Microwave Antennas Derived from the Cassegrain Telescope*

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Summary—A microwave antenna can be designed in the form of two reflecting dishes and a feed, based on the principle of the Cassegrain optical telescope. There are a variety of shapes and sizes available, all described by the same set of equations. The essential performance of a Cassegrain double-reflector system may be easily analyzed by means of the equivalent-parabola single-reflector concept.

Techniques are available for reducing the aperture blocking by the sub dish of the Cassegrain system: one method minimizes the blocking by optimizing the geometry of the feed and sub dish; other methods avoid the blocking by means of polarization-twisting schemes. The former method yields good performance in a simple Cassegrain antenna when the beamwidth is about 1° or less. The latter methods are available for any application not requiring polarization diversity, and an optimized set of polarization-operative surfaces has been developed for these twisting Cassegrain antennas.

Experimental results, presented for practical antennas of both types, illustrate the feasibility of these principles. A number of unusual benefits have been obtained in the various Cassegrain antenna designs, and additional interesting features remain to be exploited.

I. INTRODUCTION

For the design of an optical telescope, the Cassegrain double-reflector system has often been utilized [1]–[4]. Compared with the single-reflector type, it achieves a high magnification with a short focal length, and allows a convenient rear location for the observer.

Recently, a number of microwave antennas have been developed which employ double-reflector systems similar to that of the Cassegrain telescope. Each of these antennas has achieved one or more particular benefits not obtainable with the ordinary single-reflector type. While the various designs may differ from each other to a considerable degree, there are certain basic features which are common to all.

It is the purpose of this paper to outline the design principles and essential properties of the Cassegrain antenna, and to discuss its advantages and limitations. Some of the techniques available for minimizing its limitations are described, and experimental results illustrating the practical nature of two particular designs are presented. Finally, a number of interesting applications for the Cassegrain antenna are mentioned.

II. TELESCOPE VS ANTENNA

A Cassegrain telescope consists of two mirrors and an observing optical instrument, as indicated in Fig. 1. The primary mirror, which is a large concave mirror in the rear, collects the incoming light and reflects it toward the secondary mirror, which is a small convex mirror out in front. The secondary mirror then reflects the light back through a hole in the center of the primary mirror. When the incoming rays of light are parallel to the telescope axis, the final bundle of light rays is focused toward a point; at this location the observer places his eye or his camera.

The basic microwave antenna derived from the Cassegrain telescope is shown in Fig. 2. The microwave reflectors, which will be called the main dish and the sub dish, respectively, have surfaces similar in shape to those of the telescope. The microwave feed is a small antenna which, together with a transmitter or receiver, replaces the optical instrument of the telescope.

Analysis of the operation of a Cassegrain antenna system may be performed with the same semi-optical approximation commonly employed with an ordinary single-dish antenna. Usually the feed is sufficiently small so that the wave radiated by the feed can be described by the far-field pattern of the feed before reach-
ing the sub dish, and the wave incident on the sub dish appears to travel along the rays originating from a point centered on the feed. The sub dish, which must be large enough to intercept the useful portion of the feed radiation, ordinarily reflects this wave essentially according to ray optics. On reaching the main dish, the wave is again reflected according to ray optics; and because of the geometry of the antenna elements, the rays emerge parallel and the wavefront has the flat shape which is usually desired. The amplitude of the emergent wave across the aperture has a taper which is determined by the radiation pattern of the feed, modified by the additional tapering effect of the antenna geometry. The far-field pattern of the antenna is, of course, a diffraction pattern whose characteristics depend on the amplitude taper of the emergent wave.

III. Geometry

The geometry of the Cassegrain system is simple and well-known, but it is helpful to have at hand those formulas describing the dish contours in terms of the significant antenna parameters. The classical Cassegrain geometry, shown in Fig. 3, employs a parabolic contour for the main dish and a hyperbolic contour for the sub dish. One of the two foci of the hyperbola is the real focal point of the system, and is located at the center of the feed; the other is a virtual focal point which is located at the focus of the parabola. As a result, all parts of a wave originating at the real focal point, and then reflected from both surfaces, travel equal distances to a plane in front of the antenna.

To completely describe a Cassegrain system, four fixed parameters are required, two for each dish. Since seven parameters are shown in Fig. 3, three are dependent on the other four, and three equations exist which describe this dependency. In the case of the main dish, the relationship is

\[ \tan \frac{1}{2} \phi_v = \pm \frac{D_m}{4 F_m} \cdot \]  

As will be discussed later, the positive sign in the above formula applies to the Cassegrain forms, and the negative sign to the Gregorian forms. In the case of the sub dish, the relationships are

\[ \frac{1}{\tan \phi_v} + \frac{1}{\tan \phi_s} = 2 \frac{F_s}{D_s} \cdot \]  

\[ 1 - \frac{\sin \frac{1}{2} (\phi_v - \phi_s)}{\sin \frac{1}{2} (\phi_v + \phi_s)} = 2 \frac{L_v}{F_s} \cdot \]  

In a typical case, the parameters \( D_m, F_m, F_s, \) and \( \phi_s \) might be determined by considerations of antenna performance and space limitations; \( \phi_v, D_s, \) and \( L_s \) would then be calculated. It is interesting to note that a value for the parameter \( \phi_s \), which determines the beamwidth required of the feed radiation, may be specified independently of the ratio \( F_m/D_m \), which determines the shape of the main dish.

