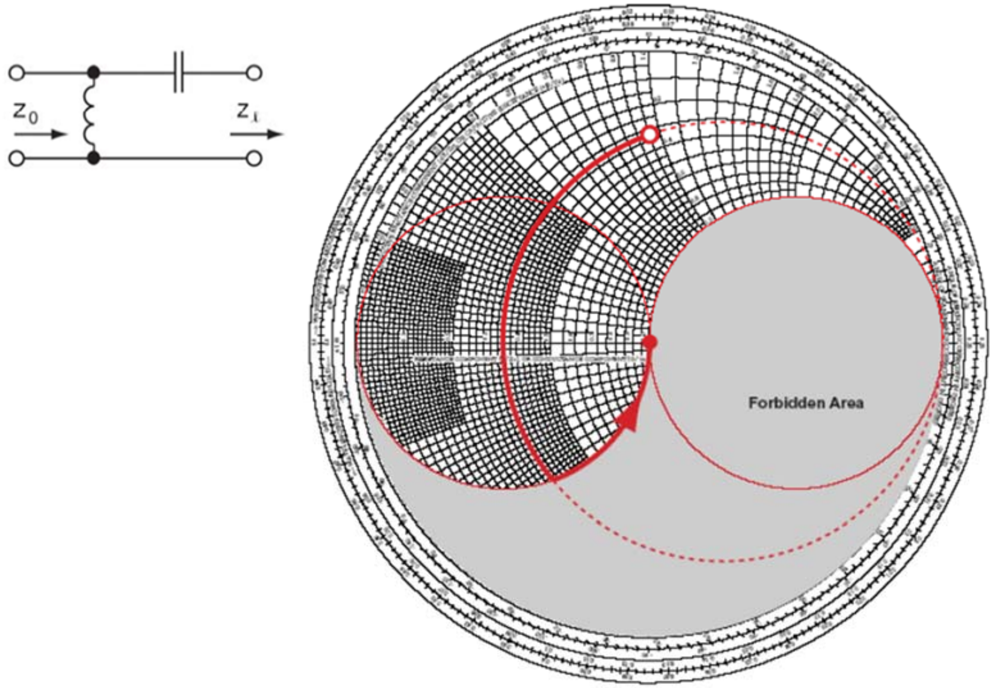


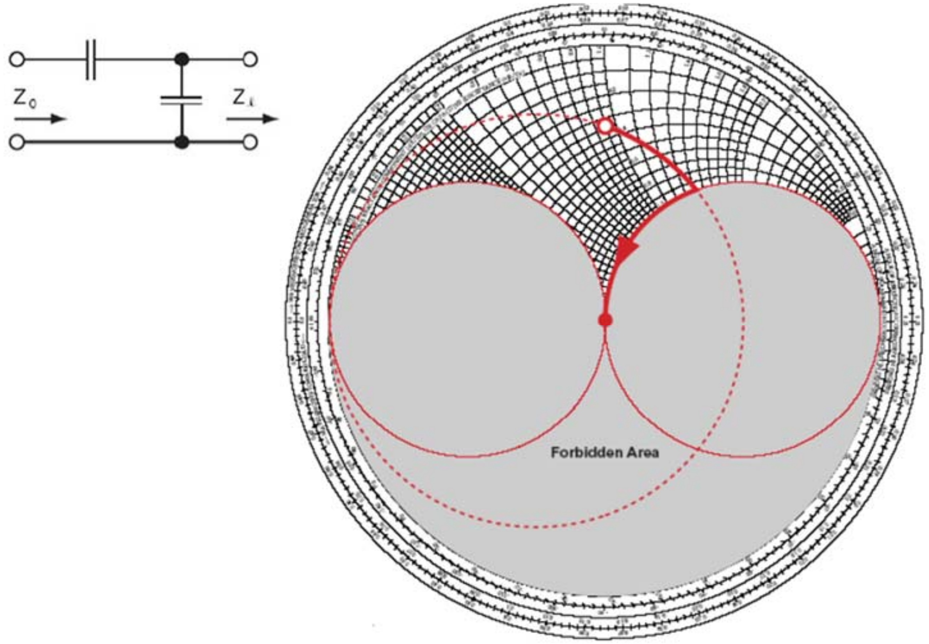
- 4 solutions

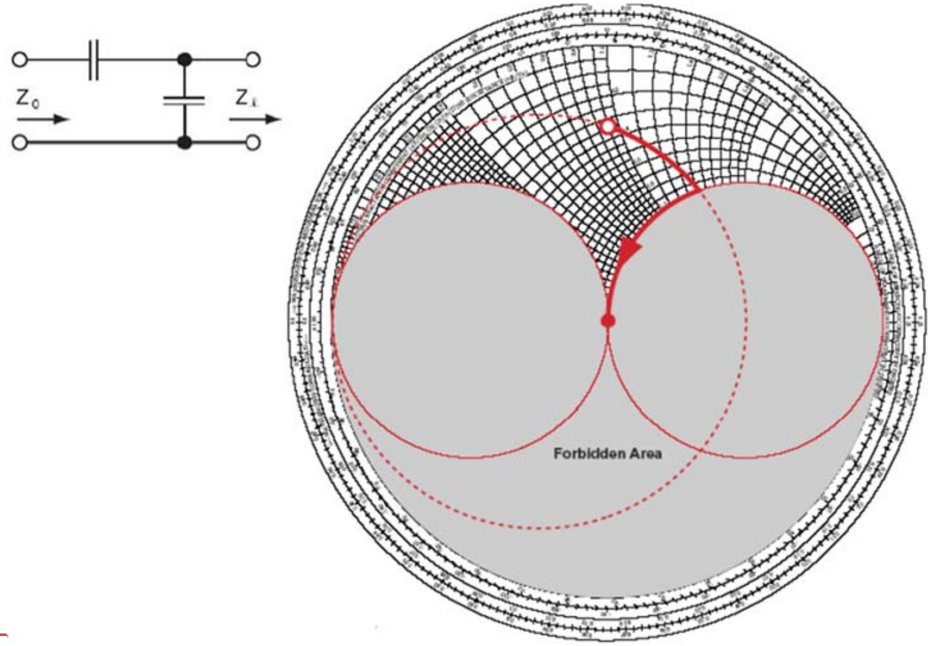


- 4 solutions

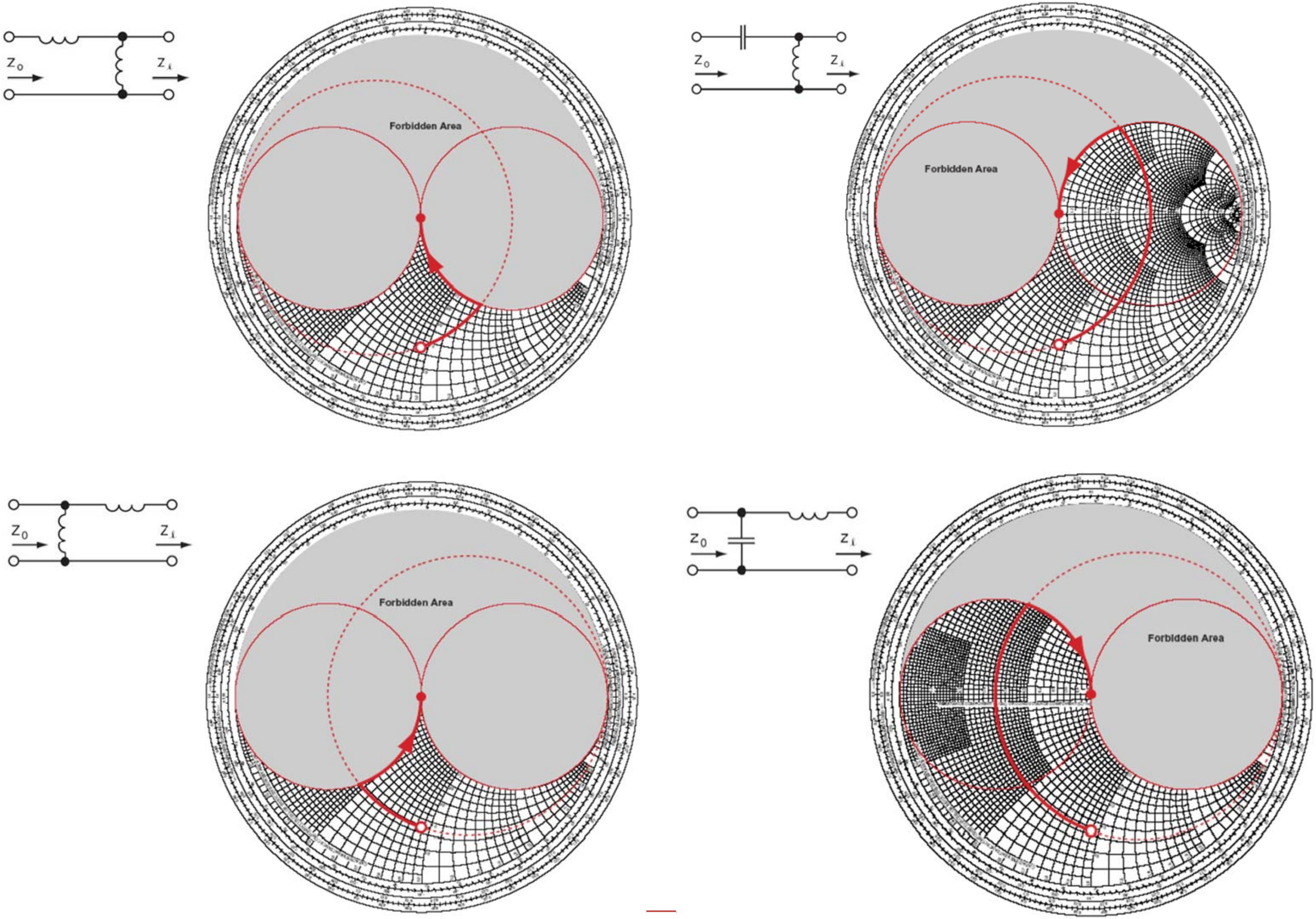

Z0
Zi
Forbidden Area


Z0
Zi
Forbidden Area

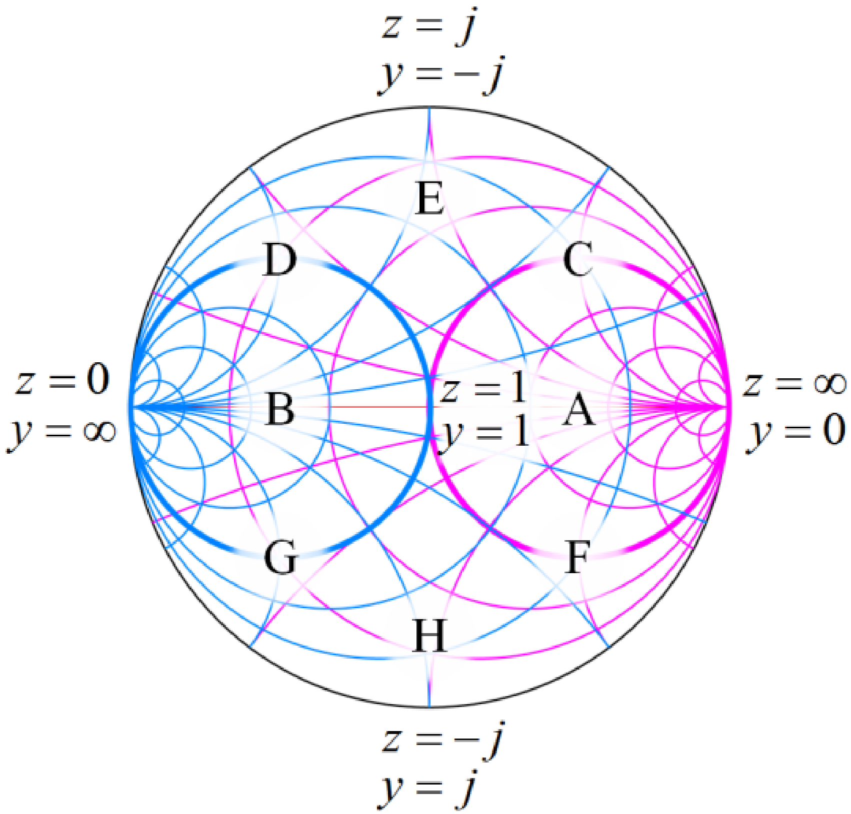

Z0
Zi
Forbidden Area

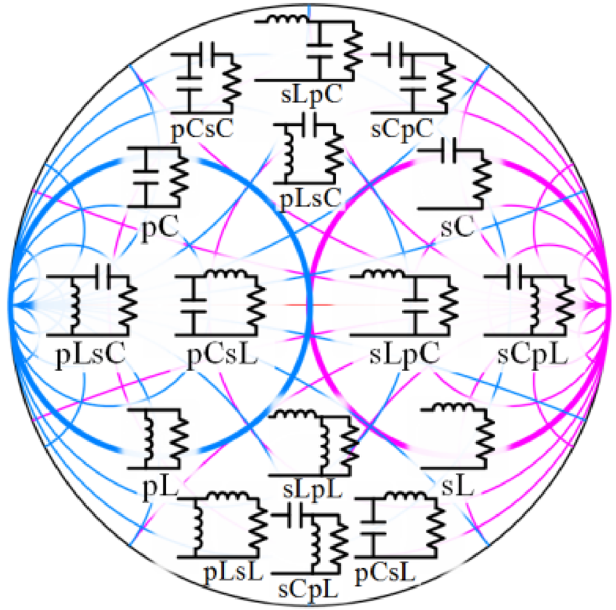

Z0
Zi
Forbidden Area

- 4 solutions


Z0
Zl
Forbidden Area
Z0
Zl
Forbidden Area
Z0
Zl
Forbidden Area
Z0
Zl
Forbidden Area

- Number of solutions vs load impedance

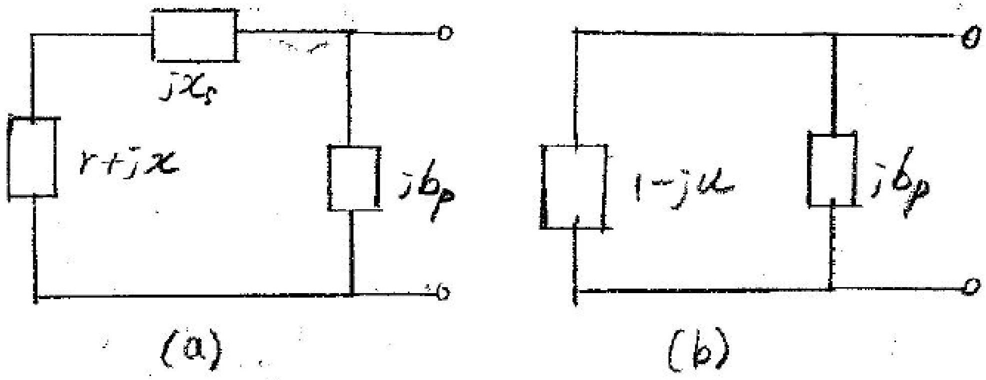




Region	Range of Normalized Port Impedance & Admittance	Available Network Topologies
A	$\mathrm{Re}\{z_P\} > 1$	sCpL and sLpC
B	$\mathrm{Re}\{y_P\} > 1$	pLsC and pCsL
C	$\mathrm{Re}\{z_P\} = 1$ and $\mathrm{Im}\{z_P\} > 0$	sC
D	$\mathrm{Re}\{y_P\} = 1$ and $\mathrm{Im}\{z_P\} > 0$	pC
E	$\mathrm{Re}\{z_P\} < 1, \mathrm{Re}\{y_P\} < 1$ and $\mathrm{Im}\{z_P\} > 0$	sCpC, sLpC, pCsC and pLsC
F	$\mathrm{Re}\{z_P\} = 1$ and $\mathrm{Im}\{z_P\} < 0$	sL
G	$\mathrm{Re}\{y_P\} = 1$ and $\mathrm{Im}\{z_P\} < 0$	pL
H	$\mathrm{Re}\{z_P\} < 1, \mathrm{Re}\{y_P\} < 1$ and $\mathrm{Im}\{z_P\} < 0$	sLsL, sCpL, pLsL and pCsL

- *LC* matching equations

- Series-shunt topology



$Z = R + jX$ (impedance to be matched to Z_0)

$$r + jx = z = \frac{Z}{Z_0}$$

If $r \le 1$, then use the following matching elements

$$r + j(x + x_s) = \frac{1}{1 - ju} = \frac{1}{1+u^2} + j\frac{u}{1+u^2}$$

$$u = \pm\sqrt{(1/r)-1},\ u_1 = \sqrt{(1/r)-1},\ u_2 = -\sqrt{(1/r)-1}$$

$$r = 1/(1+u^2),\ u/(1+u^2) = x + x_s = ru \rightarrow x_s = -x + ru$$

First solution:

$$b_{p1} = u_1 = \frac{\omega C_{p1}}{Y_0},\ Y_0 = \frac{1}{Z_0}$$

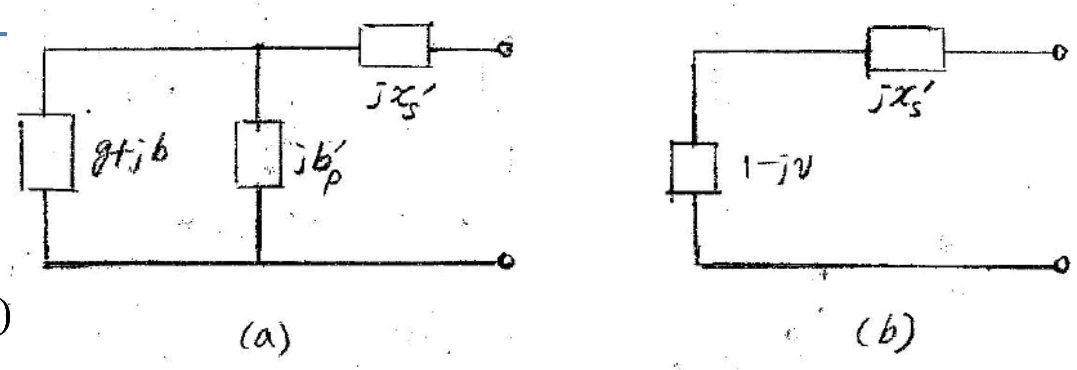
$$x_{s1} = -x + ru_1 = \begin{cases} \dfrac{\omega L_{s1}}{Z_0}, & x_{s1} \ge 0 \\ -\dfrac{1}{Z_0(\omega C_{s1})}, & x_{s1} < 0 \end{cases}$$

Second solution:

$$b_{p2} = u_2 = -\sqrt{1/r - 1} = -\frac{1}{Y_0(\omega L_{p1})}$$

$$x_{s2} = -x + ru_2 = \begin{cases} \dfrac{\omega L_{s2}}{Z_0}, & x_{s2} \geq 0 \\ -\dfrac{1}{Z_0(\omega C_{s2})}, & x_{s2} < 0 \end{cases}$$