The contour of the main dish is given by the equation

\[ x_m = \frac{y_m^2}{4F_m} \cdot \]  

The contour of the sub dish is given by the equation

\[ x_s = a \left[ \sqrt{1 + \left( \frac{y_s}{b} \right)^2} - 1 \right] \cdot \]  

where

\[ e = \frac{\sin \frac{1}{2} (\phi_v + \phi_s)}{\sin \frac{1}{2} (\phi_v - \phi_s)} \cdot \]

\[ a = \frac{F_s}{2e} \quad b = a \sqrt{e^2 - 1} \cdot \]

The quantities \( e, a, \) and \( b \) are the parameters of the hyperbola: \( e \) is eccentricity, \( a \) is half the transverse axis, and \( b \) is half the conjugate axis.

So far, only the geometry of the classical Cassegrain system has been considered. However, the system may easily be extended to include a variety of forms, all obeying the basic formulas presented above. In Fig. 4, two series are shown in which the curvature of the sub dish is modified from the classical convex shape to a flat, and finally a concave shape. As this is done, the diameter of the sub dish increases. The first series shows the case in which the main dish is held invariant; this yields a progressive increase in the required feed beamwidth, and a progressive decrease in the axial dimension of the antenna. In the second series the feed beamwidth is held invariant; in this case, the main dish becomes progressively flatter, and the axial dimension of the antenna progressively increases.
(a) Invariant main dish. (b) Invariant feed.

Fig. 4—Cassegrain modifications.

In Fig. 5, a series is presented in which the beamwidth of the feed is progressively increased while the overall dimensions of the antenna are held fixed. The range of values of some of the parameters previously mentioned are indicated alongside the sketches, together with the distinguishing characteristic of each case. (In each sketch an additional dish and some additional rays are shown in dashed lines, and one column has an additional parameter \( F_e/F_m \); these will be discussed in the next section of this paper, and should be disregarded at this point.) The first three cases are similar to those shown in Fig. 4. In the fourth case, the main dish has degenerated to a flat contour and the sub dish has degenerated to a parabolic contour; here, the flat main dish may be placed at any distance from the sub dish, out to the region where the ray-optical approximation begins to fail. The final case carries the progression to the ridiculous extreme of a concave elliptical sub dish and a convex parabolic main dish, with the former being larger than the latter. It should be mentioned that in the two cases having one flat dish, the formulas presented before, while valid, are overly complicated and contain indeterminate factors; since the focusing is accomplished entirely by the curved parabolic surface, it is preferable to employ the simple formulas for an antenna having a single parabolic dish.

A further extension of the Cassegrain system is shown in Fig. 6. Here, the focal point of the main dish moves to a region between the two dishes, and the contour of the sub dish becomes concave elliptical. In the first of the two cases shown, the system is identical with that of the Gregorian telescope; however, both cases obey the formulas given previously for the Cassegrain system, if the proper values are employed. The ranges allowable for some of the parameters are indicated in the figure. In addition, the negative sign must be employed in (1) so as to maintain a positive \( F_m \) with the negative \( \phi_s \) which occurs in the Gregorian forms. The first case, or classical Gregorian, is drawn so as to have the same over-all size and the same feed beamwidth as the classical

Fig. 5—Series of Cassegrain forms.
Cassegrain in the first case of the previous figure. Under these conditions, the Gregorian form requires a shorter focal length for the main dish. In the second of the Gregorian forms shown, the feed has been moved to a location between the main dish focus and the sub dish, with the main dish kept the same as in the first case. This form would have several major disadvantages that would make it unattractive in most antenna applications.

All of the above-mentioned forms are members of the same family, which might be called the Cassegrain family. In every case, incoming rays collected by the main dish are focused toward a point. It should be mentioned that a further extension of the Cassegrain system can be made by modifying the contours of both dishes in such a way that incoming rays collected by the main dish are not focused exactly toward a point, while the final bundle of incoming rays, after reflection from the sub dish, remain focused toward a point. Although this may be a useful technique for achieving certain kinds of performance [3]–[7], it is beyond the scope of this paper.

IV. Equivalence Concepts

A. Virtual Feed

One concept which is helpful in understanding and predicting the essential performance of a Cassegrain antenna is that of a virtual feed. As shown in Fig. 7, the combination of real feed and sub dish is considered as being replaced by a virtual feed at the focal point of the main dish. Thus the antenna becomes an ordinary single-dish design, having the same main dish but a different feed.

If both the real and virtual feeds had dimensions much larger than a wavelength, the configuration of the virtual feed could be determined by finding the optical image of the real feed in the sub dish. This condition seldom exists for a microwave antenna; however, if only the effective apertures of the feeds are considered, it is found that the imaging process yields approximately the correct results. For the classical Cassegrain configuration shown in Fig. 7, the virtual feed has an effective aperture smaller than that of the real feed, and has a correspondingly broader beamwidth. The beamwidth increase is, of course, the result of the convex curvature of the sub dish; the ratio of virtual-feed to real-feed beamwidth is indicated by the quantity $\phi_v/\phi_r$.

For the various Cassegrain modifications, the range of values that the quantity $\phi_v/\phi_r$ may achieve is given in the first column of Figs. 5 and 6, and from this the relative sizes of the effective apertures of the real and virtual feeds may be inferred. When the sub dish is flat, the virtual and real feeds are, of course, identical. For the Cassegrain system having a concave sub dish, the virtual feed has a beamwidth smaller than that of the real feed, and has a larger effective aperture. However in the classical Gregorian form, the concave sub dish results in an effective aperture of the virtual feed which is smaller than that of the real feed, just as in the classical Cassegrain system.