- Shunt-series topology



$Y = G + jB = 1/Z$ (admittance to be matched to Y_0)

$$g + jb = \frac{Y}{Y_0},\ Y_0 = 1/Z_0$$

$$g \le 1$$

$$g + j(b + b'_p) = \frac{1}{1 - jv} = \frac{1}{1+v^2} + j\frac{v}{1+v^2}$$

$$v = \pm\sqrt{1/g - 1},\ v_1 = \sqrt{1/g - 1},\ v_2 = -\sqrt{1/g - 1}$$

$$b + b'_p = gv \rightarrow b' = -b + gv$$

First solution:

$$x'_{s1} = v_1 = \frac{\omega L'_{s1}}{Z_0}$$

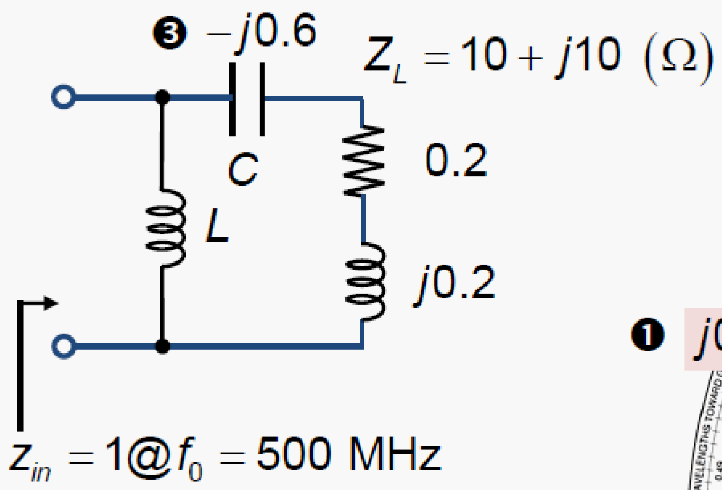
$$b'_{p1} = -b + gv_1 = \begin{cases} \dfrac{\omega C'_{p1}}{Y_0}, & b'_{p1} \ge 0 \\ -\dfrac{1}{Y_0(\omega L'_{p1})}, & b'_{p1} < 0 \end{cases}$$

Second solution:

$$x'_{s2} = v_2 = -\sqrt{1/g - 1} = -\frac{1}{Z_0(\omega C'_{s1})}$$

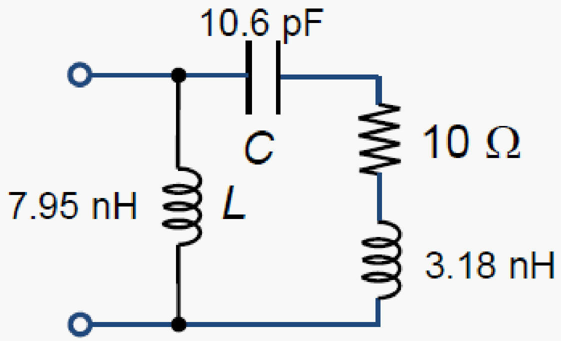
$$b'_{p2} = -b + g v_2 = \begin{cases} \dfrac{\omega C'_{p2}}{Y_0}, & b'_{p2} \geq 0 \\ -\dfrac{1}{Y_0(\omega L'_{p2})}, & b'_{p2} < 0 \end{cases}$$

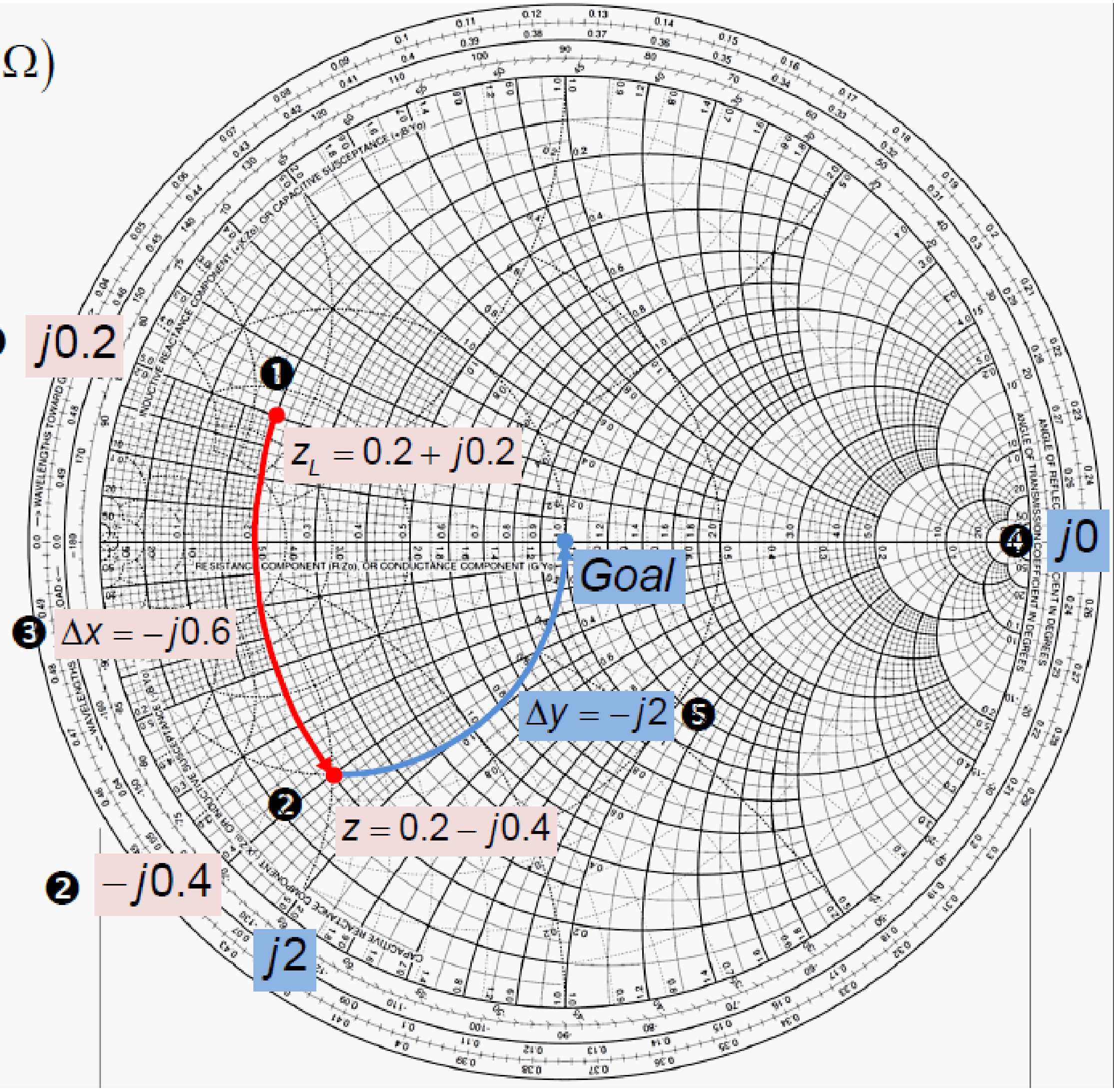
- Example



$(2\pi f_0 C)^{-1} = 0.6 \times 50\Omega = 30 \Rightarrow C = 10.6$ pF

$(2\pi f_0 L)^{-1} = 2 \times \frac{1}{50\Omega} = 0.04 \Rightarrow L = 7.95$ nH

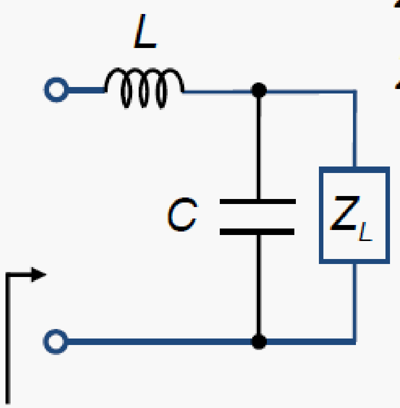




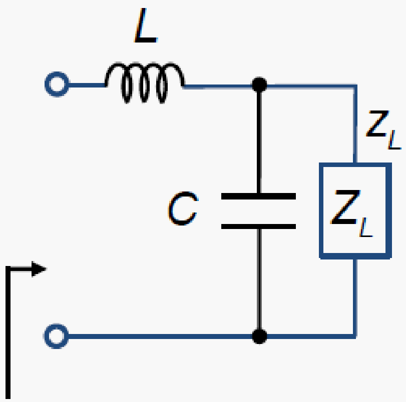
- Example

$Z_{ref} = 100\ \Omega$

$Z_L = 100 + j100\ \Omega$

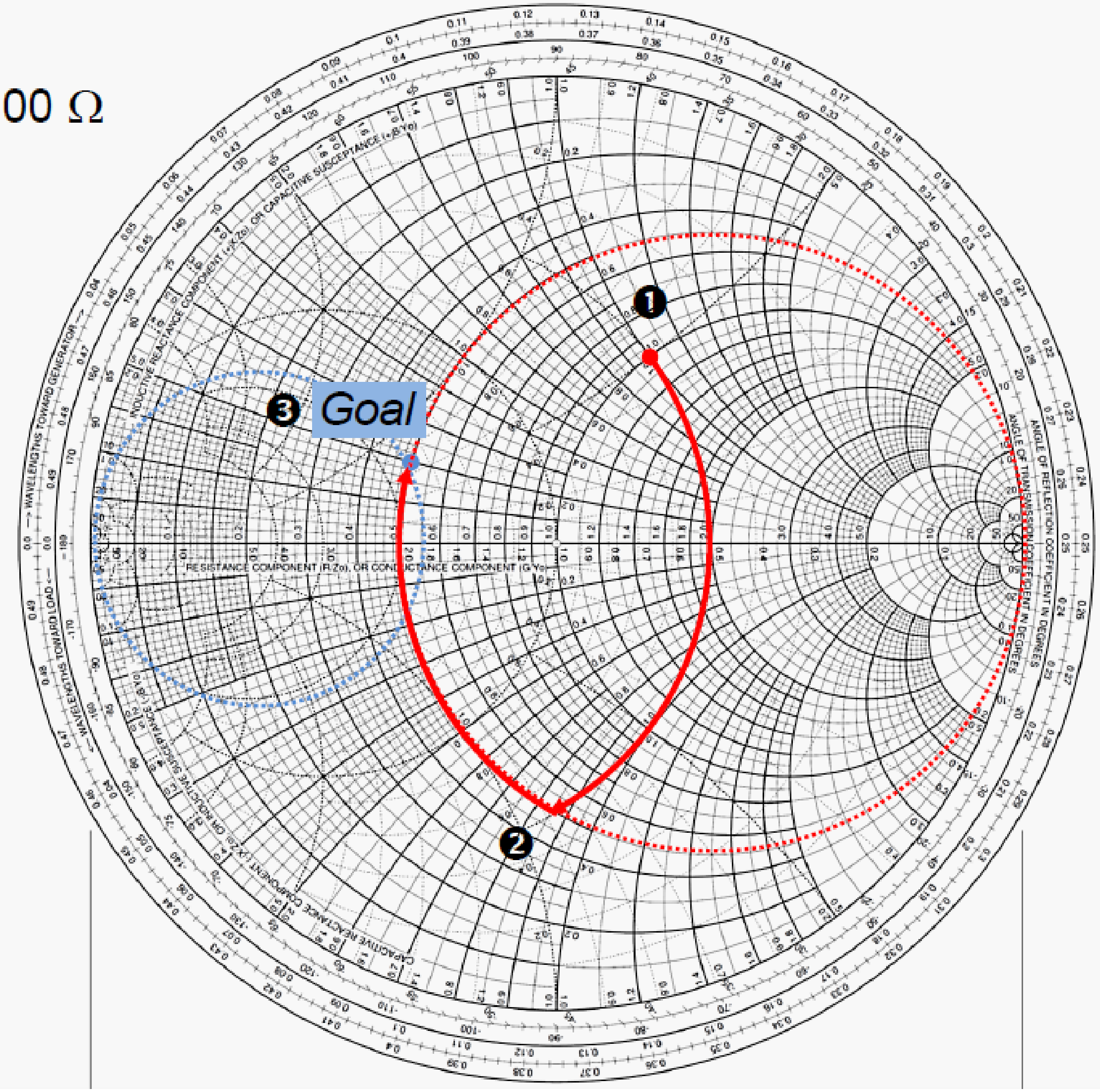


$Z_{in} = 50 + j20\ \Omega$



$Z_L = 1 + j1\ \Omega$

$Z_{in} = 0.5 + j0.2\ \Omega$



5. Coding Example

- Example

- Match impedance from $R + jX$ to Z_0 using LC-network at frequency f_0.

- Find all the solutions.