There are several situations in the design of microwave antennas in which the ability to obtain a different effective aperture of the virtual feed from that of the real feed is quite helpful. One such case occurs with a monopulse antenna, where it is difficult to reduce the overall size of the feed aperture to a wavelength or less, while maintaining efficient and wideband performance. On the other hand, a large feed aperture ordinarily requires a long focal length for effective utilization of the main aperture, thereby increasing the size of the
antenna structure. This problem may be solved by means of the classical Cassegrain system of Fig. 7, which can incorporate a large feed while employing a short focal length for the main dish. Actually, the axial dimension of such an antenna is often less than the main focal length, because the virtual feed is beyond the sub dish. In addition, of course, there are no waveguide components required in this forward region.

B. Equivalent Parabola

The concept of a virtual feed furnishes a useful qualitative means for analyzing a Cassegrain antenna, but, in general, it’s not convenient for an accurate quantitative analysis. In addition, the virtual feed assumes ridiculous proportions for certain of the Cassegrain configurations. A second concept, that of the equivalent parabola, overcomes these limitations.

As shown in Fig. 8, the combination of main dish and sub dish is considered as being replaced by an equivalent focusing surface, drawn with dashed lines in the figure, at a certain distance from the real focal point. The properties of this focusing element can be determined from a study of the “principal surface” of the Cassegrain system. This surface [8] is defined here as the locus of intersection of incoming rays parallel to the antenna axis with the extension of the corresponding rays converging toward the real focal point, as indicated in Fig. 8. It happens that for the Cassegrain system, the “principal surface” has a parabolic contour, and the focal length of this parabola exactly equals the distance from its vertex to the real focal point. As a result, this surface could be employed as a reflecting dish which would focus an incoming plane wave toward the real focal point in exactly the same manner as does the combination of main dish and sub dish. (Actually, the plane wave would have to be incident from the opposite direction; this is of no significance in the principles of this concept.) Thus the antenna again becomes an ordinary single-dish design, but this time having the same feed and a different main dish.

It should be mentioned that the equivalent parabola is based on simple ray analysis, rather than on an exact analysis of the wave action. This ray approximation is made throughout the paper, and is accurate enough for most purposes except when the sub dish is only a few wavelengths in diameter. When the wave analysis is necessary, consideration would have to be given to the Fresnel diffraction pattern formed at the main dish after reflection of the feed radiation by the sub dish.

The following equations provide the relationship between the equivalent parabola, the antenna parameters shown in Fig. 8, and some of the parameters previously mentioned:

\[
\frac{1}{4} \frac{D_m}{F_e} = \tan \frac{1}{2} \phi_e, \tag{6}
\]

\[
x_e = \frac{y_e^2}{4F_e}, \tag{7}
\]

\[
\pm \frac{F_e}{F_m} = \tan \frac{1}{2} \phi_e, \quad L_r = e + 1
\]

In (8), the positive sign applies to the Cassegrain forms, and the negative sign to the Gregorian forms. Eqs. (6) and (7) describe the equivalent parabola itself, in terms of its equivalent focal length, \(F_e\). Eq. (8) presents the various alternate expressions for the quantity \(F_e/F_m\) the ratio of equivalent focal length to focal length of the main dish. It is evident that with the classical Cassegrain system, the equivalent focal length is greater than the focal length of the main dish.

As might be expected, the equivalent-parabola concept also applies to the extended Cassegrain forms, and to the Gregorian forms, as well. The equivalent parabola for each of these cases is indicated by the dashed curves in Figs. 5 and 6, and the range of values for the quantity \(F_e/F_m\) is indicated in the first column of these figures. When the sub dish is flat, the equivalent focal length equals the focal length of the main dish. For the Cassegrain system having a concave sub dish, the equivalent focal length is shorter than that of the main dish. For the case of a flat main dish, the equivalent parabola is identical with the sub dish. In the classical Gregorian form, the equivalent focal length is greater than that of the main dish, as is also the case in the classical Cassegrain system.

In describing the magnifying properties of a Cassegrain optical telescope, it has become customary to employ the concept of the equivalent focal length [1], [2], [4]. It has also been recognized that the coma aberration of a Cassegrain telescope is the same as that of a telescope having a single parabolic mirror of focal length equal to the equivalent focal length of the Cassegrain [2]. These two aspects are readily explainable in terms of the equivalent parabola. It may be noted that since
the Cassegrain optical telescope has an equivalent focal length greater than that of its large mirror, it has greater magnifying power and reduced coma compared with that obtained with only the single large mirror.

In the case of a microwave antenna, the equivalent-parabola concept yields properties similar to those mentioned above. The effective aperture of the feed should be such that the equivalent parabola is properly illuminated; when the equivalent focal length is greater than the focal length of the main dish, the optimum feed aperture is larger than that which would be optimum for a single-dish antenna having the same focal length as the main dish. This result is analogous to the magnifying properties [1]-[3] of the optical telescope; it also corresponds to the result obtained with the virtual-feed concept. Indeed, the ratio $F_e/F_m$ is sometimes called the magnification. This is a valid approximation when applied to the relative sizes of the real and virtual feeds or images. However, it should not be confused with the magnification of an optical telescope containing an eyepiece, in which the term usually applies to the relative sizes of the image and the object.

As regards coma aberration, the equivalent-parabola concept yields the same results as in the optical case, when the off-axis beam angle is small. However, when this angle becomes appreciable, as may be necessary in a microwave antenna, the feed is sufficiently offset from the dish axes so that the principal surface is no longer closely approximated by the original equivalent parabola; therefore, the wide-angle coma may differ considerably from that calculated by the equivalent-parabola concept. Of course, other aberrations may become appreciable at the same time.

There are some significant uses of the equivalent-parabola concept in the microwave antenna which appear to have no application in the optical telescope. One such case involves the determination of amplitude taper across the main aperture of the antenna. For an ordinary single-dish antenna, the illumination is determined by the radiation pattern of the feed, modified by a "space-attenuation" characteristic which is a simple function of the $F/D$ ratio [9]. For a Cassegrain antenna, exactly the same process is applicable, with the $F/D$ ratio now being the ratio of equivalent focal length to main-dish diameter, $F_e/D_m$. In other words, the illumination is exactly the same as that which would exist across a single dish having the equivalent focal length and being illuminated with the same feed. When the equivalent focal length is greater than the diameter, the "space-attenuation" characteristic modifies the feed radiation only slightly; with a practical feed, such an antenna can have high efficiency even though it may have a physically short axial length.

V. REDUCTION OF APERTURE BLOCKING

The principal limitation on the application of the historical Cassegrain system to microwave antennas is the blocking of the main aperture by the sub dish [10], [11]. This problem has not been serious with optical telescopes because the requirements on characteristics of the diffraction pattern have not been severe, and because, for the relatively short wavelength of light, the size of the small reflector can be made very much less than that of the large reflector. With a microwave antenna, neither of these conditions ordinarily exists.

The presence of an opaque sub dish in the main aperture of the antenna creates a "hole" in the illumination which causes decreased gain and increased sidelobe levels. To analyze this effect, the resulting illumination may be resolved into two components [9], the original illumination plus a negative center, or "hole," as shown in Fig. 9(a). The resulting antenna pattern, shown in Fig. 9(b), can be determined by adding together the two pattern components, the original pattern plus a broad, low, negative pattern radiated by the "hole."

Although the above method facilitates an exact calculation of the shadowing effect for any case, it is instructive to apply the method to a particular simple case.
which approximates many practical cases. If the main aperture is circular, and is assumed to have a completely tapered parabolic illumination, a small circular obstacle in the center of the aperture will create a “hole” pattern whose peak voltage relative to the peak voltage of the original pattern is

\[
\frac{E_b}{E_m} = 2 \left( \frac{D_b}{D_m} \right)^2.
\]

where \(D_b\) is the diameter of the blocked portion of the aperture. This relative voltage is then subtracted from unity to yield the resultant relative peak voltage, and is added to the relative level of the first sidelobe to yield the resultant relative level.

The illumination hole is not the only effect created by the presence of an obstacle in the main aperture; the power which strikes the obstacle must also be accounted for. Usually this power reradiates and contributes an additional component to the sidelobes. For a particular sub dish and antenna configuration, it is often a straightforward process to estimate the amplitude pattern of this radiation. However the manner in which it combines with the original pattern is more complicated, and is likely to vary radically with a change of frequency. A further consideration of this effect is beyond the scope of this paper, and, even though it may sometimes be an important one, the effect will be neglected henceforth.

A. Minimum Blocking with Simple Cassegrain

In order to determine the degree of aperture blocking to be expected in a Cassegrain antenna having an ordinary reflecting sub dish, it is necessary to consider those factors which influence the size of the sub dish. Essentially, the minimum size of the sub dish is determined by the directivity of the feed, and the distance between the feed and the sub dish. By making the feed more directive, or by decreasing its distance to the sub dish, the size of the sub dish may be reduced without incurring a loss caused by spillover of the feed radiation beyond the edge of the sub dish. However, as indicated in Fig. 10, a continuation of this process can eventually result in the feed itself creating a shadow in the main illumination which is greater than that created by the sub dish. It is evident that there is some intermediate condition in which neither the sub-dish nor the feed shadow predominates, and which would yield the least amount of aperture blocking; this may be termed the minimum-blocking condition.

In Fig. 11, the minimum blocking condition is shown, together with some approximate equations describing the basic relations between certain parameters. By combining these equations, a relationship is obtained which specifies the geometry for the minimum-blocking condition; it is as follows:

\[
\frac{F_e}{F_m} \approx \frac{1}{2} \frac{k D_f^2}{F_e} \approx \frac{1}{k D_f'}
\]

where \(D_1\) is the physical or blocking diameter of the sub dish, \(D_f\) is the physical or blocking diameter of the feed aperture, and \(k\) is the ratio of the effective feed-aperture diameter to its blocking diameter.\(^1\) This approximate relationship assumes that the angles \(\phi_s\) and \(\phi_f\), shown in Fig. 11, are small; and that the sub dish is much closer to the focus of the main dish than it is to the feed \((F_e/F_m\) much larger than one). It also assumes that ray optics can describe the feed shadow; this is a good approximation when the feed is far from the sub dish. Within these limitations, minimum aperture blocking is obtained for a practical case in which there is essentially no spillover of the main lobe of the feed pattern past the edge of the sub dish. Although shown for the classical Cassegrain system, the above approximate analysis also applies for the classical Gregorian system [13].