- For each solution, find the reflection coefficient in dB from 0.8 f_0 to 1.2 f_0 at $0.01f_0$ interval.

```
# LC-matching network design
# Input data:
#  z0: impedance after matching (ohm) (real)
#  rr, xx: real and imaginary parts of the impedance to be matched to z0
#  f0: frequency of matching (Hz)
# Output data:
#  When the load impedance outside r=1 circle:
#  Series element first:
#   1st solution:
#     Ls or Cs: series element
#     Cp: parallel element
#   2nd solution:
#     Ls or Cp: series element
#     Lp: parallel element
#  Parllel element first:
#   1st solution:
#     Lp or Lp: parallel element
#     Ls or Cs: series element
#   2nd solution:
#     Lp or Cp: parallel element
#     Ls or Cs: series element
```

```
import cmath
import math
pi=3.141593; rad=pi/180
while True:
  z0=float(input('Impedance after matching: Z0 (ohm) (real) = '))
  rr,xx=map(float, input('Impedance to be matched: R, X (ohm) = ').split())
  f0=float(input('Frequency of matching: f0 (Hz) = '))
  r=rr/z0; x=xx/z0
  y0=1/z0
  w0=2*pi*f0

# Load impedance outside r=1 circle
  if r <= 1:
    u=math.sqrt(1/r-1)
    xs=-x+r*u
    print('SERIES ELEMENT FIRST:')

# 1st solution:
    print('1ST SOLUTION:')
    if xs >= 0:
      Ls=z0*xs/w0; iLc=1
      print('  Ls = ',Ls)
```

```
else:
  cs=-1/(z0*w0*xs); iLc=-1
  print('  Cs = ',cs)
cp=y0*u/w0
print('  Cp = ',cp)    iprint=int(input('Print frequency response of the 1st solution (0: no, 1:yes) = '))
if iprint ==1:
  f=0.8*f0-0.02*f0
  for i in range(21):
    f=f+0.02*f0
    w=2*pi*f
    if iLc == 1:
      z=complex(rr,xx)+complex(0,w*Ls)
    else:
      z=complex(rr,xx)+complex(0,-1/(w*cs))
    y=1/z+complex(0,w*cp)
    z=1/y
    gamma=(z-z0)/(z+z0)
    print('  f(Hz)=',f,', mag(S11)(dB)=',20.*math.log10(abs(gamma)),', phase(S11)(deg)=',cmath.phase(gamma)/rad)
```

```
iprint=int(input('Print frequency response of the 2nd solution (0: no, 1:yes) = '))
if iprint ==1:
  f=0.8*f0-0.02*f0
  for i in range(21):
    f=f+0.02*f0
    w=2*pi*f
    if iLc == 1:
      z=complex(rr,xx)+complex(0,w*Ls)
    else:
      z=complex(rr,xx)+complex(0,-1/(w*cs))
    y=1/z+complex(0,-1/(w*Lp))
    z=1/y
    gamma=(z-z0)/(z+z0)
    print('  f(Hz)=',f,', mag(S11)(dB)=',20.*math.log10(abs(gamma)),', phase(S11)(deg)=',cmath.phase(gamma)/rad)
```

```
# 2nd solution:
    xs=-x-r*u
    print('2ND SOLUTION:')
    if xs >= 0:
      Ls=z0*xs/w0; iLc=1
      print('  Ls = ',Ls)
    else:
      cs=-1/(z0*w0*xs); iLc=-1
      print('  Cs = ',cs)
    if u == 0: Lp=1e30
    if u > 0: Lp=1/(y0*w0*u)
    print('  Lp = ',Lp)

    iprint=int(input('Print frequency response of the 2nd solution (0: no, 1:yes) = '))
    if iprint ==1:
      f=0.8*f0-0.02*f0
      for i in range(21):
        f=f+0.02*f0
        w=2*pi*f
        if iLc == 1:
          z=complex(rr,xx)+complex(0,w*Ls)
        else:
          z=complex(rr,xx)+complex(0,-1/(w*cs))
        y=1/z+complex(0,-1/(w*Lp))
        z=1/y
        gamma=(z-z0)/(z+z0)
        print('  f(Hz)=',f,', mag(S11)(dB)=',20.*math.log10(abs(gamma)),', phase(S11)(deg)=',cmath.phase(gamma)/rad)
```

```
# Parallel element first:
  yy=1/complex(rr,xx)
  y=yy/y0
  g=y.real; b=y.imag
  if g <= 1:
    u=math.sqrt(1/g-1)
# 1st solution
    bp=-b+g*u
    print('PARALLEL ELEMENT FIRST:')
    print('1ST SOLUTION:')
    cp=0; Lp=0
    if bp >= 0:
      cp=y0*bp/w0; iLc=-1
      print('  Cp = ',cp)
    else:
      Lp=-1/(y0*w0*bp); iLc=1
      print('  Lp = ',Lp)
    Ls=z0*u/w0
    print('  Ls = ',Ls)
```

```
iprint=int(input('Print frequency response of the 1st solution (0: no, 1:yes) = '))
if iprint ==1:
  f=0.8*f0-0.02*f0
  for i in range(21):
    f=f+0.02*f0
    w=2*pi*f
    if iLc == 1:
      y=1/complex(rr,xx)+complex(0,-1/(w*Lp))
    else:
      y=1/complex(rr,xx)+complex(0,w*cp)
    z=1/y+complex(0,w*Ls)
    gamma=(z-z0)/(z+z0)
    print('  f(Hz)=',f,', mag(S11)(dB)=',20.*math.log10(abs(gamma)),', phase(S11)(deg)=',cmath.phase(gamma)/rad)

bp=-b-g*u
print('2ND SOLUTION:')
if bp >= 0:
  cp=y0*bp/w0; iLc=-1
  print('  Cp = ',cp)
else:
  Lp=-1/(y0*w0*bp); iLc=1
  print('  Lp = ',Lp)
if u == 0: cs=1e30
if u > 0: cs=1/(z0*w0*u)
```

```
print('  Cs = ',cs)

iprint=int(input('Print frequency response of the 2nd solution (0: no, 1:yes) = '))
if iprint == 1:
  f=0.8*f0-0.02*f0
  for i in range(21):
    f=f+0.02*f0
    w=2*pi*f
    if iLc == 1:
      y=1/complex(rr,xx)+complex(0,-1/(w*Lp))
    else:
      y=1/complex(rr,xx)+complex(0,w*cp)
    z=1/y+complex(0,-1/(w*cs))
    gamma=(z-z0)/(z+z0)
    print('  f(Hz)=',f,', mag(S11)(dB)=',20.*math.log10(abs(gamma)),', phase(S11)(deg)=',cmath.phase(gamma)/rad)
```