It can be seen that the minimum-blocking condition

\(^1\) Ordinarily \(k\) is slightly less than one; however, where a cluster of many feeds is employed to obtain a cluster of antenna beams, \(k\) can become quite small.
is not limited to a particular set of antenna dimensions, but includes a series ranging from the case of a feed located near the vertex of the main dish and having a diameter about equal to that of the sub dish, to the case of a feed located far in front of the main dish and having a diameter much smaller than the sub dish. In the former case, the feed should be focused approximately toward the focal point of the main dish in order that the illumination of the main aperture be characterized by a Fraunhofer diffraction pattern rather than a Fresnel pattern. In the latter case this is not necessary, but the feed must, of course, be excited by a length of transmission line and supported in its extended location; in an extreme form the latter case resembles a single-dish antenna with a splash-plate feed, although the principle of operation is quite different.

The diameter of the aperture blocking for the minimum-blocking condition is given by the following approximate equation:

$$D_{b_{\text{min}}} \approx \sqrt{\frac{2}{k} F_m \lambda}$$

(11)

where the limitations are the same as those mentioned previously. This equation also assumes that the total amount of aperture blocking is no greater than either of the two equal and coincident shadows; actually the blocking would be somewhat greater, particularly for the case of a small feed located close to the sub dish. It should also be mentioned that the approximation given in (11) implies that a classical Gregorian would have slightly less blocking than a classical Cassegrain of the same axial dimension, because of the shorter $F_m$ of the former; however, a more exact formulation would show that just the reverse is true. The significant fact to note from (11) is that the minimum blocking diameter can be computed before determining the feed size and location, these latter dimensions finally being related by (10).

It is of interest to express (11) in some alternate approximate forms which more clearly illustrate the basic relationships;

$$\left( \frac{D_{b_{\text{min}}}}{D_m} \right)^2 \approx \frac{2}{k} \frac{\lambda}{D_m} \frac{F_m}{D_m} \approx \frac{\pi}{2k} \frac{F_m}{D_m} \approx \frac{\pi}{2k} \frac{2\theta_\text{p/2}}{2\phi_v}$$

(12)

where $2\theta_\text{p/2}$ is the approximate half-power beamwidth of the antenna pattern in radians, and $2\phi_v$ is the approximate included angle formed by the main dish at the virtual feed in radians. (Actually the first and second forms of (12) are almost exactly equal when the main aperture is circular and has a completely tapered parabolic illumination, and the second and third forms are equal when $2\phi_v$ is small.) It is apparent from (12) that an antenna with a narrow beamwidth can have less relative aperture blocking than one with a wide beamwidth. This might be expected on the basis that the optical case, which has a very narrow beamwidth, has the capability for very small relative aperture blocking. Also apparent is the desirability of a small $F/D$ ratio for the main dish, and an efficient feed aperture ($k$ approaching one).

As an example, consider an antenna which is to have a pencil beam of one-degree half-power beamwidth, $F_m/D_m = 0.3$, $k = 0.7$, and which is to be optimized for the minimum-blocking condition. The second form of (12) yields a value of about 0.012 for $(D_{b_{\text{min}}}/D_m)^2$, which may then be applied in (9) to yield a value of about 0.024 for $E_b/E_m$. The aperture blocking in this antenna would therefore reduce the gain by about $\frac{3}{2}$ db and would increase a $-23$ db sidelobe\(^2\) to about $-20.5$ db. This effect might be acceptable for some applications, but not for others. Thus a one-degree beamwidth might be considered as a rough boundary above which the simple Cassegrain design, even though optimized, would be unattractive.

**B. Twisting Cassegrains for Least Blocking**

The preceding discussion of a minimum-blocking design has assumed that, similar to an optical telescope, operation in all polarizations is required. However, many microwave antennas need operate in only one polarization, and in this event a considerable reduction of aperture blocking is possible. Fig. 12 presents one scheme for accomplishing this, by means of a polarization-twisting technique which avoids the sub-dish shading.

![Fig. 12—Polarization twist for non-blocking sub dish.](image)

In this scheme as shown, the sub dish comprises a horizontal grating, called a transreflector, which reflects a horizontally-polarized wave radiated by the feed. The main dish incorporates a surface design, called a twistreflector, which twists the horizontally-polarized wave to a vertically-polarized one as it reflects the wave back. The portion of this wave which is now incident on the

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\(^2\) The $-23$ db figure is a value which is typical for the first-sidelobe level when the illumination is tapered to about 11 db for maximum gain with a circular aperture.
sub dish is transmitted through unaffected, because the sub dish is transparent to a vertically-polarized wave. Thus there is no blocking by the sub dish at all. The feed does, of course, create aperture blocking; however, its size can be made quite small, and the blocking can be comparable with that of an ordinary single-dish design. In this scheme, therefore, it is advantageous to use a large sub dish with a small feed.

While the above process is theoretically perfect in the cardinal regions of the antenna aperture, the three-dimensional geometry of the system is such that there can be some loss into cross-polarized radiation toward the outer portion of the intercardinal regions. It is beyond the scope of this paper to consider this effect in detail, but some general comments can be made. One part of this effect occurs with the wave radiated from the sub dish to the main dish; for any but the most extreme Cassegrain forms, the loss here is usually so small as to be negligible. The other part occurs with the wave radiated from the feed to the sub dish, and the results are dependent on the polarization characteristics of the feed. There is often a moderate amount of loss here; however it is usually not greatly different from the loss in an ordinary single-dish antenna caused by the same effect.

Another scheme for reducing aperture blocking is indicated in Fig. 13; here, a polarization-twisting technique is employed to render the feed invisible. Two configurations are possible, both involving a sub dish which incorporates a twistreflector. In one case, a vertically-polarized feed is located behind the main dish, and the central portion of the main dish includes a transreflector having a horizontal grating. The feed radiates through the transreflector toward the sub dish, the sub dish reflects this wave and twists its polarization to horizontal, and the horizontally-polarized wave is then completely reflected by the main dish. In the other case, the feed is composed of thin horizontal elements and is located out in front of a simple main dish. When the feed radiates toward the sub dish, the vertically-polarized wave returned by the sub dish passes through the feed unaffected, is completely reflected by the main dish, and again passes through the feed. In both of these configurations, it is evident that the sub dish causes aperture blocking but the feed does not. Consequently, the feed may be greatly enlarged so that its increased directivity allows the sub dish to become quite small. As mentioned previously, when the feed becomes equal to or larger than the sub dish, the phase front across the feed aperture should be curved so as to focus the feed toward the vicinity of the main dish focus. It should also be mentioned that when this condition exists, the simple geometry of the Cassegrain system and the equivalence concepts no longer apply; however the basic operation of the antenna remains similar.

Of the two basic polarization-twisting schemes, the one having a twistreflecting main dish is of general applicability to many antenna developments [14]–[17], while the one having a twistreflecting sub dish is useful in some special circumstances. For example, the former may be efficiently employed with any of the Cassegrain extensions shown in Fig. 5, and with the classical Gregorian form, as well. On the other hand, the latter should be limited to those forms in which the sub dish is small compared with the main dish. In either case it is essential, of course, to have suitable designs for the twistreflector and transreflector. One particular technique [16] involving thin metal wires embedded in fiberglass skins, has proven most satisfactory. While it is not the purpose of this paper to discuss these designs in detail, a brief description is in order as an indication of their practical nature.

For the transreflector design, a grating of thin wires, closely-spaced compared with a wavelength, has the property of being essentially a perfect reflector for parallel polarization, and being essentially invisible to perpendicular polarization. The cross-section of a practical structure which incorporates a quarter-wave sandwich support is indicated in Fig. 14(a). The wires may be placed all in one skin, or else they may be divided equally between the two skins as shown; either technique usually yields about the same result. For the twistreflector design, a grating of metal wires oriented at 45° to the incident polarization may be placed in front of a reflecting surface. When the spacing between the grating and the reflecting surface is about three-eighths of a wavelength, and the grating is designed to allow about one-half of the parallel-polarized power to pass through, the twistreflector operates over a broad frequency band and over a wide range of incidence angles. The cross-section of a practical structure is shown in Fig. 14(b).

It is perhaps interesting to note in passing, that there are a number of uses for a twistreflector in addition to those already discussed. One such use occurs in an ordinary single-reflector antenna during transmission, when it is desired to prevent any of the wave reflected by the dish from getting back into the feed. This can be accomplished with a twistreflector on the dish [12].
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FIBERGLASS
SKINS

DIELECTRIC
HONEYCOMB

TRANSMITTED
POLARIZATION

CLOSELY-SPACED,
THIN METAL WIRES

(a) Transreflector.

(b) Optimun twistreflector.

Metal wires, spaced
for 1/2 power transmission

INCIDENT
POLARIZATION

REFLECTED
POLARIZATION

METAL SURFACE

FIBERGLASS
SKIN

Fig. 14—Designs for polarization-operative surfaces.

Fig. 15—Photograph of a simple Cassegrain.

A number of Cassegrain antennas have been designed at Wheeler Laboratories, and their performance has been highly satisfactory. By way of illustration, the radiation patterns of two different antennas are presented; these two were designed for the Bell Telephone Laboratories on Army Ordnance projects.

One design is shown by the photograph in Fig. 15. It employs simple reflecting surfaces, and its geometry approaches that of the minimum-blocking configuration indicated in Fig. 11. Actually, as may be seen from the picture, the sub dish is appreciably larger than the feed shadow, and the blocking area is greater than the minimum possible by a factor of almost 3. The antenna is shown in location on the roof of the antenna-development facility of Wheeler Laboratories at Smithtown, Long Island. Also visible in the picture are the precision mount for the antenna, and the versatile positioning devices for the feed and sub dish; these items were provided by the Bell Telephone Laboratories.

The radiation pattern and efficiency of this Cassegrain antenna are shown by the solid curve in Fig. 16(a). The half-power beamwidth is 0.6 degrees; this is narrow enough so that the simple Cassegrain system without twisting is adequate for the intended application, even without complete optimization of the aperture blocking. As may be seen, the efficiency of this Cassegrain is fairly high in spite of aperture blocking by both the sub dish and its rigid supporting system. This is probably a result of the efficient aperture utilization obtained with a long equivalent focal length. As expected, the near sidelobes are raised several db by the sub dish blocking; however this effect would also have occurred in the intended application had an ordinary single-reflector antenna been employed.

The dashed curve in Fig. 16(a) shows the radiation pattern which is obtained when the antenna is intentionally defocused by moving the sub dish a small distance toward the main dish. If desired, this technique might be used to provide a variable beamwidth. Alternatively, by moving the sub dish away from the main dish the antenna can be focused toward a point nearer than the far field; this would permit a greater concentration of power at such a point than could otherwise be obtained. Of course these focusing techniques are also

Another use applies to a simple Cassegrain, in which it is desired to eliminate what may sometimes be an appreciable reflection by the sub dish back into the feed. Incorporation of a twistreflector on the sub dish achieves this result, and may create other advantages as well.