'''
Impedance after matching: Z0 (ohm) (real) =
50
Impedance to be matched: R, X (ohm) =
20 50
Frequency of matching: f0 (Hz) =
100e6

SERIES ELEMENT FIRST:
1ST SOLUTION:
Cs = 6.24012097548766e-11
Cp = 3.898483576298995e-11
Print frequency response of the 1st solution (0: no, 1:yes) =
1
f(Hz)= 80000000.0 , mag(S11)(dB)= -14.359119835498817 , phase(S11)(deg)= 151.05438246985145
f(Hz)= 82000000.0 , mag(S11)(dB)= -15.255989918484328 , phase(S11)(deg)= 148.77934147746228
f(Hz)= 84000000.0 , mag(S11)(dB)= -16.262121464434774 , phase(S11)(deg)= 146.55255730859523
f(Hz)= 86000000.0 , mag(S11)(dB)= -17.40660788796544 , phase(S11)(deg)= 144.37104144793682
f(Hz)= 88000000.0 , mag(S11)(dB)= -18.73194421414794 , phase(S11)(deg)= 142.23212001912736
f(Hz)= 90000000.0 , mag(S11)(dB)= -20.303891565862063 , phase(S11)(deg)= 140.1334069283544
f(Hz)= 92000000.0 , mag(S11)(dB)= -22.23248813620591 , phase(S11)(deg)= 138.0727789623073
f(Hz)= 94000000.0 , mag(S11)(dB)= -24.72386901005052 , phase(S11)(deg)= 136.0483526716371
f(Hz)= 96000000.0 , mag(S11)(dB)= -28.240630826347967 , phase(S11)(deg)= 134.05846288355445
f(Hz)= 98000000.0 , mag(S11)(dB)= -34.25860390338928 , phase(S11)(deg)= 132.1016427017397
f(Hz)= 100000000.0 , mag(S11)(dB)= -321.23599479677745 , phase(S11)(deg)= 89.99999007607968
f(Hz)= 102000000.0 , mag(S11)(dB)= -34.26101439435095 , phase(S11)(deg)= -51.717755783198555
f(Hz)= 104000000.0 , mag(S11)(dB)= -28.24559595707976 , phase(S11)(deg)= -53.58245794296694
f(Hz)= 106000000.0 , mag(S11)(dB)= -24.731678109531835 , phase(S11)(deg)= -55.41832991246969
f(Hz)= 108000000.0 , mag(S11)(dB)= -22.243577797565234 , phase(S11)(deg)= -57.226103150951346
f(Hz)= 110000000.0 , mag(S11)(dB)= -20.318848825505587 , phase(S11)(deg)= -59.006405645466316
f(Hz)= 112000000.0 , mag(S11)(dB)= -18.75151088990369 , phase(S11)(deg)= -60.75977423241218
f(Hz)= 114000000.0 , mag(S11)(dB)= -17.431686161461258 , phase(S11)(deg)= -62.48666606480024
f(Hz)= 116000000.0 , mag(S11)(dB)= -16.293780814229443 , phase(S11)(deg)= -64.18746925989508
f(Hz)= 118000000.0 , mag(S11)(dB)= -15.295475531544497 , phase(S11)(deg)= -65.86251275704204
f(Hz)= 120000000.0 , mag(S11)(dB)= -14.407862645147123 , phase(S11)(deg)= -67.51207541208241

2ND SOLUTION:

Cs = 2.136454053067665e-11

Lp = 6.497472627164993e-08

Print frequency response of the 2nd solution (0: no, 1:yes) =

1

f(Hz)= 80000000.0 , mag(S11)(dB)= -5.743505645878209 , phase(S11)(deg)= 67.76154827617734
f(Hz)= 82000000.0 , mag(S11)(dB)= -6.717930712850675 , phase(S11)(deg)= 63.263621145107464
f(Hz)= 84000000.0 , mag(S11)(dB)= -7.837101130021475 , phase(S11)(deg)= 58.83248060373801
f(Hz)= 86000000.0 , mag(S11)(dB)= -9.127836782664819 , phase(S11)(deg)= 54.497546814356504
f(Hz)= 88000000.0 , mag(S11)(dB)= -10.629652215226193 , phase(S11)(deg)= 50.28418730024743
f(Hz)= 90000000.0 , mag(S11)(dB)= -12.404825747499245 , phase(S11)(deg)= 46.21291028923126
f(Hz)= 92000000.0 , mag(S11)(dB)= -14.559625965977084 , phase(S11)(deg)= 42.29903832214077
f(Hz)= 94000000.0 , mag(S11)(dB)= -17.29633261085185 , phase(S11)(deg)= 38.55281076558243
f(Hz)= 96000000.0 , mag(S11)(dB)= -21.073770243943187 , phase(S11)(deg)= 34.979813879945205
f(Hz)= 98000000.0 , mag(S11)(dB)= -27.364208561973623 , phase(S11)(deg)= 31.58161869016395
f(Hz)= 100000000.0 , mag(S11)(dB)= -312.7603929681086 , phase(S11)(deg)= -157.85852376128585
f(Hz)= 102000000.0 , mag(S11)(dB)= -27.9342127740094 , phase(S11)(deg)= -154.69973261084203
f(Hz)= 104000000.0 , mag(S11)(dB)= -22.20845456887482 , phase(S11)(deg)= -157.59327315102385
f(Hz)= 106000000.0 , mag(S11)(dB)= -18.98506364464915 , phase(S11)(deg)= -160.3314619097645
f(Hz)= 108000000.0 , mag(S11)(dB)= -16.786505887778528 , phase(S11)(deg)= -162.92244505153144
f(Hz)= 110000000.0 , mag(S11)(dB)= -15.148795865071632 , phase(S11)(deg)= -165.37475150210338
f(Hz)= 112000000.0 , mag(S11)(dB)= -13.864692709030958 , phase(S11)(deg)= -167.69699729062017
f(Hz)= 114000000.0 , mag(S11)(dB)= -12.823302456428294 , phase(S11)(deg)= -169.8976697442771
f(Hz)= 116000000.0 , mag(S11)(dB)= -11.958226413133248 , phase(S11)(deg)= -171.98497704100618
f(Hz)= 118000000.0 , mag(S11)(dB)= -11.226520867780552 , phase(S11)(deg)= -173.9667493558944
f(Hz)= 120000000.0 , mag(S11)(dB)= -10.598806468953352 , phase(S11)(deg)= -175.85037948788045

PARALLEL ELEMENT FIRST:

1ST SOLUTION:

Cp = 4.257015442329375e-11

Ls = 1.0968996264069072e-07

Print frequency response of the 1st solution (0: no, 1:yes) =

1

f(Hz)= 80000000.0 , mag(S11)(dB)= -8.768580323148644 , phase(S11)(deg)= -5.7598078491618825
f(Hz)= 82000000.0 , mag(S11)(dB)= -9.458720648327594 , phase(S11)(deg)= -8.343944581436547
f(Hz)= 84000000.0 , mag(S11)(dB)= -10.262245912069925 , phase(S11)(deg)= -11.00203100958283
f(Hz)= 86000000.0 , mag(S11)(dB)= -11.209281175569398 , phase(S11)(deg)= -13.736585206091737
f(Hz)= 88000000.0 , mag(S11)(dB)= -12.34348611441459 , phase(S11)(deg)= -16.54926758654197
f(Hz)= 90000000.0 , mag(S11)(dB)= -13.73190805236475 , phase(S11)(deg)= -19.44069496150933
f(Hz)= 92000000.0 , mag(S11)(dB)= -15.485974712698138 , phase(S11)(deg)= -22.4102501972515
f(Hz)= 94000000.0 , mag(S11)(dB)= -17.813288110797988 , phase(S11)(deg)= -25.45589874696871
f(Hz)= 96000000.0 , mag(S11)(dB)= -21.177955063901564 , phase(S11)(deg)= -28.574026510832677
f(Hz)= 98000000.0 , mag(S11)(dB)= -27.05731707766882 , phase(S11)(deg)= -31.759316030827506
f(Hz)= 100000000.0 , mag(S11)(dB)= -309.9578959675025 , phase(S11)(deg)= -26.565048247861856
f(Hz)= 102000000.0 , mag(S11)(dB)= -26.828705638293492 , phase(S11)(deg)= 141.69871629517206
f(Hz)= 104000000.0 , mag(S11)(dB)= -20.723437547099852 , phase(S11)(deg)= 138.36143247439935
f(Hz)= 106000000.0 , mag(S11)(dB)= -17.138165275259546 , phase(S11)(deg)= 134.99545309109098
f(Hz)= 108000000.0 , mag(S11)(dB)= -14.597931068081088 , phase(S11)(deg)= 131.6139812653321
f(Hz)= 110000000.0 , mag(S11)(dB)= -12.640714189789556 , phase(S11)(deg)= 128.23105664285575
f(Hz)= 112000000.0 , mag(S11)(dB)= -11.060636333469313 , phase(S11)(deg)= 124.86113313485247
f(Hz)= 114000000.0 , mag(S11)(dB)= -9.747591111276765 , phase(S11)(deg)= 121.51862706134231
f(Hz)= 116000000.0 , mag(S11)(dB)= -8.635435707797104 , phase(S11)(deg)= 118.21746943815134
f(Hz)= 118000000.0 , mag(S11)(dB)= -7.68100761439793 , phase(S11)(deg)= 114.9706955818288
f(Hz)= 120000000.0 , mag(S11)(dB)= -6.854301388026078 , phase(S11)(deg)= 111.79010014208826

```
2ND SOLUTION:
  Cp =  1.2310854384482514e-11
  Cs =  2.3092623713829633e-11
Print frequency response of the 2nd solution (0: no, 1:yes) =
1
  f(Hz)= 80000000.0 , mag(S11)(dB)= -12.130028500521304 , phase(S11)(deg)= -100.9494005651612
  f(Hz)= 82000000.0 , mag(S11)(dB)= -13.205523668873354 , phase(S11)(deg)= -103.29032886004269
  f(Hz)= 84000000.0 , mag(S11)(dB)= -14.393246193054116 , phase(S11)(deg)= -105.56885674626757
  f(Hz)= 86000000.0 , mag(S11)(dB)= -15.721370820860036 , phase(S11)(deg)= -107.7857552790605
  f(Hz)= 88000000.0 , mag(S11)(dB)= -17.231545048889675 , phase(S11)(deg)= -109.9421249494915
  f(Hz)= 90000000.0 , mag(S11)(dB)= -18.988753484642512 , phase(S11)(deg)= -112.03933372114096
  f(Hz)= 92000000.0 , mag(S11)(dB)= -21.10232756846927 , phase(S11)(deg)= -114.07896058360092
  f(Hz)= 94000000.0 , mag(S11)(dB)= -23.777763273503307 , phase(S11)(deg)= -116.06274512865194
  f(Hz)= 96000000.0 , mag(S11)(dB)= -27.47708300262246 , phase(S11)(deg)= -117.99254329349213
  f(Hz)= 98000000.0 , mag(S11)(dB)= -33.675604500855385 , phase(S11)(deg)= -119.87028914524294
  f(Hz)= 100000000.0 , mag(S11)(dB)= -322.9681959241423 , phase(S11)(deg)= 179.99998015215937
  f(Hz)= 102000000.0 , mag(S11)(dB)= -34.03132024968978 , phase(S11)(deg)= 56.52241897313586
  f(Hz)= 104000000.0 , mag(S11)(dB)= -28.18790969544466 , phase(S11)(deg)= 54.788900475958364
  f(Hz)= 106000000.0 , mag(S11)(dB)= -24.842484973577037 , phase(S11)(deg)= 53.09949024852919
  f(Hz)= 108000000.0 , mag(S11)(dB)= -22.519104710850296 , phase(S11)(deg)= 51.45225286790762
  f(Hz)= 110000000.0 , mag(S11)(dB)= -20.75510453374562 , phase(S11)(deg)= 49.84530074885597
  f(Hz)= 112000000.0 , mag(S11)(dB)= -19.344322813144284 , phase(S11)(deg)= 48.27680264621313
  f(Hz)= 114000000.0 , mag(S11)(dB)= -18.17673412813719 , phase(S11)(deg)= 46.74498984935099
  f(Hz)= 116000000.0 , mag(S11)(dB)= -17.18662785522155 , phase(S11)(deg)= 45.248160452352025
  f(Hz)= 118000000.0 , mag(S11)(dB)= -16.331595913539395 , phase(S11)(deg)= 43.78468203735162
  f(Hz)= 120000000.0 , mag(S11)(dB)= -15.582667319622029 , phase(S11)(deg)= 42.3529930641146
Impedance after matching: Z0 (ohm) (real) =
'''
```

Fin

(End)