VI. EXPERIMENTAL RESULTS

The "efficiency" is used here as the ratio of measured gain to the gain which would be obtained if the same main aperture were uniformly illuminated, with no spillover or other losses.
available in the ordinary single-reflector antenna, as well as most other types. With this Cassegrain, however, it is possible to perform these operations by motion of a relatively small, passive device.

In Fig. 16(b), the patterns of this same antenna are shown for two cases in which the beam is scanned off axis by approximately three beamwidths. The pattern on the left is obtained by a movement of only the sub dish; the motion involves a substantial tilt about the main dish focus, plus a small axial motion to regain the focused condition in the plane of scan. It is evident that no appreciable coma is introduced by this process, since the pattern remains quite symmetrical. On the other hand, there is a considerable amount of astigmatism created; the pattern in the plane normal to the scan plane, not shown here, is quite broad. This defect is partly responsible for the decrease of gain which is apparent. The results presented above are typical of a Cassegrain system with a large $F_e/F_m$. It is perhaps instructive to mention that similar effects are obtained with this system when the scanning is accomplished by offsetting the feed and refocusing.

The pattern on the right side of Fig. 16(b) is obtained by tilting both the feed and the sub dish as a unit about the vertex of the main dish. This is equivalent to rotating the virtual feed about the same point. As can be expected, this results in a substantial degree of coma distortion, about the same as would be obtained by offsetting the feed in an ordinary single-reflector antenna of the same main focal length.

The other antenna design chosen as an example incorporates a twist-reflecting main dish and a transreflecting sub dish, such that the sub dish creates no aperture blocking, as illustrated in Fig. 12. The equivalent focal length of this antenna is designed to be just long enough so that a monopulse feed system can be employed in a size just large enough to utilize a simple cluster of four horns as the feed. The complicated monopulse plumbing is located in a convenient region behind the antenna.

The patterns of this antenna are shown in Fig. 17 for the sum and one difference channel. Also indicated are the computed points, determined from a knowledge of the feed pattern, and including a contribution from aperture blocking by the feed. The close correspondence between the two patterns is evident. Similar good agreement exists in the other properties of this antenna. The efficiency of 54 per cent in the sum pattern is rather high for a monopulse system; this is a result of the inherent advantage of a Cassegrain system with a long equivalent focal length and very small aperture blocking. All of the above results confirm the nearly lossless behavior of the polarization-twisting technique and the surface designs of Fig. 14. While the twisting type of antenna requires additional effort during design and construction of the polarization-operative surfaces, it has proven practical to build in large quantities, and has yielded the expected good performance in the field.

VII. BENEFITS OF CASSEGRAIN SYSTEMS

In concluding the discussion of Cassegrain optics applied to microwave antennas, it is appropriate to outline some of the benefits obtainable. Perhaps most important is the ability to place the feed in a convenient position, while utilizing reflectors as the focusing elements. The rear location and forward direction for the feed are most desirable in various applications involving complicated feeds and associated plumbing.

One example of this advantage occurs in the case of an antenna intended for low-noise operation, as illustrated in Fig. 18. At present, a low-noise receiver is likely to be bulky and require a number of auxiliary connections as well as occasional adjustments; it is therefore inconvenient to mount it close to the feed out in front of a single-reflector antenna. Yet this is often done, because the attenuation in a waveguide from the
feed back to a receiver located behind the dish would introduce an excessive amount of noise power. The Cassegrain system furnishes the opportunity to avoid most of these difficulties.

There is another benefit obtainable with the Cassegrain system in a low-noise application. With the ordinary single-reflector antenna, there is usually a considerable amount of wide-angle sidelobe response caused by spillover radiation from the small feed out in front. This may introduce a very substantial amount of noise power into the antenna, by coupling to the radiation from the warm ground. In the case of a Cassegrain antenna, spillover radiation from the virtual feed can be very much less. This is because of the essentially ray-optic behavior of reflection from the sub dish, which results from its relatively large diameter in wavelengths. There remains to be considered, of course, spillover from the real feed past the edge of the sub dish. Although the total amount of this spillover power may be comparable with that in a single-reflector antenna, it is likely to be confined to direction relatively close to the antenna axis. There is also to be considered the sidelobe radiation created by the aperture blocking by the sub dish; here again, this is usually appreciable only in forward directions. As a result of these directional properties, the spillover and aperture blocking couple to the ground only when the antenna is pointed at a low elevation angle. In comparison, the ordinary single-reflector antenna is likely to have appreciable coupling to the ground even at high elevation angles.

It is possible in the case of a polarization-twisting Cassegrain system to reduce even this relatively narrow-angle sidelobe response. Fig. 19 illustrates this effect, for the scheme which involves a twistreflector at the main dish and a transreflector at the sub dish. Since the feed is horizontally polarized, it is essentially isolated from any vertically-polarized source, such as the normal ground reflection of the incoming wave, or one component of thermal ground radiation. If the transreflector is extended from the sub dish to the main dish in the lower portion of the antenna, isolation may also be achieved for the other polarization. Such an antenna, then, has effectively only those sidelobes which would be inherent in the illumination distribution of its main aperture. As a result, the antenna could provide accurate tracking of a target, as well as low-noise performance, down to elevation angles determined only by the decay rate of the inherent sidelobes.

In continuing the outline of benefits obtainable with a Cassegrain system, mention can be made of the ability to obtain an equivalent focal length much greater than the physical length; as discussed previously in this paper, various advantages may be obtained in this way. A third aspect is the capability for scanning or broadening the beam by moving one of the antenna surfaces. One case involving a small moving sub dish has been described here; there have also been designs utilizing a moving flat main dish for wide-angle scanning [15], [17].

The existence of two dishes and two focal points in the Cassegrain system gives rise to interesting methods for incorporating the separate functions of two antennas into one structure. On the left side of Fig. 20 a simple scheme is shown which provides a full-size plus a re-
that the essential performance can be calculated by means of simple equivalence concepts. The basic defect, aperture blocking by the sub dish, can be minimized or virtually eliminated by certain techniques. The Cassegrain system has proven both practical and advantageous in a number of operational antennas, and the tested performance has agreed closely with the computed predictions. A variety of benefits are obtainable with the Cassegrain system, and it provides a highly versatile form of microwave antenna capable of achieving good performance in a number of unusual applications. The second reflecting surface which is available in this system provides an extra degree of freedom to the antenna designer for application to his particular problem.

VIII. CONCLUSION

To summarize the discussion of the principles and features of Cassegrain antennas, it has been shown that a simple set of formulas describe a number of forms, and that the essential performance can be calculated by means of simple equivalence concepts. The basic defect, aperture blocking by the sub dish, can be minimized or virtually eliminated by certain techniques. The Cassegrain system has proven both practical and advantageous in a number of operational antennas, and the tested performance has agreed closely with the computed predictions. A variety of benefits are obtainable with the Cassegrain system, and it provides a highly versatile form of microwave antenna capable of achieving good performance in a number of unusual applications. The second reflecting surface which is available in this system provides an extra degree of freedom to the antenna designer for application to his particular problem.

IX. SYMBOLS

\[ D_m = \text{effective diameter of circular main dish (to edge rays).} \]
\[ D_n = \text{effective diameter of circular sub dish (to edge rays).} \]
\[ D_b = \text{blocking diameter of sub dish.} \]
\[ D_f = \text{diameter of feed.} \]
\[ D_a = \text{diameter of aperture blocking.} \]
\[ D_b_{\text{min}} = \text{diameter of aperture blocking for minimum-blocking geometry.} \]
\[ F_m = \text{focal length of main dish.} \]
\[ F_n = \text{distance between foci of sub dish.} \]
\[ F_e = \text{equivalent focal length of Cassegrain system.} \]
\[ L_e = \text{distance from virtual focus (or main dish focus) to sub dish.} \]
\[ L_f = \text{distance from real focus (or feed) to sub dish.} \]
\[ \phi_a = \text{angle between axis and edge ray, at virtual focus.} \]
\[ \phi_b = \text{angle between axis and edge ray, at real focus.} \]
\[ 2\phi_a = \text{included angle between rays from real focus to physical edges of sub dish, in radians.} \]
\[ 2\phi_b = \text{included angle between rays from virtual focus to edges of feed, in radians.} \]
\[ e = \text{eccentricity of conic section.} \]
\[ a = \text{transverse half-axis of conic section.} \]
\[ b = \text{conjugate half-axis of conic section.} \]
\[ x_m, y_m = \text{coordinates of main dish (axial, radial).} \]
\[ x_n, y_n = \text{coordinates of sub dish (axial, radial).} \]
\[ x_e, y_e = \text{coordinates of equivalent parabola (axial, radial).} \]
\[ \lambda = \text{wavelength.} \]
\[ f = \text{frequency.} \]
\[ k = \text{ratio of effective diameter to blocking diameter of the feed.} \]
\[ \theta = \text{antenna pattern angle, in radians.} \]
\[ 2\theta_{\text{hp}} = \text{half-power beamwidth of antenna, in radians.} \]
\[ E = \text{pattern voltage.} \]
\[ E_b = \text{peak voltage of the supplemental negative pattern caused by aperture blocking.} \]
\[ E_m = \text{peak voltage of the pattern of the main aperture without blocking.} \]
X. ACKNOWLEDGMENT

The principles and geometry of the double-reflector systems described in this paper are based entirely on the principles developed for the optical telescope. This work originated in the 17th century, with particular forms being attributed to Gregory, Newton, and Cassegrain. It seems very probable that the simple concept of an equivalent parabola has long been known to those involved in the design of another antenna for the Bell Telephone Laboratories. The system involving a twist-reflecting sub dish and a transreflecting main dish was developed in the theory of optical reflecting telescopes; however the writer, having only a limited acquaintance with the optical literature, has not found any reference to this.

The existence of a minimum-blocking design was suggested by H. A. Wheeler, for the case of a feed located at the vertex of the main dish. This suggestion led to the general case of a feed located anywhere, and the basic relationship of minimum blocking as a function of beamwidth.

During the design of a radar antenna for the Bell Telephone Laboratories, the system involving a trans-reflecting sub dish and twist-reflecting main dish was conceived [16] as the solution to the problem at hand. Later it was learned that the basic concept had already occurred to C. A. Cochrane [14] of Elliott Brothers, London. The scheme involving a twist-reflecting sub dish and a trans-reflecting main dish or feed was devised in connection with the design of another antenna for the Bell Telephone Laboratories.

The use of thin wires in fiberglass for the polarization-operational surfaces was worked out in cooperation with K. B. Woodward of the Bell Telephone Laboratories. The particular set of dimensions which achieve wideband, wide-angle twistreflector performance was derived by H. Jasik, as a consultant to Wheeler Laboratories.

Of the four dual-antenna schemes mentioned in Section VII, it is of interest to note that the first has also been perceived and utilized by the Ryan Aeronautical Co., and the second by both Sperry Gyroscope Co., and Melpar, Inc.

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XI. BIBLIOGRAPHY